 Behaviour of synchronous machine during a short-circuit
(a simple example of electromagnetic transients)

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Objectives of this lecture

- Recall the Park model of synchronous machines
- Give an example of electromagnetic transient simulation
- Prepare the derivation of the corresponding model under the “phasor approximation”
System modelling

Network represented by a simple Thévenin equivalent:
- resistance $R_e$ and inductance $L_e$ in each phase
- no magnetic coupling between phases, for simplicity

Machine:
- only the field winding $f$ in the $d$ axis
- only one damper winding $q1$ in the $q$ axis
- rotor speed $\dot{\theta}_r$ assumed constant
  - focus on short-lasting electromagnetic transients: speed has no time to change
- constant excitation voltage $V_f$
  - it is assumed that the automatic voltage regulator has no time to react
Network equations

\[ v_a - e_a = R_e i_a + L_e \frac{di_a}{dt} \quad \text{with} \quad e_a = \sqrt{2}E \cos(\omega_N t + \theta_e) \]  
(1)

\[ v_b - e_b = R_e i_b + L_e \frac{di_b}{dt} \quad \text{with} \quad e_b = \sqrt{2}E \cos(\omega_N t + \theta_e - \frac{2\pi}{3}) \]  
(2)

\[ v_c - e_c = R_e i_c + L_e \frac{di_c}{dt} \quad \text{with} \quad e_b = \sqrt{2}E \cos(\omega_N t + \theta_e - \frac{4\pi}{3}) \]  
(3)

Park transformation

\[
\begin{bmatrix}
  v_d \\
v_q \\
v_o
\end{bmatrix} = \mathcal{P} \begin{bmatrix}
  v_a \\
v_b \\
v_c
\end{bmatrix}
= \mathcal{P} \begin{bmatrix}
  i_d \\
i_q \\
i_o
\end{bmatrix}
\]

with \( \mathcal{P} = \sqrt{\frac{2}{3}} \begin{bmatrix}
  \cos \theta_r & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r - \frac{4\pi}{3}) \\
  \sin \theta_r & \sin(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r - \frac{4\pi}{3}) \\
  \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix} \)  
(5)

where \( \theta_r = \theta_r^o + \omega_N t \)
Machine equations

\[ \psi_d = L_{dd}i_d + L_{df}i_f \quad (6) \]
\[ \psi_q = L_{qq}i_q + L_{qq1}i_{q1} \quad (7) \]
\[ \psi_o = L_{oo}i_o \quad (8) \]
\[ \psi_f = L_{ff}i_f + L_{df}i_d \quad (9) \]
\[ \psi_{q1} = L_{q1q1}i_{q1} + L_{qq1}i_q \quad (10) \]
\[ v_d = -R_ai_d - \dot{\theta}_r \psi_q - \frac{d\psi_d}{dt} \quad (11) \]
\[ v_q = -R_ai_q + \dot{\theta}_r \psi_d - \frac{d\psi_q}{dt} \quad (12) \]
\[ v_o = -R_ai_o - \frac{d\psi_o}{dt} \quad (13) \]
\[ V_f = R_f i_f + \frac{d\psi_f}{dt} \quad (14) \]
\[ 0 = R_{q1}i_{q1} + \frac{d\psi_{q1}}{dt} \quad (15) \]
Variable - equations balance

- 19 variables: $v_a, v_b, v_c, i_a, i_b, i_c, v_d, v_q, v_o, i_d, i_q, i_o, \psi_d, \psi_q, \psi_o, \psi_f, \psi_{q1}, i_f, i_{q1}$
- 19 equations: $(1 - 3)$, 6 eqs. in $(4)$, $(6 - 15)$

Remarks

- The model is made up of Differential-Algebraic Equations (DAEs)
- some of the variables and some of the equations could be eliminated but the additional computational effort of keeping all of them is negligible
- $\theta_r$ being known, the equations are linear with respect to the unknowns
- some coefficients in these equations vary with time.

\footnote{not to mention the risk of introducing mistakes in analytical manipulations!}
Passing the equations in per unit

In each phase of the stator (a, b, c):
- base voltage \( V_B = \) nominal RMS phase-to-neutral voltage (kV)
- base power \( S_B = \) three-phase apparent power (MVA)
- base current \( I_B = \frac{S_B}{3V_B} \), base flux \( \psi_B = \frac{V_B}{\omega_N} \), etc.

