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Flexibility services in the electrical system

by

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Contents

Abstract

The work presented in this thesis considers the electrical flexibility from the electric load to its usage as a commodity. The conception of the European electrical system has led to a large amount of actors that are impacted by flexibility exchanges. This thesis proposes approaches to assess the impact of exchanging flexibility in the electrical system and analyzes the complex interactions resulting from these exchanges. The modeling techniques used to carry the analysis are optimization, game theory and agent-based modeling. The impacts on different parts of the electrical system are presented: the day-ahead energy market, the secondary reserve and the distribution system. Since flexibility is the base block of this thesis, two methods to obtain flexibility from actual consumption processes are broached: direct control of the loads and dynamic pricing. One chapter provides an example of how flexibility can be obtained by the direct control of a portfolio of heat pumps and another chapter studies the control of electric heaters and boilers via the use of a simple price signal.

Acronyms

ANM	Active network management
BRP	Balancing responsible party
CAPEX	Capital expenditure
COP	Coefficient of performance
DSIMA	Distribution system interaction model analysis
DSO	Distribution system operator
FSP	Flexibility services provider
FSU	Flexibility services user
HV	High voltage
LV	Low voltage
MCP	Market clearing price
MV	Medium voltage
OPEX	Operating expense
PV	Photovoltaic
RAM	Random-access memory
TCL	Thermostatically controlled load
TSO	Transmission system operator

Chapter 1

Introduction

Since its conception, the electric system has tremendously evolved to one of the most complex machinery of our everyday life. The initial power system feeding intermittently electricity to a few lamps now runs most of our appliances and industries nearly without interruptions. To achieve this impressive reliability, the *electric* system relies on contracts, markets and ancillary services, which as a whole shapes the nowadays *electrical* system. This is an occasion to clarify the distinction between the adjectives *electric* and *electrical*. An English dictionary provides the following definitions [27].

Definition 1. *Electric – of, derived from, produced by, producing, transmitting, or powered by electricity.*

Definition 2. *Electrical – of, relating to, or concerned with electricity*

Electric is therefore more related to devices or to the grid as an electric component and electrical to the system or its engineers. This thesis studies the electrical system, as well as the behaviors and interactions of the actors within this system.

This chapter introduces the context and the motivations behind this thesis. To this end, a sketch of the functioning of the European electrical system is described. Note that the concepts detailed in this introduction may slightly differ in some countries, for instance considering different gate closures, longer delays for some actions or granularities of the decision time steps e.g. quarters or hours. However, the European Union is willing to reach a single electricity market for Europe [58]. We consider here a generic electrical system as its often done in as done in academic books dedicated to the description of power systems, for instance in [82]. This synthesis also highlights a potential drawback of the unbundling of the historical system into a market based system. A second part defines the notion of flexibility in electrical systems and highlights the main difficulties associated by considering flexibility as a service and its exchange as a commodity.

1.1 The European electrical system

Electricity is one of the most complex commodity to exchange. Since electricity flows quasi-instantaneously through the network, the production must be equal to the consumption at every moment. This equality is ensured by a lot of interactions between the actors of the electrical system. The definition of an actor in this work is an entity which takes part in the electrical system by fulfilling one or more roles. Some roles are well-known to the public while others, despite being necessary for the survival of the system, are usually unknown from the end user of electricity Hereafter are defined roles that may be taken by actors of the electrical system. Most definitions are inspired from a publication of the ENTSO-E, the European Network of Transmission System Operators for Electricity [49] and from the Directive 2009/72/EC from the European Commission [58]. Note that real-life electrical systems are extremely complex and differ significantly from one country to another. This chapter presents a general picture of the European electricity market closer from a simplified version of the Belgian electrical system. However, there is a trend in Europe to harmonize the whole electrical system driven by the European commission [48, 56].

Like every commodity, the main roles of the electrical systems are producers and consumers.

Definition 3. *Producer – Role of an actor that produces electricity.*

Definition 4. *Consumer – Role of an actor that consumes electricity.*

Small consumers do not buy their electricity individually but make use of the services of intermediation provided by retailers.

Definition 5. *Retailer – Role of an actor that sells electricity to ultimate consumers, usually in small quantity.*

Electricity is traded before its actual delivery. Two mechanisms are mainly used: bilateral trades and pool markets. Bilateral trading involves only a buyer and a seller without involvement, interference or facilitation from a third party [82]. The most common bilateral trade is the long term contract, typically a few years in advance, between a producer and a retailer or a big consumer willing to secure its minimum energy needs. Producers use these contracts to ensure them safer financial returns as they obtain a guarantee that their production units will be worth to be running.

The second mechanism, pool markets, involves a third party named the market operator.

Definition 6. *Market Operator – Operator of a market in which energy bids are traded.*

The principle of these markets is common, all offers are collected and the cheapest selling offers are matched with most expensive buying offers. The most important pool market in Europe is the day-ahead energy market. Each country has its own day-ahead markets, like Belpex for Belgium or EPEX for France which is coupled with other countries. Currently, the day-ahead energy markets of most European countries are coupled together allowing energy to be exchanged between the following countries: Austria, Belgium, Czech Republic, Denmark, Estonia, Finland, France, Germany, Italy, Latvia, Lithuania, Luxembourg, the Netherlands, Norway, Poland, Portugal, Spain, Slovenia, Sweden, Switzerland and the Great Britain [51]. The document also details the different products that may be exchanged within the market. Figure 1.1 shows the status of the day-ahead energy markets coupling in 2015 [26]. One could expect this coupling trend to grow worldwide towards a global grid [24]. The common day-ahead energy market settles, for each hour of the day and each geographic zone, a unique price called the system marginal price. This price is based on supply and demand offers which are respectively aggregated into an offer curve and a demand curve. The system marginal price is obtained at the intersection of these two curves. Supply offers at price lower than the system marginal price and demand offers at higher prices are accepted at this unique price. Figure 1.2 shows the production and consumption aggregated curves of the French spot market for the first hour of the 1st April 2014 [50]. The intersection of the non-decreasing offer curve with the demand curve leads to the system marginal price of 37€/MWh which corresponds to a volume of 10.735MWh. Every bids to the left-hand side of this volume are accepted. This clearing method gives incentive to the participants to bid at their marginal costs and therefore discourage gaming on the price they require [82]. The complete description of the day-ahead energy market clearing procedure can be found in the public description of Euphemia, the market clearing algorithm [51].

Once traded, electricity needs to be conveyed from the production units to the electric devices. The physical link between production and consumption is called the network or the grid.

Definition 7. *Grid user – Role of an actor physically connected to the electric network.*

The term network not only includes cables and lines, but also necessary equipment such as transformers, breakers, etc. The network is divided into two layers dependent on the voltage-level: the distribution network and the transmission network. The distribution network is the low voltage (LV) part of the network below 1kV and the medium voltage (MV) part below 36kV [87]. This network is operated by the distribution system operator.

Definition 8. *Distribution system operator (DSO) – Role of an actor responsible of operating, ensuring the maintenance of, and, if necessary, developing*

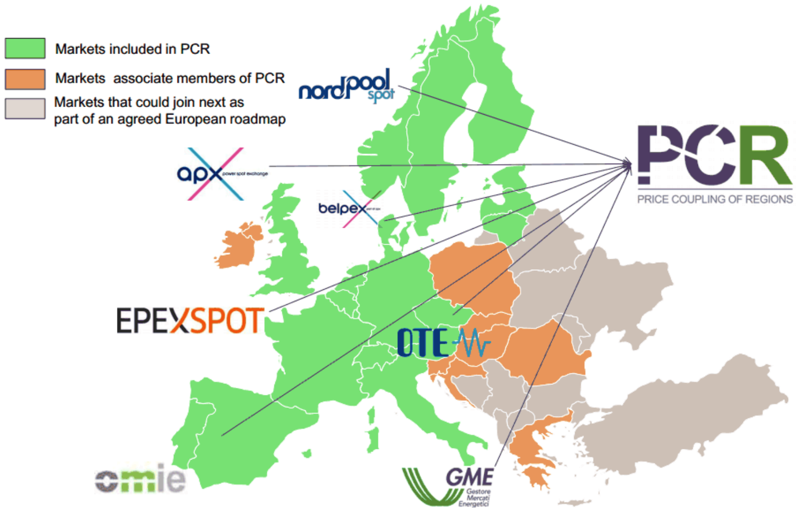


Figure 1.1: Status of the day-ahead energy markets coupling in 2015 [26].

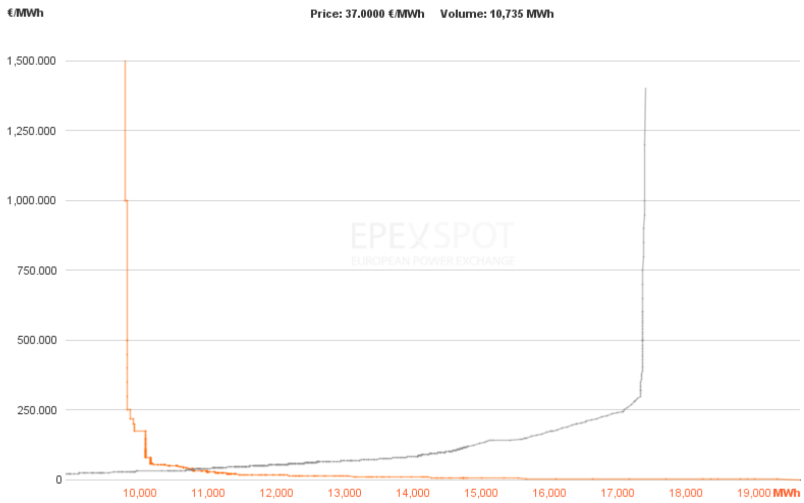


Figure 1.2: Aggregated curves of the market clearing of the 1st April 2014 in the period 0h-1h [50].

the distribution system.

European DSOs are also required to facilitate effective and well-functioning retail markets and to collect the metering of the electricity customers [54].

The transmission network is the high-voltage (HV) part of the network above 36kV. This network is operated by the transmission system operator.

Definition 9. *Transmission system operator (TSO) – Role of an actor responsible of operating, ensuring the maintenance of, and, if necessary, developing the transmission system and managing the balance of its system.*

One of the most important task of the TSO is to maintain the balance at every moment between production and consumption. This equilibrium is first ensured on the trades with the obligation for each participant to submit a planning of production and consumption to the TSO for each period of the day, which may be quarter, hours, etc. This planning is called a *baseline*. This information is provided by the balancing responsible parties which may regroup multiple producers, consumers and retailers.

Definition 10. *Balancing responsible party (BRP) – Role of an actor that is responsible for the balance of injections and offtakes at different access point of the transmission network.*

The TSO denies the access to its network to any grid user which is not covered by a BRP [49]. Therefore, many producers and retailers are also their own BRP. In real-time, the realization may deviate from the baseline and the TSO has the responsibility to compensate the overall imbalance, i.e. the mismatch between injections and offtakes, which is not balanced by BRPs. Figure 1.3 provides an example of baseline and the corresponding realization.

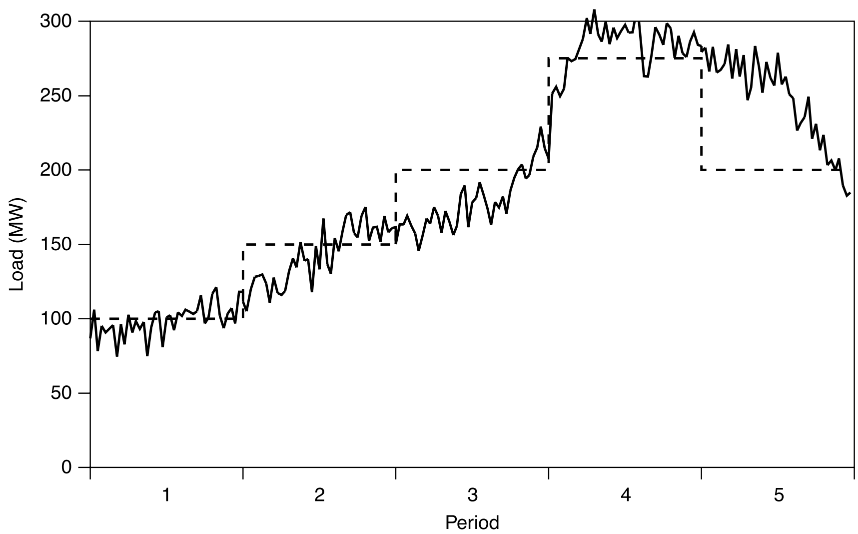


Figure 1.3: Example of baseline given by the dashed stroke and realization given by the continuous stroke [82].

Due to the unbundling of the electrical system [58], the TSO does not own production or consumption assets. The first mechanism to ensure the balance is the primary frequency control which is provided by some generating units able to quickly modify their production. They react to deviations of the frequency of the system from the traditional 50Hz. If the frequency is above, there is too much production and the generating units decrease their production. If the frequency is below, there is too much consumption and the generating units increase their production. More details about primary frequency control can be found in [115]. This mechanism is designed to handle deviations such that, on average, no energy is provided by the generating units for primary frequency control.

The energy needed to relieve the generating units of the primary frequency control is provided by the balancing mechanism [46] which capacity is given by the secondary and tertiary reserves. The TSO contracts balancing services to balance service providers.

Definition 11. *Balance service provider – Role of an actor that provides flexibility services to reduce the difference between traded volumes and realizations.*

The balance service provider proposes flexibility services to the TSO which activates them according to the needs of the system. The most expensive activated service defines the price of imbalance in €/MWh which serves as a basis for the payment or compensation of BRPs penalizing or helping the balance of the system. Figure 1.4 shows an example of imbalance prices and volumes.