In each park winding (d, q, o):
- base voltage \( V_{PB} = \sqrt{3}V_B \)
- base power = \( S_B \)
- base current \( I_{PB} = \frac{S_B}{V_{PB}} = \sqrt{3}I_B \), base flux \( \psi_B = \frac{V_{PB}}{\omega_N} \), etc.

In the rotor windings \( f \) and \( q1 \):
- we use the reciprocal “Equal Mutual Flux Linkages” per unit system
- the latter is such that \( L_{dd} = L_{df} + L_\ell \) and \( L_{qq} = L_{qq1} + L_\ell \) in per unit, where \( L_\ell \) is the leakage inductance (same for both d and q windings)

After passing in per unit:
- \( \dot{\theta}_r = 1 \) pu in Eqs. (11, 12)
- the equations are unchanged, except that each time derivative is multiplied by \( 1/\omega_N \), since we keep the time in second (not in pu)
Equations converted in per unit and rearranged

\[
\frac{1}{\omega_N} \frac{di_a}{dt} = -\frac{R_e}{L_e} i_a + \frac{1}{L_e} v_a - \frac{1}{L_e} e_a \tag{16}
\]

\[
\frac{1}{\omega_N} \frac{di_b}{dt} = -\frac{R_e}{L_e} i_b + \frac{1}{L_e} v_b - \frac{1}{L_e} e_b \tag{17}
\]

\[
\frac{1}{\omega_N} \frac{di_c}{dt} = -\frac{R_e}{L_e} i_c + \frac{1}{L_e} v_c - \frac{1}{L_e} e_c \tag{18}
\]

\[
0 = \frac{\sqrt{2}}{3} \left[ \cos(\theta_r) v_a + \cos(\theta_r - \frac{2\pi}{3}) v_b + \cos(\theta - \frac{4\pi}{3}) v_c \right] - v_d \tag{19}
\]

\[
0 = \frac{\sqrt{2}}{3} \left[ \sin(\theta_r) v_a + \sin(\theta_r - \frac{2\pi}{3}) v_b + \sin(\theta - \frac{4\pi}{3}) v_c \right] - v_q \tag{20}
\]

\[
0 = \frac{1}{3} (v_a + v_b + v_c) - v_o \tag{21}
\]

\[
0 = \frac{\sqrt{2}}{3} \left[ \cos(\theta_r) i_a + \cos(\theta_r - \frac{2\pi}{3}) i_b + \cos(\theta_r - \frac{4\pi}{3}) i_c \right] - i_d \tag{22}
\]

\[
0 = \frac{\sqrt{2}}{3} \left[ \sin(\theta_r) i_a + \sin(\theta_r - \frac{2\pi}{3}) i_b + \sin(\theta_r - \frac{4\pi}{3}) i_c \right] - i_q \tag{23}
\]

\[
0 = \frac{1}{3} (i_a + i_b + i_c) - i_o \tag{24}
\]
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System modelling

0 = \( L_{dd} i_d + L_{df} i_f - \psi_d \) (25)

0 = \( L_{qq} i_q + L_{qq1} i_{q1} - \psi_q \) (26)

0 = \( L_{ff} i_f + L_{df} i_d - \psi_f \) (27)

0 = \( L_{q1q1} i_{q1} + L_{qq1} i_q - \psi_{q1} \) (28)

0 = \( L_{oo} i_o - \psi_o \) (29)

\[
\frac{1}{\omega_N} \frac{d\psi_d}{dt} = -R_a i_d - \psi_q - v_d
\] (30)

\[
\frac{1}{\omega_N} \frac{d\psi_q}{dt} = -R_a i_q + \psi_d - v_q
\] (31)

\[
\frac{1}{\omega_N} \frac{d\psi_f}{dt} = -R_f i_f + V_f
\] (32)

\[
\frac{1}{\omega_N} \frac{d\psi_{q1}}{dt} = -R_{q1} i_{q1}
\] (33)

\[
\frac{1}{\omega_N} \frac{d\psi_o}{dt} = -R_a i_o - v_o
\] (34)
Model in compact form