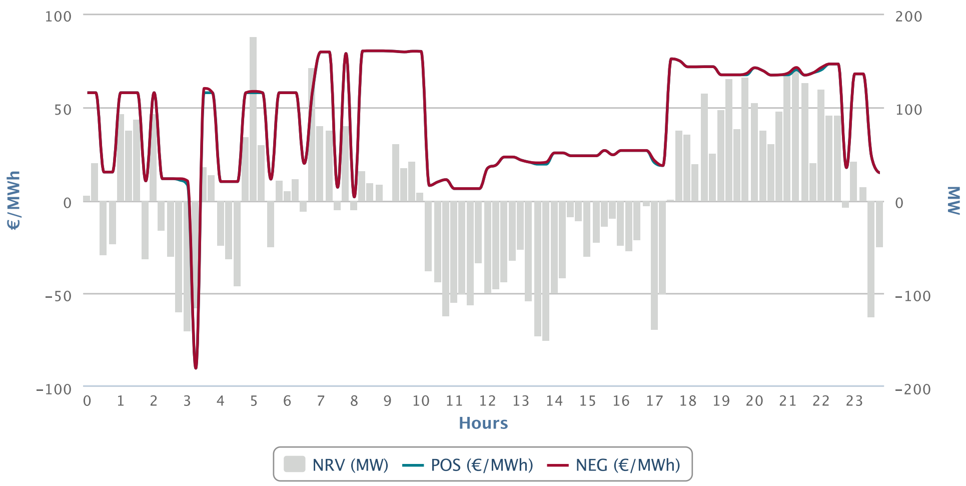


Figure 1.4: Imbalance prices and volumes on June 1, 2015 of Elia, the Belgian TSO [43].

1.2 Flexibility in electrical system

In the previous section, we broached the notions of flexibility and flexibility services which are the main topics of this thesis. The following definitions are inspired from [53], written by the Eurelectric association which represents the common interests of the electricity industry at pan-European level [52].

Definition 12. *Flexibility in electrical system is the modification of generation injection and/or consumption patterns in reaction to an external price or activation signal in order to provide a service within the electrical system.*

This flexibility is offered on the form of a service provided by flexibility services providers.

Definition 13. *A flexibility service is a definition of the parameters and signals describing precisely how flexibility is activated.*

Definition 14. *Flexibility service provider (FSP) – Role of an actor that provides flexibility services.*

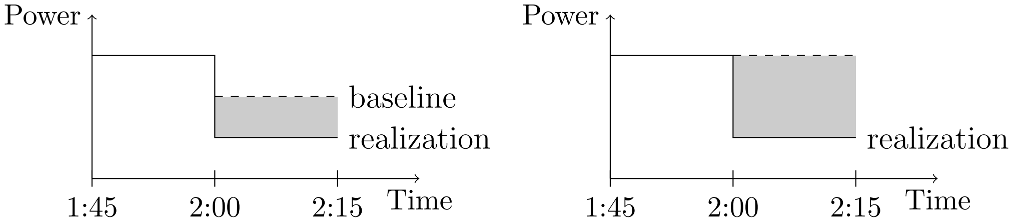
Note that, following that definition, the balance service provider is a specific FSP providing flexibility for balancing purposes. The parameters used to characterize flexibility include the amount of power modulation, the duration, the rate of change, the response time, the location, etc. [53].

The needs for flexibility can be triggered by a congestion in a line of the distribution or the transmission network, the loss of a production unit, an unexpected over-production of a wind turbine, etc. One example of flexibility service is the primary frequency control offered by the production units taking the frequency of the network as a trigger signal. From the point of view of the network, this is the simplest solution as the responsible units reacts automatically. The production units in charge of the dedicated service change their outputs proportionally to the deviation of the frequency of the network with respect to the 50Hz. To participate to primary frequency control, these units only needs to be qualified by the TSO to make sure that they are able to provide primary frequency control. This service is paid not only at the activation but also at its reservation to cover the cost of availability. For instance, a 300MW unit providing primary frequency control operates at 250MW to be able to ramp up by 50MW. These not produced 50MW could have been sold as energy. The reservation cost aims at covering this loss of opportunity. This reservation of the capacity is not well suited for renewable production which bears nearly no marginal cost. For instance, reserving 1MW of a wind turbine can be done by changing the pitch of the blades. However, reserving this megawatt is equivalent to throwing it away since the energy not produced could have been obtained for free.

The same philosophy of reservation and activation is applied to the balancing mechanism where the TSO coordinates the activation of the flexibility services. The balancing flexibility service consists in well defined deviations, called in this thesis modulations, with respect to a given reference.

Definition 15. *A modulation is a change of active power with respect to a reference.*

The notion of reference has a major importance to quantify and therefore charge a flexibility service. In the case of the balancing mechanism, this reference is clearly identified as the baseline provided by the BRP of the provider of the service. A modulation provided by a FSP can be defined with respect to multiple references. Figure 1.5 illustrates two reference choices: the baseline and the realization of the last period. This example illustrates that the reference choice affects the volume of modulation which will be paid by the user of the flexibility service for the same final effect in terms of power. The reason for which the baseline could not be taken as reference is that it may not exist. For instance, small consumers do not communicate to their retailer when they intend to iron or use their microwave. If a wind turbine is shut down, it is not possible to accurately known what would be its production if it was turned on. The prediction of the production of a single wind farm is as difficult as the prediction of the wind. To give an order of magnitude, the mean absolute error of the energy produced by a wind turbine with respect to a day-ahead prediction is about 55% [74]. The question of the choice of reference for flexibility services is further investigated in [69]. In this thesis, the choice of reference will be clearly stated when a flexibility service is defined.



(a) Baseline as reference (b) Realization of the last period as reference

Figure 1.5: Two flexibility services for the same realization defined by their reference choice.

Current usage of flexibility services made the European electrical system extremely reliable to a point where loosing electricity for one hour in an household became an exceptional event. To maintain this level of service with the current evolution of the network, flexibility services must evolve. The driver of this evolution is the increasing share of the renewable production. Figure 1.6 shows the evolution of the installed power generating *capacity* per year and the share of renewable production. This increasing amount of renewable energy aims to reach the 20% share of the European Union energy consumption produced for 2020 [57]. The European commission intends to continue this evolution to reach at least a 27% share of renewable energy consumption for 2030 [55]. This ongoing trend for renewable energy has two major consequences on the electrical system: an increase of the production within the distribution network and the shutdown of many thermal power plants. These two consequences respectively imply the needs to the *use flexibility within distribution network* to ensure its safe operation and to find alternatives to provide flexibility other than the one provided by thermal generation units. One alternative is to harness the flexibility of the consumption.

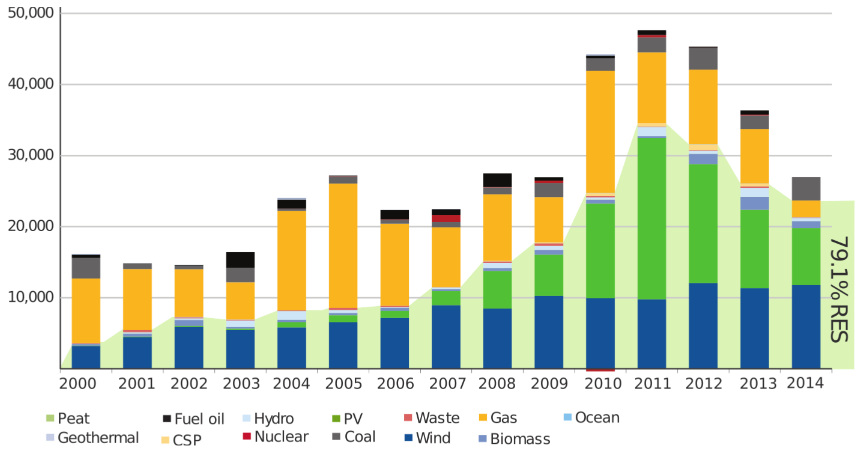


Figure 1.6: Installed power generating capacity in Europe per year in MW and renewable share [59].

1.2.1 Flexibility of the consumption

The share of renewable energy increases at the expense of the loss of thermal generation units more capable to provide flexibility. Figure 1.6 pictures this fact and shows that gas production capacity is particularly affected. The European commission expects this phenomenon to continue in the coming decades [21]. As explained previously, using renewable energy to provide flexibility implies wasting nearly free energy. Using the flexibility of the consumption has therefore earned a lot of interest in the previous years under the term *demand side management* [118, 134]. Even if a part of our consumption, such as lights, microwaves and televisions, is not flexible, a large part of it can be controlled in a smarter way. Electrical heaters, air-conditioners, dishwashers, fridges, etc. The consumption of these appliances can be modulated or differed to provide flexibility to the electrical system. The load aggregator is responsible of making the link between the consumption of the appliances and flexibility services. Therefore, the aggregator is a FSP providing coherent flexibility services from the aggregation of the consumption of a set of devices.

Definition 16. *Aggregator - Role of an actor which directly or indirectly controls the consumption of a set of devices.*

The control of loads gives rise to new challenges not encountered with the flexibility of production units [134]. One of them is to manage the heterogeneity of the consumption. One aggregator may have in its portfolio heat pumps, electric cars, air-conditioners, commercial fridges, washing machines; or all of them. The aggregator must develop solutions which considers the constraints of all processes behind the electrical consumption while providing flexibility. One of the most common constraints is to consider the *payback effect* inherent to energy consumption. Assume that an aggregator controls supermarket fridges and decreases their electrical consumption for one hour. As a result, the temperature inside the fridges rises. In the second hour, the fridges must consume more to restore the temperature to the initial set point. There is therefore an energy recovery phenomenon which does not appear with energy production. In the case of a gas production unit, decreasing its production is as simple as decreasing the gas input and has no consequences on the production in the following hour.

The activation cost of fueled production unit is naturally its fuel cost as increasing its production is a result of increasing the fuel input. Assessing the activation cost of the flexibility is not as easy for the consumption. Shifting or modulating the consumption does not require any other fuel than electricity. *If the energy is shifted, does it have a cost and which one? If the consumption pattern of a heat pump is changed, is there an overconsumption and what is its cost?*

1.2.2 Flexibility in distribution networks

The electric network has been conceived to carry the electricity from production units connected to the transmission network to the consumption devices connected to the distribution network. The current grid grows apart from its traditional conception as the number of solar panels and wind turbines connected to the distribution network increases. DSOs already need to compute estimates of the maximum generation that can be connected to appropriate substations of they network [30]. This *distributed generation* may cause over-voltage problems, congestions in lines and important reverse flows going from the distribution network to the transmission network [104]. Over-voltages are already happening often in some low-voltage networks with a high concentration of solar panels [95]. Currently this problem is tackled by a protection mechanism in solar panels which switches off the production if the voltage is too high. The increase of the distributed generation may also cause congestions in lines within the distribution network and important reverse flows, saturating the transformer between the distribution and the transmission network. Current practice is to invest in distribution networks such that the capacity of these networks is sufficient to face the peak production and consumption. This method does not comply well with renewable energy whose production is most of the time significantly lower than the maximum capacity. Therefore, designing the network for the exceptional event where all production units in the network produce at their maximum capacity would lead to unbearable investments for the society. An alternative is active network management (ANM) which aims at in increasing the efficiency of distribution systems by operating the system using all control means available and the flexibility of the grid users [64].

The DSO may resort to different options to obtain flexibility. The first one would be to change the tariff of usage of the distribution network to incentivize the shifting of the consumption such that over-production is consumed locally. Another solution is to use flexibility services in the distribution network. To this end, the regulators should first establish the rules allowing the exchange of flexibility services. These rules are called interaction models in this manuscript. Numerous interaction models can be imagined and one natural question is: *Which interaction model should be used?* To answer such question, the impact of choosing one interaction model over another should be evaluated on each actor.

1.3 Contributions and outline of the thesis

This thesis studies the flexibility from the demand side and its integration into the electrical system. Flexibility is broached starting from a high level view of the system and ending with detailed models of the loads. The first three chapters aim at studying the impact of flexibility on electrical systems of increasing complexity. The last two chapters focus on methods to obtain flexibility from the demand side using dynamic pricing or direct control of heat pumps. The following gives as brief summary of the contributions in each chapter.

Chapter 2 studies the impact of load flexibility on the day-ahead energy market prices. A system composed of multiple electricity retailers determining their buying offers in each hour is mapped to a game theory problem: the atomic splittable flow congestion game. The first contribution of this chapter is this mapping which allows benefiting from the theoretical results of the game theory literature. New results for this game theory problem are also provided in this chapter. Finally, a simple method to compute the price of load flexibility is proposed based on a game theory analysis.

Chapter 3 considers the effect of opening the reserve market to flexible loads. To this end, a specific flexibility service taking into account the payback effect of the loads is proposed. The resulting reserve market is analyzed taking into account the gaming possibilities of producers and retailers, controlling load flexibility, in the day-ahead energy and reserve markets, and in imbalance settlement. This analysis is carried out by an agent-based approach where, for every round, each actor uses optimization problems to maximize their individual profits according to forecasts of the prices.

Chapter 4 continues with agent-based techniques to model the exchange of flexibility within a distribution system. Interaction models guiding the exchange of flexibility are proposed. They are analyzed using the open-source tool DSIMA [37] developed in this thesis to evaluate quantitatively interaction models. The tool is an agent-based code in Python accessible by a high-level dedicated webpage. The open source-code of the project is available at the address http://www.montefiore.ulg.ac.be/~dsima.

Chapter 5 provides a mean to obtain flexibility within the distribution network in short term. This mean consists in using the tariff of distribution as a price signal to shift the consumption and alleviate problems in the distribution networks. This solution has the major advantage that, in most countries, the infrastructure is already in place with the system of off-peak and on-peak tariffs. This chapter demonstrates that placing off-peak hours from noun to 2 p.m. could increase the production of residential solar panels while decreasing the peak power flows in the medium-voltage network.

Chapter 6 presents one example of flexibility service that could be delivered by an heat pumps aggregator. This flexibility service consists in a modulation in one quarter followed by a well defined energy payback in the following hour. Accurately defining this payback is the key element that allows flexibility to be used in one quarter without risking creating other issues in following periods. The baseline of the aggregator is obtained by optimizing the consumption to minimize the energy cost. A second optimization problem determines the available quantity of flexibility and shows how the payback following the modulation of heat pumps can be quantified and minimized. The optimization problems use accurate thermal models of heat pumps and buildings.

Chapter 7 provides the key conclusions of this thesis and suggestions of future works.

1.4 List of publications

Hereafter are the publications written during my thesis ordered by submission date. These publications summarize the contributions presented in this document. Two papers are outside the scope of this manuscript and relates to the data mining of the optimal parameters of an industrial process.

1. Sébastien Mathieu, Damien Ernst, and Quentin Louveaux. "An efficient algorithm for the provision of a day-ahead modulation service by a load aggregator". In: *Innovative Smart Grid Technologies Europe (ISGT EUROPE), 2013 4th IEEE/PES*. IEEE, 2013 [98].

2. Quentin Louveaux and Sébastien Mathieu. "A combinatorial branch-and-bound algorithm for box search". In: *Discrete Optimization* 13 (2014), pp. 36 –48. ISSN: 1572-5286. DOI: 10.1016/j.disopt.2014.05.00 [89].

3. Sébastien Mathieu, Quentin Louveaux, Damien Ernst, and Bertrand Cornélusse. "A quantitative analysis of the effect of flexible loads on reserve markets". In: *Proceedings of the 18th Power Systems Computation Conference (PSCC)*. IEEE, 2014 [99].

4. Luca Merciadri, Sébastien Mathieu, Damien Ernst, and Quentin Louveaux. “Optimal assignment of off-Peak hours to lower curtailments in the distribution network”. In: *Innovative Smart Grid Technologies Europe (ISGT EUROPE), 2014 5th IEEE/PES*. IEEE, 2014 [106].

5. Sébastien Mathieu, Damien Ernst, and Bertrand Cornélusse. “Macroscopic analysis of interaction models for the provision of flexibility in distribution systems”. In: *International Conference on Electricity Distribution, CIRED 2015*. 2015 [97].

6. Sébastien Mathieu, Quentin Louveaux, Damien Ernst, and Bertrand Cornélusse. “DSIMA: A testbed for the quantitative analysis of interaction models within distribution networks”. In: *Sustainable Energy, Grids and Networks* 5 (2016), pp. 78 –93. ISSN: 2352-4677. DOI: 10.1016/j.segan.2015.11.004 [100].

7. Quentin Louveaux, Axel Mathei, and Sébastien Mathieu. “A combinatorial branch-and-bound algorithm for box search”. In: *Intelligent Data Analysis* 20 (2016) [88].

8. Emeline Georges, Bertrand Cornélusse, Damien Ernst, Quentin Louveaux, Vincent Lemort, and Sébastien Mathieu. “Direct control service from residential heat pump aggregation with specified payback”. In: *Proceedings of the 19th Power Systems Computation Conference (PSCC)*. IEEE, 2016 [66].

9. Quentin Louveaux and Sébastien Mathieu. “Electricity markets with flexible consumption as atomic splittable flow congestion games”. In: *Submitted*. 2016 [90].

Chapter 2

Day-ahead energy market and congestion games

This chapter studies the impact of load flexibility on the day-ahead energy market. Electricity retailers are assumed to control their flexible consumption in order to minimize their own energy costs. Shifting their consumption from one hour of the day to another influences the corresponding market prices and consequently their costs. The total cost for all retailers may be far from the system optimum that would be obtained if only one actor would control the whole flexible demand. The purpose of this chapter is to evaluate the implications of having multiple retailers gaming on the market to optimize their personal costs. To this end, the previous problem is assimilated to an atomic splittable flow congestion game with players sending flow in arcs linking a unique source and destination. Some concepts of the game theory literature are therefore directly applicable to this case. This chapter also provides new contributions for games with affine cost functions. We focus on laminar Nash equilibria where the constraints on the minimum and maximal flow that a player must send in a given arc are not binding. We show that the flow sent by a player at a laminar Nash equilibrium does not depend on the demand of other players. In laminar flow, we bound the price of anarchy and the ratio between the maximum and the minimum arc cost. Finally, we propose a simple method based on the property of a laminar Nash equilibrium to compute the price of flexibility, i.e. the price at which energy flexibility should be remunerated in electrical power systems.

2.1 Nomenclature

The index i is used for retailers also referred to in this chapter as players. Indexes t and u are used for periods or arcs. The superscript N is used for a Nash equilibrium solution.

Sets

$\mathcal{K}$	Players
$\mathcal{T}$	Edges

Parameters

a_t	Slope of the affine cost function of arc t
$\alpha_t, \beta, \delta_{t,u}, \sigma_t, \omega$	Parameters dependent of a_t
b_t	Y-intercept of the affine cost function of arc t.
$c_t(x_t)$	Cost function of arc t
D	Total demand
D_i	Demand of player i
Δ_t	Imposed deviation from the Nash equilibrium
γ	Constant dependent of k.
k	Number of players
$[x_{i,t}^{min}, x_{i,t}^{max}]$	Bounds on the flow

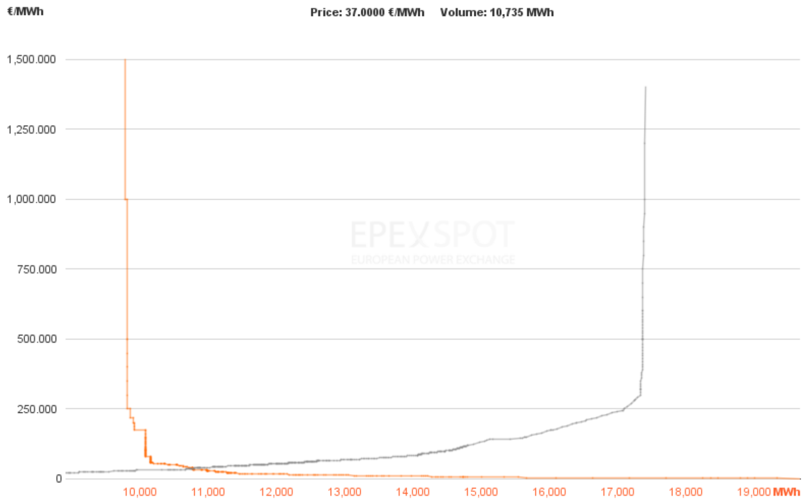
Variables

$\epsilon_{i,t}$	Deviation from the Nash equilibrium
$\kappa_{i,t}, \nu_{i,t}$	Dual variables on the bounds on the flow
λ_i	Dual variable of the constraint on the demand of a player
$\mu_{i,t}$	Dual variable associated with an imposed deviation
x_t	Flow in arc t
$x_{i,t}$	Flow of player i in arc t

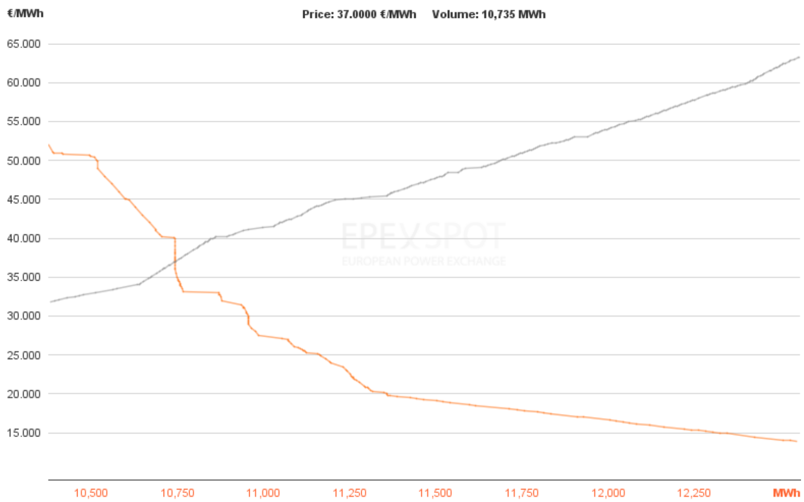
2.2 Introduction

Since electricity flows instantaneously through the network and only few quantities can be stored, electricity production must always be equal to consumption at every moment. To this end, electricity is traded before its delivery. One Part of the trade is conducted years or months ahead in long term contracts while the rest is cleared on energy spot markets. The most common is the day-ahead energy market whose prices are taken as references. This market divides the day into periods, typically twenty-four, and provides a unique price for each of these periods. Participants to these markets submit bids to supply or consume a certain amount of electric energy at a given cost for the period under consideration. The bids are ranked to form the demand and the offer curves. The intersection of the two curves defines the system marginal price. This unique price for each hour is the price paid by every accepted participant whatever the cost they submitted. This scheme gives incentive to the participants to bid at their marginal costs and therefore discourage gaming on the cost they require [82]. Figure 2.1 shows the production and consumption aggregated curves of the French spot market for the first hour of the 1st April 2014 [50]. The intersection of the non-decreasing offer curve with the demand curve leads to the system marginal price of 37€/MWh.

With the ongoing trend for connected electric appliances, electricity retailers now not only retail electricity bought on the energy market but also control flexible consumption. More and more consumers, from retail customers to industrials, accurately monitor their consumption in order to control their final electricity bill. This trend for energy awareness leads to an increasing number of connected electric appliances which may not only be monitored but also controlled by electricity retailers [119]. This additional activity allows retailers to shift the consumption in cheaper hours to decrease their energy procurement cost. The purpose of this chapter is to study an electrical system where each electricity retailer managing flexible consumption chooses independently the total consumption of its portfolio in each hour. The aggregated consumption of all retailers in one period determines a unique market price for this period. Given the prices in each hour, retailers aim at minimizing their own retailing cost. One of our results is to show that this problem can be mapped to an atomic splittable flow congestion game. Therefore, some concepts of the game theory literature are directly applicable to this electrical systemto answer questions such as: *Does the electrical system converge to stable electricity prices? If it is the case, what are these prices, in particular what is the ratio between the maximum and the minimum price? How inefficient is this system with respect to a case where there would be only a single retailer that manages all the demand? Suppose that a power system operator asks a retailer to change its load in order to solve an issue in the electric network, what would be the cost incurred by the retailer?* In the gaming theory literature, the first question



(a) 0h-1h



(b) Zoom on the market clearing price

Figure 2.1: Aggregated curves of the market clearing of the 1st April 2014 [50].

is equivalent to showing that there exists a Nash equilibrium.

Definition 17. *A Nash equilibrium is a strategy profile such that, for each player, its strategy is its best response with the perfect knowledge of the strategy of the others [62].*

The second can be found under the term *unfairness* [31] while the third is obtained by the price of anarchy [85].

In addition to the reduction of energy procurement costs, demand side flexibility can also be used to provide services to other actors of the electrical system. These services may target the relief of a congestion in a line or cover the unexpected loss of a production unit. The first case is related to active network management which is studied extensively in the literature [10, 64, 151].

The second is handled through reserve markets and the methods to provide reserve services by the demand side flexibility has also been broadly investigated [98, 99, 118]. One important unknown in these works and others is the price at which demand side flexibility should be sold. The price of the flexibility from production units is easy, it is proportional to the fuel cost and the unit maintenance and operating cost. The provision of flexibility from production unit is also easier to handle as increasing the production in an hour has barely any impact on what can be done in the following hours. Conversely, changing the consumption of an electric appliance in an hour impacts its consumption in the following one. For instance, a retailer controlling supermarket fridges can interrupt them for one hour. The internal temperature of the fridges increases and consequently the fridges consume more later. The consumption is therefore shifted from one hour to another. Following these thoughts, the cost associated to this shifting is related to the difference of prices between the market prices in the periods where the energy consumption is modified. In this chapter, we provide a simple method to compute the price at which the flexibility of the demand side should be remunerated. This method only depends on public data of the clearing of the day-ahead energy market. The result of the method is supported by its link to the Nash equilibrium of the corresponding congestion game.

The work of this chapter is based on atomic splittable flow congestion games. As a reminder, congestion games are non-cooperative games usually related to traffic problems where each user, or player, tries to find the path to its destination which minimizes its travel time. The travel time is given by a cost function dependent on the total traffic of the path. In the classic congestion game, a player is seen as an infinitesimal quantity of traffic to carry in the network [128]. Nonatomic congestion games are generalization of congestion games where some players, among the infinite amount of them, form coalitions and aim at minimizing the cost of their coalition [108]. One decision taken by the coalition on the strategy space can be discrete, e.g. assigning one user to a given path, or continuous, e.g. assigning some flow to a given arc. Games where the number of players is finite and where these players may split their flow along any number of paths are known as atomic splittable flow games [70, 130]. This chapter focuses on a particular regime of atomic splittable flow congestion games with affine arc cost functions that we name *laminar* flow. This name is motivated by the analogy with the fluid mechanics regime where the flow is organized in layers without interactions. One of our results is that, if the Nash equilibrium of an atomic splittable flow congestion game is laminar, i.e. the demand of all players are within specific bounds, the strategy of each player depends only on its own demand. The motivation for studying laminar flows is that it leads to a clean analysis. Assuming laminar flow, we provide four contributions stated in the following theorems.

Theorem 1. *Consider an atomic splittable flow congestion game with affine* *cost functions. If the Nash equilibrium is laminar then the flow of each player is independent on other players.*

Two byproducts of this theorem are that the flows and arc costs are only dependent on the total demand and that a game can be checked to have a laminar equilibrium only based on the individual total demands of the players.

Theorem 2. *Consider an atomic splittable flow congestion game with affine cost functions and parallel arcs. If the Nash equilibrium is laminar then the flow of each player in each arc is independent from the demand of the other players.*

Two byproducts of this theorem are that the flows and arc costs are only dependent on the total demand and that a game can be checked to have a laminar equilibrium only based on the individual total demands of the players.