With a proper reordering of equations and states, the model can be rewritten in compact form as:

\[
\begin{align*}
(1/\omega_N) \dot{x} &= A_{xx} x + A_{xy} y + u \\
0 &= A_{yx} x + A_{yy} y
\end{align*}
\]

(35) \hspace{1cm} (36)

where:

\[
\begin{align*}
x &= \begin{bmatrix} i_a & i_b & i_c & \psi_d & \psi_q & \psi_f & \psi_{q1} & \psi_{o} \end{bmatrix}^T \\
y &= \begin{bmatrix} v_a & v_b & v_c & v_d & v_q & v_o & i_d & i_q & i_o & i_f & i_{q1} \end{bmatrix}^T \\
u &= \begin{bmatrix} -\frac{e_a}{L_e} & -\frac{e_b}{L_e} & -\frac{e_c}{L_e} & 0 & 0 & V_f & 0 & 0 \end{bmatrix}^T
\end{align*}
\]
Numerical solution of the DAEs

Let $k$ denote the discrete time ($k = 0, 1, 2, \ldots$), and $h$ the time step size.

A popular numerical integration formula is the \textit{Trapezoidal Method}:

$$x_{k+1} = x_k + \frac{h}{2} (\dot{x}_{k+1} + \dot{x}_k)$$

Replacing $\dot{x}_{k+1}$ by its expression (35):

$$x_{k+1} = x_k + \frac{h}{2} \omega_N A_{xx} x_{k+1} + \frac{h}{2} \omega_N A_{xy} y_{k+1} + \frac{h}{2} \omega_N u_{k+1} + \frac{h}{2} \dot{x}_k$$

Dividing by $\frac{h \omega_N}{2}$ and rearranging the various terms:

$$\left[ A_{xx} - \frac{2}{h \omega_N} \mathbf{I} \right] x_{k+1} + A_{xy} y_{k+1} = -\frac{2}{h \omega_N} x_k - \frac{1}{\omega_N} \dot{x}_k - u_{k+1}$$

(37)

where $\mathbf{I}$ is the unit matrix of same dimension as $x$. 

On the other hand, from Eq. (36) we have:

$$A_{yx}x_{k+1} + A_{yy}y_{k+1} = 0$$  \hspace{1cm} (38)

Grouping Eqs. (37) and (38), the linear system to solve at each time step is:

$$
\begin{bmatrix}
    A_{xx} - \frac{2}{h\omega_N} I & A_{xy} \\
    A_{yx} & A_{yy}
\end{bmatrix}
\begin{bmatrix}
    x_{k+1} \\
    y_{k+1}
\end{bmatrix}
= 
\begin{bmatrix}
    -\frac{2}{h\omega_N} x_k - \frac{1}{\omega_N} \dot{x}_k - u_{k+1} \\
    0
\end{bmatrix}$$  \hspace{1cm} (39)
Numerical example and comments on the results

**Network and machine data**

\[ f_N = 50 \text{ Hz} \]
\[ L_e = 0.20 \text{ pu} \quad R_e = 0.01 \text{ pu} \]
\[ Ra = 0.005 \text{ pu} \]
\[ L_{dd} = 2.4 \text{ pu} \]
\[ L_{df} = 2.2 \text{ pu} \]
\[ L_{ff} = 2.42 \text{ pu} \]
\[ L_{qq} = 2.4 \text{ pu} \]
\[ L_{qq1} = 2.2 \text{ pu} \]
\[ L_{q1q1} = 2.2512 \text{ pu} \]
\[ R_f = 0.0011 \text{ pu} \]
\[ R_{q1} = 0.0239 \text{ pu} \]
\[ L_{oo} = 0.1 \text{ pu} \]

**Initial operating point**

\[ P = 0.5 \text{ pu} \]
\[ Q = 0.1 \text{ pu} \]
\[ \bar{V}_a = 1.000 \text{ pu} \angle 0 \]

A MATLAB script to simulate this system is available in `emt.m` and `linmodel.m`. 
Simulation results

A three-phase short-circuit is simulated by setting $E$ to zero at $t = 0.05$ s.