Theorem 3. *Consider a k-player atomic splittable flow congestion game with affine cost functions and parallel arcs. If the Nash equilibrium is laminar then the ratio between the maximum and the minimum arc cost is bounded by a constant dependent on k and the y-intercept of the cost functions.*

Theorem 4. *Consider an atomic splittable flow congestion game with parallel arcs. At the laminar Nash equilibrium, the price of flexibility for an imposed small deviation in an arc is at least twice the first derivative of the corresponding arc cost at the Nash equilibrium without deviation times the squared deviation.*

The *price of flexibility* is a concept introduced in this paper which in our power system problem corresponds to the price of shifting the electric consumption from one period to others.

Theorem 5. *The price of anarchy of a k-player atomic splittable flow congestion game with a laminar Nash equilibrium and affine cost functions with positive coefficients is at most*

$$\frac{4k^2}{(k+1)(3k-1)}. \tag{2.1}$$

This upper bound is smaller than the general bound of 1.5 given in [70], but only valid for laminar Nash equilibria.

This chapter is organized as follows. The relevant literature is reviewed in Section 2.3. Section 2.4 sets our notations, describes laminar flows in atomic splittable flow congestion games and links atomic splittable flow congestion games with retailers managing flexible electric consumption. In laminar flow, we obtain a bound on the ratio between the minimum and maximum arc cost in Section 2.5. Section 2.6 details a method to obtain the price of flexibility. A bound on the price of anarchy of atomic splittable flow congestion games with laminar Nash equilibrium is proven in Section 2.7. Finally, Section 2.8 concludes.

2.3 Literature review

Many theoretical results on congestion games can be found in the literature. The existence of equilibria in atomic splittable flow congestion games has been proven by Rosen [127]. The price of anarchy of atomic splittable flow congestion games with affine latency functions and positive coefficients in games with only parallel edges between a single source and a single destination is bounded by 3/2 [70]. The matching lower bound is given in [129]. Since atomic players may also be modeled as nonatomic players forming coalitions, the nonatomic congestion games literature with coalitions is also of interest [25, 28, 29, 61, 71, 144]. Forming coalitions in symmetric nonatomic games reduces the overall costs with respect to the Nash equilibrium [71]. Even the individual costs decrease when the size of the coalition of the individual increases [144]. The price of anarchy is known to be bounded by the minimum between k and a constant dependent on the number of edges [61]. For symmetric and asymmetric congestion games, the price of anarchy can be bounded by a constant dependent on the number of players. The authors show that the price of anarchy is bounded by $\frac{5k+2}{2k+2}$ in the affine case and that this bound is tight. The price of anarchy is bounded in the affine case with symmetric players by $\frac{4k^2}{(k+1)(3k-1)}$ and that this bound is tight [28].

Several power systems problems are related to game theory as attested in the literature survey [60]. Distributed load management in smart grid infrastructure to control the power demand at peak hours using dynamic pricing strategies has been studied as a network congestion game [75]. One close but different problem of flexible consumption management to the one considered in this paper is addressed in [3]. They study the energy transaction between a single retailer and multiple consumers with a total energy constraint. The authors map the problem to an atomic splittable flow game. They prove the existence of a Nash equilibrium using the communication network paper [116].

2.4 Laminar flow in congestion game

Consider k players sending flow in a set of arcs $\mathcal{T}$ with the same source and destination. Such a game is depicted in Figure 2.2. The goal of a player $i \in \mathcal{K}$ is to minimize its total cost by choosing which quantity to send in each arc $t \in \mathcal{T}$, $x_{i,t}$ such that the flow sent is equal to the demand of the player D_i. The total flow is $D = \sum_{i\in\mathcal{K}} D_i$. The aggregated flow in one arc is given by $x_t = \sum_{i\in\mathcal{K}} x_{i,t}$ and define the price in one arc using the cost function $c_t(x_t) : \mathbb{R}_+ \to \mathbb{R}$. Therefore, the prices are independent on the identity of the player. The total cost incurred by player i, $C_i(\mathbf{x}_i)$ is given by

$$C_i(\mathbf{x}_i) = \sum_{t\in\mathcal{T}} c_t(x_t)x_{i,t}. \tag{2.2}$$

where $\mathbf{x}_i = \{x_{i,t} | \forall t \in \mathcal{T}\}$. Note that this cost depends on the actions of the other players through the term x_t. The total system cost, $C(\mathbf{x})$, is only dependent on the aggregated flows

$$C(\mathbf{x}) = \sum_{t \in \mathcal{T}} c_t(x_t) x_t = \sum_{i \in \mathcal{K}} C_i(\mathbf{x}_i) \tag{2.3}$$

where $\mathbf{x} = \{x_t | \forall t \in \mathcal{T}\}$. Note that to one $\mathbf{x}$ can correspond more than one solution of same total system cost in terms of $\mathbf{x}_i$. Bounds on the flow can be

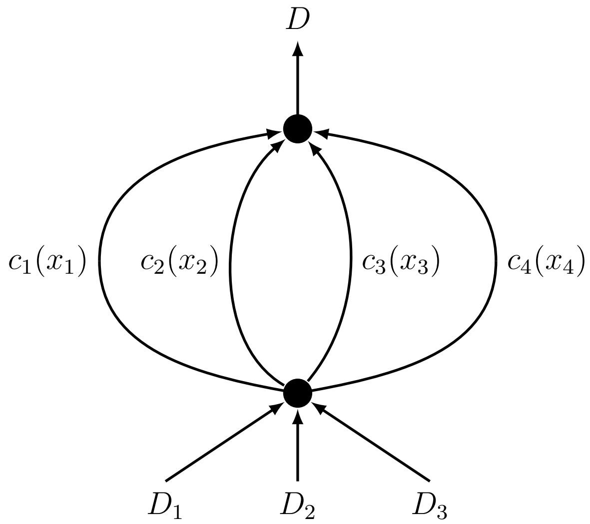


Figure 2.2: Visual representation of an atomic splittable flow congestion game with three players and four arcs.

added to the model as additional constraints:

$$x_{i,t}^{\min} \leq x_{i,t} \leq x_{i,t}^{\max}. \tag{2.4}$$

The classic version of atomic splittable flow congestion games takes $x_{i,t}^{\min} = 0$ and $x_{i,t}^{\max} = +\infty$. This chapter focuses on Nash equilibriums of atomic splittable flow congestions games where this constraint is either non existing or not active.

Definition 18. *The equilibrium of an atomic splittable flow congestion game is laminar if, for each player $i \in \mathcal{K}$ and each arc $t \in \mathcal{T}$, the flow at the equilibrium $x_{i,t}$ is such that*

$$x_{i,t}^{\min} < x_{i,t} < x_{i,t}^{\max}. \tag{2.5}$$

In fluid mechanics, a streamline is an imaginary line with no flow normal to it, only along it. When the flow is laminar, the streamlines are parallel and for flow between two parallel surfaces, we may consider the flow as made up of parallel laminar layers. In a laminar flow, no mixing occurs between adjacent layers [39]. As we see in Theorem 2, if the Nash equilibrium of a congestion game is laminar, the strategy of a player at the equilibrium does not depend on the demand of the others.

The mapping to an atomic splittable flow congestion game is as follows. To each market period t, e.g. each hour, corresponds one arc between a single source and a single destination. The cost function of an arc, c_t is the offer curve of the market at the corresponding period. Note that the offer curve of Figure 2.1b may be well approximated by a linear regression. The outcome of the market is the system marginal price, defined in each period as the cost of the corresponding arc at the Nash Equilibrium, c_t^N and in practice determined by the wholesale market operator. Each retailer i is a player with a total flow equal to the energy needs of its clients, D_i. At the end of the time horizon, D_i is the total energy that must be bought by the retailer. Retailers minimize their own energy procurement cost which is the sum over the periods of the electricity price, c_t^N, times the energy consumed in the corresponding period, $x_{i,t}$. The base load of the retailer is given by $x_{i,t}^{min}$ and its maximum flexibility by $x_{i,t}^{max}$. Note that several modeling assumptions are taken to make this mapping. The practical system is much more complex. For instance, the flexibility of the retailer could be modeled in more details or block bids in the real day-ahead energy market introduce a correlation between periods.

We denote $\mathbf{x}^*$ the optimal flow which minimizes the total cost: $\forall \mathbf{x} \in X, C(\mathbf{x}^*) \leq C(x)$. Note that if there is more than one player, the solution in terms of $\mathbf{x}_i$ is not unique. The Nash equilibrium is denoted by $\mathbf{x}_i^N$ $\forall i \in \mathcal{K}$ and the resulting aggregated flows by $\mathbf{x}^N$. At the Nash equilibrium, no player has any incentive to change its flows given the flows of the others. Note that if there is only one player, $\mathbf{x}^N = \mathbf{x}^*$. To shorten the notation, we define $c_t^N = c_t(x_t^N)$. The system is at a Nash equilibrium $\mathbf{x}_i^N$ if no retailer i can improve its strategy given the strategy of the others. As a result, the strategy $\mathbf{x}_i^N$ is a solution of the following optimization problem.

$$\min_{\mathbf{x}_i} \quad \sum_{t\in\mathcal{T}} c_t(x_t)x_{i,t} \tag{2.6a}$$

$$\text{s.t.} \quad \sum_{t\in\mathcal{T}} x_{i,t} = D_i \qquad : \lambda_i \tag{2.6b}$$

$$x_{i,t} \geq x_{i,t}^{min} \qquad : \kappa_{i,t} \geq 0 \qquad \forall t \in \mathcal{T} \tag{2.6c}$$

$$x_{i,t} \leq x_{i,t}^{max} \qquad : \nu_{i,t} \geq 0 \qquad \forall t \in \mathcal{T} \tag{2.6d}$$

Taking the Karush-Kuhn-Tucker conditions of (2.6) provide the following necessary optimality condition

$$\lambda_i^N + \kappa_{i,t} - \nu_{i,t} = c_t^N + \frac{\partial c_t^N}{\partial x_{i,t}} x_{i,t}^N = c_t^N + \frac{\partial c_t^N}{\partial x_t} x_{i,t}^N \qquad \forall t \in \mathcal{T} \tag{2.7}$$

As the prices do not depend on the identity of the buyer, $\frac{\partial c_t}{\partial x_{i,t}} = \frac{\partial c_t}{\partial x_t}$. By complementarity slackness, either $\kappa_{i,t} = 0$ or $x_{i,t} = x_{i,t}^{min}$ and either $\nu_{i,t} = 0$ or $x_{i,t} = x_{i,t}^{max}$. In most of the chapter, we consider affine cost functions $c_t(x_t) = a_t x_t + b_t$ with $a_t > 0$. The optimality conditions (2.7) are in this case

$$\lambda_i^N + \kappa_{i,t} - \nu_{i,t} = (a_t x_t^N + b_t) + a_t x_{i,t} = 2a_t x_{i,t}^N + a_t \sum_{j \in \mathcal{K}\setminus\{i\}} x_{j,t}^N + b_t \tag{2.8}$$

We now focus on the affine game represented in Figure 2.3 with three players and three arcs. We fix the total demand of the two last players and analyze how the equilibrium changes with respect to the total demand of the first player. The last two players have an identical total demand and therefore play an identical strategy. In the following, we write only the results of player two. The first arc is the most expensive which gives less incentive to the players to send flow in this arc. The bounds on the flow are $x_{i,t} \in [0, +\infty[$ for all players. Computations are performed using the open-source software Maxima [101].

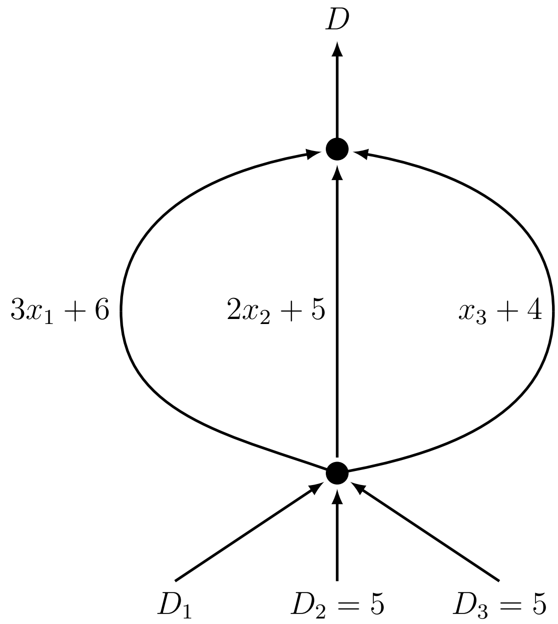


Figure 2.3: Example of affine game.

First, we consider that D_1 is such that the game is laminar, $x_{i,t} > 0 \ \forall i \in \mathcal{K}, t \in \mathcal{T}$. The analytic solution of the Nash equilibrium is

$$\mathbf{x}_1 = \left(\frac{8D_1 - 5}{44}, \frac{6D_1 - 1}{22}, \frac{24D_1 + 7}{44}\right) \tag{2.9}$$

$$\mathbf{x}_2 = (35/44, 29/22, 127/44)\,. \tag{2.10}$$

Note that only the flows of the first player are dependent on D_1 at the laminar Nash equilibrium. This observation is the object of Theorem 2. The cost of the first player is $(24D_1^2 + 444D_1 - 3)/44$ and the one of players 2 and 3 is $(120D_1{+}2217)/44$. The Nash equilibrium of the game is laminar if $D_1 \geq 5/8$. If $D_1 < 5/8$, the flow is not laminar anymore and $x_{1,1} = 0$. The new equilibrium is

$$\mathbf{x}_1 = \left(0, \frac{4D_1 - 1}{12}, \frac{8D_1 + 1}{12}\right) \tag{2.11}$$

$$\mathbf{x}_2 = \left(\frac{2D_1 + 25}{33}, \frac{527 - 8D_1}{396}, \frac{1153 - 16D_1}{396}\right) \tag{2.12}$$

where $\mathbf{x}_2$ now depends on D_1. This equilibrium is valid for $1/4 \leq D_1 \leq 5/8$. The cost of the first player is given by $(928D_1^2 + 15824D_1 - 33)/1584$ and the one of the other players by $(-64D_1^2 + 13280D_1 + 239261)/4752$. Figure 2.4 shows the equilibrium as a function of D_1. The total cost and player's costs are represented in Figure 2.4a. Prices and flows in each arc are respectively given in Figure 2.4b and 2.4c. The individual flows of players one and two in the two first arcs are plotted in Figure 2.4d.

As highlighted by the example, we have the following result for an atomic splittable flow congestion game which equilibrium is laminar:

Theorem 2. *Consider an atomic splittable flow congestion game with affine cost functions. If the Nash equilibrium is laminar then the flow of each player is independent on other players.*

Proof. In the case of affine prices and laminar flow, the equilibrium point of the game can be computed by solving the following system of equations:

$$\sum_{t\in\mathcal{T}} x_{i,t} = D_i \qquad \forall i \in \mathcal{K} \tag{2.13a}$$

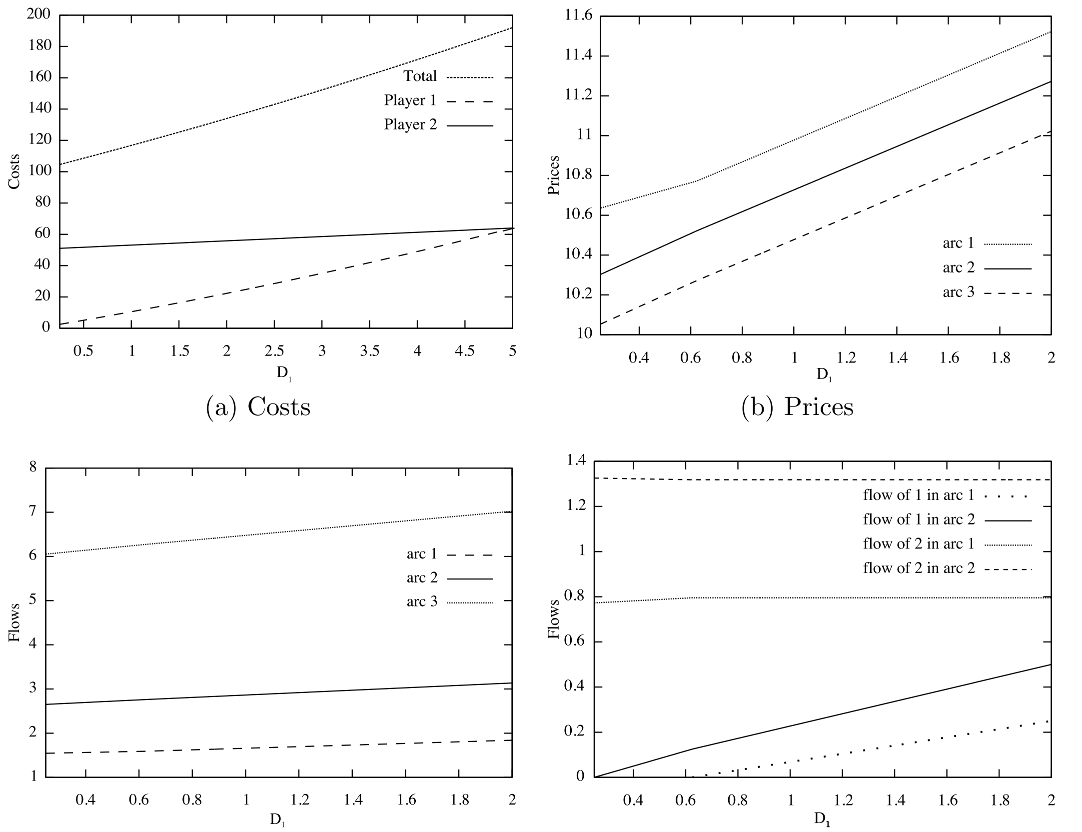
$$2a_t x_{i,t} + a_t \sum_{j\in\mathcal{K}\setminus\{i\}} x_{j,t} - \lambda_i = -b_t \qquad \forall i \in \mathcal{K}, t \in \mathcal{T} \tag{2.13b}$$

We denote this system $Ay = d$ where $y = [\mathbf{x}_1 \dots \mathbf{x}_k \ \lambda_1 \dots \lambda_k]^T$. The sketch of the proof is as follows: we provide the analytical formula of A^{-1}. The inverse is used to obtain $y = A^{-1}d$ which leads to the analytical formula of $x_{i,t}$. We introduce the following convenient notations:

$$\beta = \sum_{t\in\mathcal{T}} \prod_{v\in\mathcal{T}\setminus\{t\}} a_v \tag{2.14a}$$

$$\alpha_t = \frac{\prod_{v\in\mathcal{T}\setminus\{t\}} a_v}{\beta} \tag{2.14b}$$

$$\delta_{t,u} = \frac{\prod_{v\in\mathcal{T}\setminus\{t,u\}} a_v}{\beta(k+1)} = \delta_{u,t} \tag{2.14c}$$



(a) Costs (b) Prices

(c) Flows (d) Player's flows

Figure 2.4: Equilibrium of a three players and three arcs game.

$$\sigma_t = \sum_{u \in \mathcal{T} \setminus \{t\}} \delta_{t,u} \tag{2.14d}$$

Observe that $\beta, \alpha_t, \delta_{t,u}$ and σ_t only depend on k, a_t and b_t. Using the analytical form of A^{-1} provided in the appendix, we obtain

$$x_{i,t} = D_i \alpha_t - b_t \sigma_t - \sum_{u \in \mathcal{T} \setminus \{t\}} b_u \delta_{t,u} \tag{2.15}$$

$$= \frac{D_i (k+1) \prod_{v \in \mathcal{T} \setminus \{t\}} a_v - b_t \sum_{u \in \mathcal{T} \setminus \{t\}} \prod_{v \in \mathcal{T} \setminus \{t,u\}} + \sum_{u \in \mathcal{T} \setminus \{t\}} b_u \prod_{v \in \mathcal{T} \setminus \{t,u\}} a_v}{(k+1) \sum_{t \in \mathcal{T}} \prod_{v \in \mathcal{T} \setminus \{t\}} a_v} \tag{2.16}$$

The complete proof is available in the appendix. □

The motivation for studying laminar flows is that it leads to a clean analysis. The following results are consequences from the previous theorem.

Corollary 1. *Consider an atomic splittable flow congestion game with affine cost functions whose Nash equilibrium is laminar. At this equilibrium, the arc flows and the costs depend only on the total demand.*

Corollary 2. *If each player $i \in \mathcal{K}$ demand D_i is such that, $\forall t \in \mathcal{T}$*

$$x_{i,t}^{\min} < D_i\alpha_t - b_t\sigma_t - \sum_{u\in\mathcal{T}\setminus\{t\}} b_u\delta_{t,u} < x_{i,t}^{\max} \tag{2.17}$$

then the Nash equilibrium of the congestion game is laminar.

2.5 Ratio between the maximum and minimum arc cost

We are now interested in the ratio between the maximum cost of sending flow in one arc with respect to the minimum cost. In the following, we make the hypothesis that the Nash equilibrium of the game is laminar with affine costs functions $c_t(x_t) = a_t x_t + b_t$ and $a_t, b_t \in \mathbb{R}_+$.

The following theorem proves a bound on this ratio depending only on the number of players and the constant terms b_t which can be itself bounded by $\frac{k+1}{k}$.

Theorem 3. *Consider a k-player atomic splittable flow congestion game with affine cost functions of the form $a_t x_t + b_t$ and $a_t, b_t \in \mathbb{R}_+$. If the Nash equilibrium is laminar then the ratio between the maximum and the minimum arc cost, occurring respectively in arcs t and u, is bounded by*

$$\frac{(k+1)b_t}{b_u + kb_t} \leq \frac{k+1}{k} \tag{2.18}$$

Proof of Theorem 3. If the Nash equilibrium is laminar, the equilibrium strategy of player i is obtained by solving the system

$$\sum_{t\in\mathcal{T}} x_{i,t} = D_i : \lambda_i \qquad \forall i \in \mathcal{K} \tag{2.19}$$

$$\lambda_i = 2a_t x_{i,t} + a_t \sum_{j\in\mathcal{K}\setminus\{i\}} x_{j,t} + b_t \qquad \forall t \in \mathcal{T}, i \in \mathcal{K} \tag{2.20}$$

The set of equations given by (2.20) can be written on the form

$$a_t \begin{pmatrix} 2 & 1 & 1 \\ 1 & \ddots & 1 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} x_{1,t} \\ \vdots \\ x_{i,t} \\ \vdots \\ x_{k,t} \end{pmatrix} = \begin{pmatrix} \lambda_1 - b_t \\ \vdots \\ \lambda_i - b_t \\ \vdots \\ \lambda_k - b_t \end{pmatrix} \tag{2.21}$$

which can be concisely written as

$$a_t(\mathbb{1}_k + \mathbb{I}_k)\mathbf{x}_t^K = \lambda^K - b_t \tag{2.22}$$

where $\mathbb{I}_k$ is an identity matrix of dimension k and $\mathbb{1}_k$ a square matrix of ones of dimension k. For $a_t > 0$,

$$\mathbf{x}_t^K = (\mathbb{1}_k + \mathbb{I}_k)^{-1} \frac{\lambda^K - b_t}{a_t} \tag{2.23}$$

$$= \left(\mathbb{I}_k - \frac{1}{k+1} \mathbb{1}_k \right) \frac{\lambda^K - b_t}{a_t} \tag{2.24}$$

using Lemma 5 available in the Appendix. In particular for player i,

$$x_{i,t} = \frac{k}{k+1} \frac{\lambda_i - b_t}{a_t} + \sum_{j \in \mathcal{K} \setminus \{i\}} \frac{-1}{k+1} \frac{\lambda_j - b_t}{a_t} \tag{2.25}$$

$$= \frac{k\lambda_i - \sum_{j \in \mathcal{K}} \lambda_j - b_t}{(k+1)a_t} \tag{2.26}$$

$$x_t = \sum_{i \in \mathcal{K}} x_{i,t} = \frac{\sum_{i \in \mathcal{K}} \lambda_i - kb_t}{(k+1)a_t} \tag{2.27}$$

The sums of the dual variables λ_i can be bounded independently of x_t using (2.27) and $a_t, x_t \geq 0$.

$$\sum_{i \in \mathcal{K}} \lambda_i = (k+1)a_t x_t + kb_t \geq kb_t \tag{2.28}$$

The following observation is used later to bound the ratio.

Observation 1. *Given $a, b, c, d \in \mathbb{R}_+$. If $a \geq c$ and $b \geq d$ then*

$$\frac{a+b}{c+b} \leq \frac{a+d}{c+d}. \tag{2.29}$$

For convenience, we define that the maximum cost is obtained in arc t and the minimum in arc u. The ratio between the maximum and the minimum arc cost in the case where $a_t, a_u > 0$ is given by

$$\frac{\max\{c_t^N | t \in \mathcal{T}\}}{\min\{c_t^N | t \in \mathcal{T}\}} = \frac{c_t^N}{c_u^N} = \frac{b_t + a_t x_t}{b_u + a_u x_u} \tag{2.30}$$

$$= \frac{b_t + \frac{\sum_{i \in \mathcal{K}} \lambda_i - kb_t}{k+1}}{b_u + \frac{\sum_{i \in \mathcal{K}} \lambda_i - kb_u}{k+1}} \tag{2.31}$$

$$= \frac{b_t + \sum_{i \in \mathcal{K}} \lambda_i}{b_u + \sum_{i \in \mathcal{K}} \lambda_i} \tag{2.32}$$

$$\leq \frac{(k+1)b_t}{b_u + kb_t} \leq \frac{k+1}{k} \tag{2.33}$$

where the last inequality is obtained using (2.28) and Observation 1. The previous bound is also valid for the case where $a_t = 0$. The proof is straightforward using (2.27) and that (2.20) simplifies into $\lambda_i = b_t$.

We now focus on the case where $a_u = 0$. In this period, (2.20) simplifies into $\lambda_i = b_u$ and we also have $b_t \leq b_u$ as $c_t^N \geq c_u^N$. The ratio between the maximum and the minimum arc cost can be bounded by

$$\frac{\max\{c_t^N | t \in \mathcal{T}\}}{\min\{c_t^N | t \in \mathcal{T}\}} = \frac{c_t^N}{c_u^N} = \frac{b_t + a_t x_t}{b_u} \tag{2.34}$$

$$= \frac{b_t + \frac{\sum_{i \in \mathcal{K}} \lambda_i - k b_t}{k+1}}{b_u} \tag{2.35}$$

$$= \frac{b_t + \sum_{i \in \mathcal{K}} \lambda_i}{(k+1) b_u} \tag{2.36}$$

$$= \frac{b_t + k b_u}{(k+1) b_u} \leq 1 \tag{2.37}$$

Obviously, if $a_t = a_u = 0$ all the prices are equal. □

Note that this result applies also for symmetric players with the additional constraints $x_{i,t} \geq 0$ by removing the arcs in which $x_{i,t} = 0$.

The example of Section 2.7 taken from [28] and given in Figure 2.5 shows that this bound is tight. At the Nash equilibrium, each player sends the flow $(0, 1)$ resulting in the prices $(1, \frac{k}{k+1})$. Note that the bound of $\frac{k+1}{k}$, without the constants b_u and b_t, can be obtained directly from the marginal costs at the laminar Nash equilibrium. If the equilibrium is laminar, for each player i and any edges u, t, the marginal cost is equal on each edge, and hence $a_u x_{i,u} + c_u^N = a_t x_{i,t} + c_t^N$. Adding for all players gives $(1 + 1/k) a_u x_u + b_u = (1 + 1/k) a_t x_t + b_t$, and hence $c_u^N \leq (1 + 1/k) c_t^N$.