Important remark

The fault is not cleared in order to show the various time constants present in the current evolution.

However, in practice:

- the fault must be cleared fast enough, e.g. after 5 - 10 cycles (0.1 - 0.2 s)
- beyond that time, the model is no longer valid:
  - rotor speed would not remain constant
  - $v_f$ would be adjusted by the Automatic Voltage Regulator
  - etc.
Behaviour of synchronous machine during a short-circuit

Numerical example and comments on the results

decrease of AC voltage magnitude under the effect of the fault

presence of a residual voltage due to some emf inside the generator

remark:

such an emf does not exist in generators connected to the network through power electronic interfaces (dispersed generation in MV distribution grid)

the latter do not participate to the short-circuit capacity!
• increase of amplitude of alternating current under the effect of the fault
• the envelop of the current wave varies with time (more details in the sequel)
• presence of a small *aperiodic* or *unidirectional* or *DC* component
  • typical of transients in an RL circuit due to switchings
  • much more visible in the other two phases: see next slide
the magnitude of the aperiodic components decrease with a time constant $\simeq 0.10 \text{ s}$ (in this example)

the aperiodic components are not the same in all three phases, because the rotor is not in the same position with respect to each stator winding

once they have vanished, the three phase currents become again sinusoidal and balanced
the magnitude of the alternating component of $i_a$ shows two time constants:
- a short one (a few cycles), resulting in a slightly higher initial amplitude of the current: of subtransient type - caused by damper winding $q1$
- a much longer one ($\approx 1.5$ s in this example): of transient type - caused by field winding $f$
the current that the breakers have to interrupt is much higher than the one which would prevail in steady-state!
the machine behaves initially as if it had a smaller internal reactance
Magnetic fields

The alternating components of the stator currents $i_a, i_b$ and $i_c$
- are shifted by $\pm 2\pi/3$ rad. They create a magnetic field $H_{AC}$ which rotates at the angular speed $\omega_N$
- this field is fixed with respect to the rotor windings
- under the effect of the fault, the amplitude if $i_a, i_b$ and $i_c$ increases significantly. So does the magnetic field $H_{AC}$
- this induces aperiodic current components in the rotor windings.

The aperiodic components of the stator currents $i_a, i_b$ and $i_c$
- create a magnetic field $H_{DC}$ which is fixed with respect to the stator
- hence it rotates at angular speed $\omega_N$ with respect to the rotor windings
- this induces alternating components of angular frequency $\omega_N$ in the rotor windings.

This is confirmed by the plots in the next slides.
• the flux linkage $\psi_f$ in the field winding changes very little (large “magnetic inertia”) in spite of the large increase of stator currents!
• large “magnetic inertia” due to the long time constant $L_{ff}/R_f (= 7 \text{ s in this example})$
Lenz law: additional current components appear in the field winding in order to keep \( \psi_f \) (almost) constant.

- The oscillatory component is due to the magnetic field \( H_{DC} \)
  - Check: time constant of decay = time constant of aperiodic component of stator currents \( \approx 0.10 \) s

- The aperiodic component is due to the magnetic field \( H_{AC} \)
  - Time constant \( \approx 1.5 \) s: see slide # 18
- the flux linkage in the damper winding $q_1$ is comparatively more “volatile”
- indeed, the field and the damper windings are constructively very different: field coil vs. damper bars in rotor slots
the damper current $i_{q1}$ has a zero initial (and final) value
the oscillatory component is due to the magnetic field $H_{DC}$
the aperiodic component decreases much faster than the aperiodic component of $i_f$
it corresponds to the initial, fast decaying, increment of the stator current amplitude (see slides # 16 and 18)
the oscillatory component of $i_d$ (resp. $i_q$) corresponds to the oscillatory component of $i_f$ (resp. $i_{q1}$) which lies on the same axis

it can be shown that $i_q$ goes to almost zero due to the predominantly inductive nature of the short-circuit

the aperiodic component of $i_d$ evolves with the long time constant observed for the aperiodic component of $i_f$
- fluxes $\psi_d$ and $\psi_q$ in Park windings vary comparatively much faster
- since $i_q$ and $i_{q1}$ tend to zero, so does $\psi_q$. 