2.6 Price of flexibility

Since the electrical network is not a copper plate, flexibility of the electrical consumption may be required in the electrical system. For instance, the electric distribution network may not be able to handle a large wind power production which therefore needs to be consumed locally. The system operator may request an increase of the consumption to one of the retailer in one period and its up to the retailer to decrease the consumption in other periods to consume the same energy. This consumption shift needs to be paid as a flexibility service by the system operator to a price which reflects the costs of the action. Before defining this price, one should define a reference to quantify the increase or decrease of the consumption. In this chapter, we take as reference the baseline given by the positions of the players at the unperturbed Nash equilibrium.

Definition 19. *The price of flexibility in one arc is the price that reflects the cost of imposing a specified flow deviation in that arc with respect to the unperturbed Nash equilibrium.*

To estimate the price of flexibility, we compute the perturbation of the Nash equilibrium if we impose to one player to change its consumption in one period. For a small perturbation, the values of a laminar Nash equilibrium c_t^N and $\partial_t^N = \frac{\partial c_t(x_t^N)}{\partial x_t} \geq 0$ are taken as data.

Theorem 4. *Consider an atomic splittable flow congestion game with a laminar Nash equilibrium, the price of flexibility for an imposed small deviation in an arc is at least twice the first derivative of the corresponding arc cost at the Nash equilibrium without deviation times the squared deviation.*

Proof of Theorem 4. Let us fix a player $i \in \mathcal{K}$ and a period $u \in \mathcal{T}$. At this equilibrium, player i has no incentive to deviate from the strategy $\mathbf{x}_i^N$. Assume we impose a small deviation Δ_u to player i in a single arc u such that $x_{i,u} = x_{i,u}^N + \Delta_u$. Player i can solve the following optimization problem to modify its strategy:

$$\min_{\mathbf{x}_i} \quad \sum_{t\in\mathcal{T}} (c_t^N + \partial_t^N (x_{i,t} - x_{i,t}^N)) x_{i,t} \tag{2.38a}$$

$$\text{s.t.} \quad \sum_{t\in\mathcal{T}} x_{i,t} = D_i \qquad : \lambda_i \tag{2.38b}$$

$$x_{i,u} = x_{i,u}^N + \Delta_u \qquad : \mu_{i,u} \tag{2.38c}$$

Which can be reformulated, taking the new solution with respect to the Nash equilibrium, by introducing the variables $\epsilon_{i,t}$ such that $x_{i,t} = x_{i,t}^N + \epsilon_{i,t}$. The optimization problem using the variables $\epsilon_{i,t}$ is

$$\min \quad \sum_{t\in\mathcal{T}} \left(\partial_t^N \epsilon_{i,t}^2 + (c_t^N + \partial_t^N x_{i,t}^N)\epsilon_{i,t}\right) \tag{2.39a}$$

$$\text{s.t.} \quad \sum_{t\in\mathcal{T}} \epsilon_{i,t} = 0 \qquad : \lambda_i \tag{2.39b}$$

$$\epsilon_{i,u} = \Delta_u \qquad : \mu_{i,u} \tag{2.39c}$$

Note that this problem is convex as $\partial_t^N \geq 0$. The Lagrangian reads

$$L_{i,u} = \sum_{t\in\mathcal{T}} \left(\partial_t^N \epsilon_{i,t}^2 + (c_t^N + \partial_t^N x_{i,t}^N)\epsilon_{i,t}\right) - \lambda_i \sum_{t\in\mathcal{T}} \epsilon_{i,t} - \mu_{i,u}(\epsilon_{i,u} - \Delta_u). \tag{2.40}$$

Canceling the derivative of the Lagrangian with respect to $\epsilon_{i,u}$ gives

$$\lambda_i + \mu_{i,u} = c_u^N + \partial_u^N x_{i,u}^N + 2\partial_u^N \Delta_u \tag{2.41}$$

$$= \lambda_i^N + 2\partial_u^N \Delta_u \tag{2.42}$$

Canceling the derivative of the Lagrangian with respect to $\epsilon_{i,t}$ with $t \neq u$ gives

$$\lambda_i = c_t^N + \partial_t^N x_{i,t}^N + 2\partial_t^N \epsilon_{i,t} \tag{2.43}$$

$$= \lambda_i^N + 2\partial_t^N \epsilon_{i,t}. \tag{2.44}$$

Lemma 1. *Assume that arc costs are fixed at the values of a laminar Nash equilibrium with* c_t^N *and* $\partial_t^N = \frac{\partial c_t(x_t^N)}{\partial x_t} \geq 0$ *are taken as data. Assume an imposed deviation* Δ_u *in period* u *and writes the deviation in other periods* t, $\epsilon_{i,t}$*. Then* $\epsilon_{i,t}$ *is of opposite sign that* Δ_u *and such that* $|\epsilon_{i,t}| \leq \Delta_u,\ \forall t \in \mathcal{T} \setminus \{u\}$.

Proof of Lemma 1. Taking equation (2.44) for two arcs $t, v \in \mathcal{T} \setminus \{u\}$ yields $\partial_t^N \epsilon_{i,t} = \partial_v^N \epsilon_{i,v}$. As $\partial_t^N, \partial_v^N \geq 0$, $\epsilon_{i,t}$ and $\epsilon_{i,v}$ have the same sign. Using (2.39c) in (2.39b) gives $\sum_{t \in \mathcal{T} \setminus \{u\}} \epsilon_{i,t} = -\Delta_u$ which implies the lemma. □

Using Lemma 1 and (2.44) yields

$$\lambda_i \in \left[\lambda_i^N - 2\Delta_u \max_{t \in \mathcal{T} \setminus \{u\}} \partial_t^N, \lambda_i^N\right] \tag{2.45}$$

Injecting the latter in (2.42) yields

$$\mu_{i,u} \in \left[2\Delta_u \partial_u^N, 2\Delta_u \left(\partial_u^N + \max_{t \in \mathcal{T} \setminus \{u\}} \partial_t^N\right)\right] \tag{2.46}$$

which provides bounds on the flexibility price for each period u. Note that the dual variable $\mu_{i,u}$ is not directly dependent on the player's i data. Therefore, $\mu_{i,u}$ defines a single flexibility price in each arc u for any player i. Note also that the minimum bound is only dependent on the period under consideration. To match with the needs of simplicity of real life applications, we advise to take this minimum bound as the reference flexibility price. Note that in (2.46), $\mu_{i,u}$ depends on Δ_u and therefore the cost of the flexibility service, which is given by $\mu_{i,u}\Delta_u$ depends on the square of Δ_u. □

For instance, based on the market clearing of Figure 2.1b the price of flexibility for this period would be of 5.478 €cent/MWh^2.

2.7 Price of anarchy

This section provides a bound on the price of anarchy for an atomic splittable flow congestion game with laminar Nash equilibrium and affine cost functions $c_t(x_t) = a_t x_t + b_t$ and $a_t, b_t > 0$.

Definition 20. *The price of anarchy is the ratio between the worst Nash equilibrium and the overall optimum solution [85], such that*

$$\frac{C(x^N)}{C(\mathbf{x}^*)} \leq \textit{price of anarchy} \tag{2.47}$$

for all congestion game.

The following lemma provides necessary and sufficient conditions on the laminar Nash equilibrium.

Lemma 2. *In laminar flow, the quantities* $\mathbf{x}^N$ *are at Nash equilibrium if and only if,* $\forall t, u \in \mathcal{T}$,

$$\frac{k+1}{k} a_t x_t^N + b_t = \frac{k+1}{k} a_u x_u^N + b_u \tag{2.48}$$

where the last constraint is given by (2.7) in the affine case.

Proof of Lemma 2. Applying the optimality conditions (2.7) to the case of laminar flow and affine cost functions yields,

$$\lambda_i = a_t x_{i,t} + a_t x_t + b_t. \tag{2.49}$$

Summing over the players and dividing by k gives

$$1/k \sum_{i \in \mathcal{K}} \lambda_i = \frac{k+1}{k} a_t x_t^N + b_t \tag{2.50}$$

As the right member is independent on t, (2.50) may be applied to particular arcs t and u to obtain (2.48). □

A special case of Lemma 2 worth to be highlighted.

Corollary 3. *The optimal flows* x^* *in laminar flow with affine cost functions satisfies the following condition:* $\forall t, u \in \mathcal{T}$,

$$2 a_t x_t^* + b_t = 2 a_u x_u^* + b_u \tag{2.51}$$

The optimal flow corresponds to the case $k = 1$. Note that in the case $k = +\infty$, the cost of each arc is equal at Nash equilibrium.

The following of the proof follows the same steps as in [130].

Lemma 3. *Note* $\mathbf{x}^N$ *the Nash equilibrium of a* k *players game in laminar flows and affine cost functions with a total flow of* D*. The flow* $\gamma \mathbf{x}^N$ *is optimal for the same game with a total flow of* γD *where* $\gamma = \frac{k+1}{2k}$.

Proof of Lemma 3. As $\mathbf{x}^N$ satisfies equation (2.48), the demand allocation γx^N satisfies equation (2.51). □

The following lemma is taken from [130] and adapted to our notations.

Lemma 4. *Suppose an instance of a total flow of* D *for which* $\mathbf{x}^*$ *is an optimal flow. Let* $l_t(x_t)$ *be the minimum marginal cost of increasing the flow in arc* t *with respect to* x_t*. Then, for any* $\delta \geq 0$*, a feasible flow for the same instance with of total flow* $(1+\delta)D$ *has cost at least*

$$C(\mathbf{x}^*) + \delta \sum_{t \in \mathcal{T}} l_t(x_t^*) x_t^* \tag{2.52}$$

Proof of Lemma 4. See Lemma 4.4 of article [130]. □

The main result can be obtained using the previous lemmas.

Theorem 5. *The price of anarchy of a k-players atomic splittable flow congestion game with a laminar Nash equilibrium and affine cost functions with positive coefficients is at most*

$$\frac{4k^2}{(k+1)(3k-1)}. \tag{2.53}$$

Proof of Theorem 5. The laminar Nash equilibrium flow $\mathbf{x}^N$ for the game of total demand D is such that the flow $\gamma\mathbf{x}^N$ with $\gamma = \frac{k+1}{2k}$ is optimal for the same game with a total demand of γD. The cost of the optimal flow $\mathbf{x}^*$ can be bounded with respect to $\gamma\mathbf{x}^N$ using Lemma 4:

$$\begin{aligned}
C(\mathbf{x}^*) &\geq C(\gamma\mathbf{x}^N) + \frac{1-\gamma}{\gamma}\sum_{t\in\mathcal{T}} l_t(\gamma x_t^N)\gamma x_t^N && (2.54)\\
&= \sum_{t\in\mathcal{T}} \left(a_t\gamma^2(x_t^N)^2 + b_t\gamma x_t^N\right) + (1-\gamma)\sum_{t\in\mathcal{T}}(2a_t\gamma x_t^N + b_t)x_t^N && (2.55)\\
&= \sum_{t\in\mathcal{T}}[a_t\left(\frac{k+1}{2k}\right)^2(x_t^N)^2 + b_t\frac{k+1}{2k}x_t^N\\
&\qquad + \frac{k-1}{2k}\left(2a_t\frac{k+1}{2k}(x_t^N)^2 + b_t x_t^N\right)] && (2.56)\\
4k^2C(\mathbf{x}^*) &\geq \sum_{t\in\mathcal{T}}\left[\left(3k^2+2k-1\right)a_t(x_t^N)^2 + 4k^2b_tx_t^N\right] && (2.57)\\
4k^2C(\mathbf{x}^*) &\geq (3k^2+2k-1)C(x^N) = (k+1)(3k-1)C(x^N) && (2.58)
\end{aligned}$$

where the transition from (2.57) to (2.58) is given by $4k^2 \geq 3k^2+2k-1$ for $k \in [1, +\infty[$. □

Note that for $k = 1$, we get that $C(\mathbf{x}^*) \leq C(\mathbf{x}^N)$ and for $k \to \infty$ the result tends to the price of anarchy of 4/3 found in [130]. In a two player game system with affine prices and positive coefficients, the price of anarchy is at most 16/15. Figure 2.5 shows an example taken from [28] of k players controlling a demand of 1 where the bound on the price of anarchy is tight. The optimum flow is $(\frac{k-1}{2}, \frac{k+1}{2})$ with a total cost of $\frac{3k-1}{4}$. At the Nash equilibrium, each player games $(0,1)$ resulting in the prices $(1, \frac{k}{k+1})$ and a total cost of $\frac{k^2}{k+1}$. Note that this equilibrium is not laminar since the flow in arc 1 is 0. However, on can slightly modify the game taking $c_1(x) = 1+\epsilon$ and $c_2(x) = \frac{x}{k+1}+\epsilon$ with $\epsilon > 0$ such that every coefficients defining the costs are strictly positive. At the Nash equilibrium of this new game, each player games $(\frac{\epsilon}{\epsilon(k+1)+1}, 1-\frac{\epsilon}{\epsilon(k+1)+1})$ which for ϵ tending to 0, tends to the solution $(0,1)$ while staying laminar.

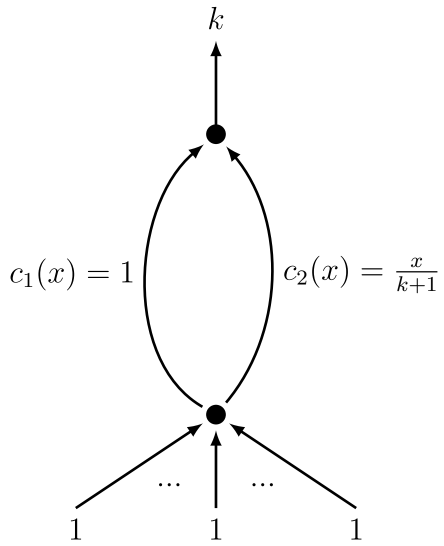


Figure 2.5: Example of an atomic splittable flow congestion game for which the bound on the price of anarchy and on the ratio between the maximum and minimum arc cost is tight.

2.8 Conclusion

This chapter studies a system where electricity retailers control flexible consumption in order to minimize their own energy costs. By shifting their consumption from one hour of the day to another, retailers influence the corresponding market prices. This system can be seen as an atomic splittable flow congestion game with a network composed of a single source and destination linked by parallel arcs corresponding to each market period. Aside from this mapping, this chapter provides new contributions for games with affine cost functions. We focus on laminar Nash equilibrium where the constraints on the minimum and maximal flow that a player must send in a given arc are not binding. We show that the flow sent by a player at a laminar Nash equilibrium does not depend on the demand of other players. In laminar flow, we bound the price of anarchy and the ratio between the maximum and the minimum arc cost. Finally, we propose a simple method based on the property of a laminar Nash equilibrium to compute the price of flexibility to which energy flexibility should be remunerated in electrical power systems.

The results obtained in this chapter suppose that the equilibrium of the game is laminar. Future research could try to obtain similar results relaxing this hypothesis. Other costs functions could be investigated: piece-wise linear, quadratic, etc. One may be interested in analyzing the simultaneous gaming of the producers along with the one of the retailers. Finally, more complexities of the real power system could be considered leading to a more complex game theory problem.

2.9 Appendix

Lemma 5. *The inverse of* $\mathbb{1}_k + \mathbb{I}_k$*, where* $\mathbb{I}_k$ *is an identity matrix of dimension* k *and* $\mathbb{1}_k$ *a square matrix of ones of dimension* k*, is* $\mathbb{I}_k - \frac{1}{k+1}\mathbb{1}_k$.

Proof. The proof is obtained by showing than multiplying $\mathbb{1}_k + \mathbb{I}_k$ by the candidate inverse yields the identity matrix.

$$(\mathbb{1}_k + \mathbb{I}_k)(\mathbb{I}_k - \frac{1}{k+1}\mathbb{1}_k) = \mathbb{1}_k + \mathbb{I}_k - \frac{1}{k+1}\mathbb{1}_k\mathbb{1}_k - \frac{1}{k+1}\mathbb{1}_k \tag{2.59}$$

$$= \mathbb{I}_k + \mathbb{1}_k(1 - \frac{k}{k+1} - \frac{1}{k+1}) \tag{2.60}$$

$$= \mathbb{I}_k \tag{2.61}$$

where (2.60) is obtained using the fact that $\mathbb{1}_k\mathbb{1}_k = k\mathbb{1}_k$. □

Theorem 1. *Consider an atomic splittable flow congestion game with affine cost functions. If the Nash equilibrium is laminar then the flow of each player is independent on other players.*

Proof of Theorem 2. In the affine case where $x_{i,t} > 0 \ \forall i \in \mathcal{K}, t \in \mathcal{T}$, the laminar equilibrium point can be computed by solving the following system of equations:

$$\sum_{t\in\mathcal{T}} x_{i,t} = D_i \qquad \forall i \in \mathcal{K} \tag{2.62a}$$

$$2a_t x_{i,t} + a_t \sum_{j\in\mathcal{K}\setminus\{i\}} x_{j,t} - \lambda_i = -b_t \qquad \forall i \in \mathcal{K}, t \in \mathcal{T} \tag{2.62b}$$

We solve this linear system (2.62) of the form $Ay = d$ where

$$y = \left(\mathbf{x}_1^T, \dots, \mathbf{x}_k^T, \lambda_1, \dots, \lambda_k\right)^T \tag{2.63}$$

$$d = \left(D_1, \dots, D_k, -b_1, \dots, -b_T, \dots, -b_1, \dots, -b_T\right)^T. \tag{2.64}$$

As an illustration, we provide an example of (2.62) with three arcs and two players. We have

$$d = \left(D_1\, D_2 - b_1 - b_2 - b_3 - b_1 - b_2 - b_3\right)^T \tag{2.65}$$

$$A = \left(\begin{array}{ccc|ccc|cc} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ \hline 2a_1 & 0 & 0 & a_1 & 0 & 0 & -1 & 0 \\ 0 & 2a_2 & 0 & 0 & a_2 & 0 & -1 & 0 \\ 0 & 0 & 2a_3 & 0 & 0 & a_3 & -1 & 0 \\ \hline a_1 & 0 & 0 & 2a_1 & 0 & 0 & 0 & -1 \\ 0 & a_2 & 0 & 0 & 2a_2 & 0 & 0 & -1 \\ 0 & 0 & a_3 & 0 & 0 & 2a_3 & 0 & -1 \end{array}\right). \tag{2.66}$$

The horizontal line delimits the constraints (2.62a) which corresponds to the line indexes $m \leq k$. We define

$$\beta = \sum_{t \in \mathcal{T}} \prod_{v \in \mathcal{T} \setminus \{t\}} a_v \tag{2.67a}$$

$$\alpha_t = \frac{\prod_{v \in \mathcal{T} \setminus \{t\}} a_v}{\beta} \tag{2.67b}$$

$$\delta_{t,u} = \frac{\prod_{v \in \mathcal{T} \setminus \{t,u\}} a_v}{\beta(k+1)} = \delta_{u,t} \tag{2.67c}$$

$$\sigma_t = \sum_{u \in \mathcal{T} \setminus \{t\}} \delta_{t,u} \tag{2.67d}$$

$$\omega = \frac{\prod_{t \in \mathcal{T}} a_t}{\beta}. \tag{2.67e}$$

We claim that the inverse of the matrix A defined in (2.66) is given by

$$B = \left(\begin{array}{cc|ccc:ccc}
\alpha_1 & 0 & 2\sigma_1 & -2\delta_{1,2} & -2\delta_{1,3} & -\sigma_1 & \delta_{1,2} & \delta_{1,3} \\
\alpha_2 & 0 & -2\delta_{2,1} & 2\sigma_2 & -2\delta_{2,3} & \delta_{2,1} & -\sigma_2 & \delta_{2,3} \\
\alpha_3 & 0 & -2\delta_{3,1} & -2\delta_{3,2} & 2\sigma_3 & \delta_{3,1} & \delta_{3,2} & -\sigma_3 \\
\hdashline
0 & \alpha_1 & -\sigma_1 & \delta_{1,2} & \delta_{1,3} & 2\sigma_1 & -2\delta_{1,2} & -2\delta_{1,3} \\
0 & \alpha_2 & \delta_{2,1} & -\sigma_2 & \delta_{2,3} & -2\delta_{2,1} & 2\sigma_2 & -2\delta_{2,3} \\
0 & \alpha_3 & \delta_{3,1} & \delta_{3,2} & -\sigma_3 & -2\delta_{3,1} & -2\delta_{3,2} & 2\sigma_3 \\
\hdashline
2\omega & \omega & -\alpha_1 & -\alpha_2 & -\alpha_3 & 0 & 0 & 0 \\
\omega & 2\omega & 0 & 0 & 0 & -\alpha_1 & -\alpha_2 & -\alpha_3
\end{array}\right). \tag{2.68}$$

The vertical line delimits column indexes $n \leq k$. The analytical solution for $x_{i,t}$ can be obtained by taking the corresponding element of Bd. For instance, we have for the first player in the second arc

$$x_{1,2} = \alpha_2 D_1 + 2\delta_{2,1} b_1 - 2\sigma_2 b_2 + 2\delta_{2,3} b_3 - \delta_{2,1} b_1 + \sigma_2 b_2 - \delta_{2,3} b_3 \tag{2.69}$$

$$= \frac{3D_1 a_1 a_3 - b_2(a_1 + a_3) + b_1 a_3 + b_3 a_1}{3(a_1 a_2 + a_1 a_3 + a_2 a_3)}. \tag{2.70}$$

We now consider the general case of k players and T arcs and derive a complete description of A. Let us fix a row index m and a column index n. We define for $m > k$, indexes dependent on m

$$i(m) = \lfloor (m - 1 - k)/T \rfloor + 1 \tag{2.71}$$

$$t(m) = (m - 1 - k) \bmod T + 1 \tag{2.72}$$

which for the sake of conciseness are denoted i and t. Observe that i represents the player corresponding to the choice of the row m and t corresponds to the period. In the following, m is a row index and n a column index. The element

(m, n) of a matrix A is denoted $A(m, n)$, its m^{th} row $A(m, :)$ and its n^{th} column $A(:, n)$. The non-zero elements of A are

$$A(m, n) = 1 \qquad \forall m \le k, n \in \{T(i-1)+1, Ti\} \tag{2.73a}$$
$$A(m, (i-1)T + t) = 2a_t \qquad \forall m > k \tag{2.73b}$$
$$A(m, (l-1)T + t) = a_t \qquad \forall m > k, l \in \mathcal{K} \setminus \{i\} \tag{2.73c}$$
$$A(m, kT + i - 1) = -1 \qquad \forall m > k \tag{2.73d}$$

In order to define the elements of B, the candidate inverse matrix, we need two further sets of indices for columns $n > k$:

$$j(n) = \lfloor (n-1-k)/T \rfloor + 1 \tag{2.74}$$
$$u(n) = (n-1-k) \bmod T + 1 \tag{2.75}$$

which for the sake of conciseness are denoted j and u. Observe that j represents the player corresponding to the choice of the column n and u corresponds to the period. We define

$$B(m, n) = \alpha_{(m-1) \bmod T+1} \qquad \forall n \le k, \lfloor (m-1)/T \rfloor + 1 = n \tag{2.76a}$$
$$B(m, n) = 0 \qquad \forall n \le k, m \le kT : \lfloor (m-1)/T \rfloor + 1 \ne n \tag{2.76b}$$
$$B(m, n) = 2\omega \qquad \forall n \le k, m : m - kT = n \tag{2.76c}$$
$$B(m, n) = \omega \qquad \forall n \le k, m > kT : m - kT \ne n \tag{2.76d}$$
$$B(m, n) = k\gamma_u \qquad \forall n > k, m \le kT : i = j, t = u \tag{2.76e}$$
$$B(m, n) = -\gamma_u \qquad \forall n > k, m \le kT : i \ne j, t = u \tag{2.76f}$$
$$B(m, n) = -k\delta_{t,u} \qquad \forall n > k, m \le kT : i = j \tag{2.76g}$$
$$B(m, n) = \delta_{t,u} \qquad \forall n > k, m \le kT : i \ne j \tag{2.76h}$$
$$B(m, n) = -\alpha_u \qquad \forall n > k, m : m - kT = j \tag{2.76i}$$
$$B(m, n) = 0 \qquad \forall n > k, m > kT : m - kT \ne j \tag{2.76j}$$

We claim that B is the inverse of A. To prove this claim, we perform the inner product of rows of A with columns of B and show that we obtain the element of an identity matrix. The reader is advised to use the matrices of the example given in (2.66) and (2.68) as support.

$\mathbf{m = n \le k}$**:** In the example, this case corresponds to the inner product of row 1 of (2.66) and column 1 of (2.68).

$$A(m, :)B(:, n) = \sum_{v \in \mathcal{T}} (2.73a)(2.76a) \tag{2.77}$$
$$= \sum_{v \in \mathcal{T}} \alpha_v = 1 \tag{2.78}$$

$\mathbf{m = n \leq k}$**:** In the example, this case corresponds to the inner product of row 1 of (2.66) and column 1 of (2.68).

$$A(m,:)B(:,n) = \sum_{v\in\mathcal{T}} (2.73\text{a})(2.76\text{a}) \tag{2.79}$$

$$= \sum_{v\in\mathcal{T}} \alpha_v = 1 \tag{2.80}$$

$\mathbf{m, n \leq k, m \neq n}$**:** This case corresponds to the inner product of row 1 of (2.66) and column 2 of (2.68).

$$A(m,:)B(:,n) = \sum_{v\in\mathcal{T}} (2.73\text{a})(2.76\text{b}) = 0 \tag{2.81}$$

$\mathbf{m \leq k, n > k}$**:**

- $m = j$: This case corresponds to the inner product of row 1 of (2.66) and column 3 of (2.68) with $j = 1$ and $u = 1$.

$$A(m,:)B(:,n) = (2.73\text{a})(2.76\text{e}) + \sum_{v\in\mathcal{T}\setminus\{u\}} (2.73\text{a})(2.76\text{g}) \tag{2.82}$$

$$= k\gamma_u - \sum_{v\in\mathcal{T}\setminus\{u\}} k\delta_{u,v} = 0 \tag{2.83}$$

- $m \neq j$: This case corresponds to the inner product of row 1 of (2.66) and column 6 of (2.68) with $j = 2$ and $u = 1$.

$$A(m,:)B(:,n) = (2.73\text{a})(2.76\text{e}) + \sum_{v\in\mathcal{T}\setminus\{u\}} (2.73\text{a})(2.76\text{h}) \tag{2.84}$$

$$= -\gamma_u + \sum_{v\in\mathcal{T}\setminus\{u\}} \delta_{u,v} = 0 \tag{2.85}$$

$\mathbf{m > k, n \leq k}$**:**

- $n = i$: This case corresponds to the inner product of row 3 of (2.66) and column 1 of (2.68) with $i = 1$ and $t = 1$.

$$A(m,:)B(:,n) = (2.73\text{b})(2.76\text{a}) + \sum_{l\in\mathcal{K}\setminus\{i\}} (2.73\text{c})(2.76\text{b}) + (2.73\text{d})(2.76\text{c}) \tag{2.86}$$

$$= 2a_t\alpha_t + 0 - 2\omega = 0 \tag{2.87}$$

- $n \neq i$: This case corresponds to the inner product of row 3 of (2.66) and column 2 of (2.68) with $i = 1$ and $t = 1$.

$$A(m,:)B(:,n) = (2.73b)(2.76b) + (2.73c)(2.76a) + \sum_{l \in \mathcal{K} \setminus \{i,j\}} (2.73c)(2.76b) + (2.73d)(2.76d) \quad (2.88)$$

$$= 0 + a_t \alpha_t + 0 - \omega = 0 \quad (2.89)$$

as $a_t \alpha_t = \omega$.

$\mathbf{m = n > k}$**:** This case corresponds to the inner product of row 3 of (2.66) and column 3 of (2.68) with $i = j = 1$ and $t = u = 1$. Note that $a_t \delta_{t,u} = \frac{\alpha_u}{k+1}$ and consequently $a_t \sigma_t = \frac{\sum_{u \in \mathcal{T} \setminus \{t\}} \alpha_u}{k+1}$. We have,

$$A(m,:)B(:,n) = (2.73b)(2.76e) + \sum_{l \in \mathcal{K} \setminus \{i\}} (2.73c)(2.76f) + (2.73d)(2.76i) \quad (2.90)$$

$$= 2a_t k \sigma_t - \sum_{l \in \mathcal{K} \setminus \{i\}} a_t \sigma_t + \alpha_t \quad (2.91)$$

$$= (k+1) a_t \sigma_t + \alpha_t \quad (2.92)$$

$$= \sum_{u \in \mathcal{T}} \alpha_u = 1 \quad (2.93)$$

$\mathbf{m, n > k, m \neq n}$**:**

- $t = u$ and $i \neq j$: This case corresponds to the inner product of row 1 of (2.66) and column 6 of (2.68) with $i = 1$, $j = 2$ and $t = u = 1$.

$$A(m,:)B(:,n) = (2.73b)(2.76f) + (2.73c)(2.76e) + \sum_{l \in \mathcal{K} \setminus \{i,j\}} (2.73c)(2.76f) + (2.73d)(2.76j) \quad (2.94)$$

$$= -2a_t \sigma_t + a_t k \sigma_t - \sum_{l \in \mathcal{K} \setminus \{i,j\}} a_t \sigma_t + 0 = 0 \quad (2.95)$$

- $t \neq u$ and $i = j$: This case corresponds to the inner product of row 1 of (2.66) and column 4 of (2.68) with $i = j = 1$, $t = 1$ and $u = 2$.

$$A(m,:)B(:,n) = (2.73b)(2.76g) + \sum_{l \in \mathcal{K} \setminus \{i\}} (2.73c)(2.76h) + (2.73d)(2.76i) \quad (2.96)$$

$$= -2a_t k \delta_{t,u} + \sum_{l \in \mathcal{K} \setminus \{i\}} a_t \delta_{t,u} + \alpha_u \quad (2.97)$$

$$= -(k+1) a_t \delta_{t,u} + \alpha_u = 0 \quad (2.98)$$

- $t \neq u$ and $i \neq j$: This case corresponds to the inner product of row 1 of (2.66) and column 7 of (2.68) with $i = 1$, $j = 2$, $t = 1$ and $u = 2$.

$$\begin{aligned} A(m,:)B(:,n) &= (2.73b)(2.76h) + (2.73c)(2.76g) + \sum_{l \in \mathcal{K} \setminus \{i,j\}} (2.73c)(2.76h) \\ &\quad + (2.73d)(2.76j) && (2.99) \\ &= 2a_t \delta_{t,u} - a_t k \delta_{t,u} - \sum_{l \in \mathcal{K} \setminus \{i,j\}} a_t \delta_{t,u} + 0 = 0 && (2.100) \end{aligned}$$

The analytical form of $x_{i,t}$ is obtained by taking the corresponding row of Bd and therefore

$$\begin{aligned} x_{i,t} &= D_i \alpha_t - b_t \sigma_t - \sum_{u \in \mathcal{T} \setminus \{t\}} b_u \delta_{t,u} && (2.101) \\ &= \frac{D_i(k+1) \prod_{v \in \mathcal{T} \setminus \{t\}} a_v - b_t \sum_{u \in \mathcal{T} \setminus \{t\}} \prod_{v \in \mathcal{T} \setminus \{t,u\}} + \sum_{u \in \mathcal{T} \setminus \{t\}} b_u \prod_{v \in \mathcal{T} \setminus \{t,u\}} a_v}{\beta(k+1)} && (2.102) \end{aligned}$$

which is not dependent on the demand of other players than i. □

Chapter 3

Load flexibility in the reserve market

This chapter presents and analyzes a day-ahead reserve market model that handles bids from flexible loads. This pool market model takes into account the fact that a load modulation in one direction must usually be compensated later by a modulation of the same magnitude in the opposite direction. Our analysis takes into account the gaming possibilities of producers and retailers, controlling load flexibility, in the day-ahead energy and secondary reserve market, and in imbalance settlement. This analysis is carried out by an agent-based approach where, for every round, each actor uses linear programs to maximize its profit according to forecasts of the prices. The procurement of a reserve is assumed to be determined, for each period, as a fixed percentage of the total consumption cleared in the energy market for the same period. The results show that the provision of reserves by flexible loads has a negligible impact on the energy market prices but markedly decreases the cost of reserve procurement. However, as the rate of flexible loads increases, the system operator has to rely more and more on non-contracted reserves, which may cancel out the benefits made in the procurement of reserves.

3.1 Nomenclature

The indexes a,t, i and j are respectively used for agents, time periods, flexibility bids and flexible loads or generation units. A forecast of x is denoted $\hat{x}$.

Sets

$\mathcal{F}$	Modulation reserve bids
$\mathcal{F}_a$	Modulation reserve bids of retailer a
$\mathcal{B}$	Reserve Bids : $\mathcal{B} = \mathcal{F} \cup_{t=1}^{T} (\mathcal{S}_t^+ \cup \mathcal{S}_t^-)$
$\mathcal{G}_a$	Generation units of producer a
$\mathcal{N}_i$	Periods covered by the modulation bid $i \in \mathcal{F}$
$\mathcal{S}_t$	Classical reserve bids in period t
$\mathcal{T}$	Periods

Parameters

c_i^E	Cost of activation of reserve bid $i \in \mathcal{B}$
c_t^o	Fictive cost penalizing reserve contracted over the requirements
Δt	Duration of a time period
γ_t	Fictive price to incentivize producers to provide reserve
$\mathcal{M}_a$	Set of flexible load of retailer a
$\nu_{a,t}$	Inelastic demand of retailer a in period t
π^{cap}	Maximum energy market clearing price
π^F	Regulated capacity price of modulation bids
π^L	Regulated capacity price of classical downward reserve bids
π^{nc}	Cost of activating non-contracted reserve
π^U	Regulated capacity price of classical upward reserve bids
Q_i	Volume of reserve bid $i \in \mathcal{B}$
R_t^-	Quantity of downward reserve to contract in period t
R_t^+	Quantity of upward reserve to contract in period t
T	Number of periods per day
ξ	Efficiency factor of modulation bids wrt. classical bids
ζ_i	Efficiency ratio of reserve bid $i \in \mathcal{B}$

Variables

$d_{j,t}$	Consumption of flexible load i in period t
$D_{a,t}$	Total demand of retailer a in period t
$D_{a,t}^{max}$	Threshold demand in period t
$I_{a,t}$	Imbalance of agent a in period t
I_t	Imbalance of the system
$L_{a,t}$	Downward reserve proposed by producer a in period t
s_t^+	Over-contracted upward reserve: $s_t^+ \geq 0$

s_t^-	Over-contracted downward reserve: $s_t^- \geq 0$
n_t^+	Non-contracted upward reserve: $s_t^+ \geq 0$
n_t^-	Non-contracted downward reserve: $s_t^- \geq 0$
x_i	Activation of a bid $i \in \mathcal{S}^+{}_t \cup \mathcal{S}^-{}_t,\ \forall t$: $x_i \in [0,1]$
$U_{a,t}$	Upward reserve proposed by producer a in period t
$v_{i,t}$	Activation upward of a modulation bid $i \in \mathcal{F}$ in period $t \in \mathcal{N}_i$: $v_{i,t} \in [0,1]$
$w_{i,t}$	Activation downward of a modulation bid $i \in \mathcal{F}$ in period $t \in \mathcal{N}_i$: $w_{i,t} \in [0,1]$
y_t^+	Non-contracted upward reserve: $y_t^+ \geq 0$
y_t^-	Non-contracted downward reserve: $y_t^- \geq 0$
x_i	Determine if a bid $i \in \mathcal{B}$ is accepted totally $(= 1)$, partially $(\in]0,1[)$ or rejected $(= 0)$

3.2 Introduction

In the previous chapter, flexibility is used solely to reduce the energy procurement cost by shifting the consumption in the hours with the cheapest market prices. To recover the cost of turning loads into smart appliances [16], an aggregator may need more than the benefits it could obtain from the day-ahead energy market. One business opportunity for the aggregator is to participate to the secondary reserve market. As a reminder, the secondary reserve is an ancillary service coordinated by the Transmission System Operators (TSO) to ensure the equality between production and consumption on a time-frame of one quarter. Using the frequency of the system as a measure of the imbalance between production and consumption, the TSO activates flexibility contracted on the secondary reserve market to restore the balance. The cost of the most expensive flexibility service activated defines a unique imbalance price in €/MWh for each quarter which is paid or received by each actor connected to the transmission network.

In this chapter, we model the behavior of the actors (producers and retailers acting on load flexibility) of the electricity market and the market mechanisms governed by the system operator. The scope of this chapter being larger than the previous one, the system to model is too complex to obtain an analytical solution of the corresponding market equilibrium. To harness this complexity, this chapter makes use of agent-based modeling. This approach to modeling systems comprised of interacting autonomous agents [91]. In this case, the autonomous agents are the actors of the electrical market which want to maximize their individual profit or to minimize their costs. For the sake of simplicity, we consider only three types of actors: retailers with flexible demand in their portfolio, producers of energy, and the TSO which has to buy in the retail market well-defined amounts of flexibility. Using the agent-based framework, the trajectory of the system are analyzed so as to provide answers concerning the properties of the electricity market. This chapter is a first step towards answering questions such as the following. How is the money paid for reserve procurement affected by allowing more flexible loads to participate in the reserve market? How would it affect energy market prices? How would a retailer behave when it could exploit its load flexibility both in the energy market and the reserve market?

The agent-based simulation is based on three main stages. In the first stage, producers and retailers submit their bids to the energy market for each period of the next day. The market is cleared by a market operator that decides on a price and a set of bids that are accepted for each time period. In the second stage, bids are submitted to the reserve market. In a first setting, we only allow producers to submit bids whereas in a second setting, both producers and retailers are allowed to bid in the secondary reserve market. Again the market is cleared with a price and a set of accepted bids. Finally, the third stage gives the imbalance fee that each actor pays or receives depending on its position at the end of each market period compared to the position announced one day ahead corrected if activation of flexibility has been requested by the TSO. We make several assumptions to model this system. The main one is that they optimize their positions using forecasts of the prices of the energy and reserve markets, and the imbalance tariffs. In the simulations described in this chapter, we arbitrarily suppose that they use a weighted average of the previous prices to forecast prices.

The rest of the chapter is organized as follows. A review of the literature is presented in Section 3.3. Section 3.4 focuses on the case where flexible loads can only be exploited in the energy market. It defines the models that are used by the actors, and presents simulation results that serve for comparison with the results of the next section. Section 3.5 proposes a reorganization of the system to allow retailers to bid in the secondary reserve market, and computational results are compared to the first setting. Section 3.6 concludes.

3.3 Literature review

Many authors have studied the effects of load flexibility in the electricity and reserve market on the payoffs to different actors in the electrical system [7, 80, 146]. In [7], demand is represented by constant elasticity curves independent from one market period to another. The results show that demand-side reserve provision leads to lower operating costs. Reference [146] proposes a market model where the demand-side is directly controlled so as to shift consumption and provide upward and downward reserves. They also conclude that demand-side reserve offers can lead to significant gains in economic efficiency. Load reduction periods are typically followed by load recovery periods [80]. This observation leads the author to conclude that first, the demand-side should not be seen as a pure alternative to the provision of reserves, and second, the participation of demand-side resources could increase the overall required levels of reserve. Nevertheless, the fact that the system operator can exploit demand flexibility can reduce operating costs. These results are based on globally optimized systems and do not capture gaming possibilities coming from the individual optimization of each actor.

Prior any modification of the regulation framework, the quantitative analysis of the effect of such modifications should be evaluated, as advocated in [92]. The complex interactions that happen in electricity markets demand a flexible computational environment where designs can be tested and sensitivities to power system and market rule changes can be explored [149]. Agent-based modeling has been extensively used in the literature to model electric systems [102, 103, 137, 143, 152]. A tutorial about agent-based modeling can be found in article [91]. For instance, [86] deploys an agent-based model to compare different investment criteria and transmission technologies to determine where and how the network should be reinforced. This agent-based model takes into account generation companies which strategically submits bids to the marketplace deviating from their true marginal cost. The review [148] concludes that most agent-based models of the electricity market represent the demand side as a fixed and price-insensitive load. For instance, [18] models the electricity trading arrangements in the United Kingdom. In this system, retailers supply inelastic loads but may game on an intentional imbalance to maximize their profit. Nowadays, retailers already have access to flexible loads to optimize their costs. In this work, we assume that retailers have direct control over the flexible loads in their portfolio. Control with real-time pricing is investigated in [153], where retailers optimize the real-time pricing to minimize their retailing and imbalance costs. The results show that a retailer has an incentive to shift the demand using a time-dependent price to reduce its imbalance. These models do not consider the provision of secondary reserves by load aggregation. The OPTIMATE open platform is an example of an agent-based tool used in power systems. It can be used for evaluating and comparing various existing and potential new market designs in several regional European power markets [147]. Alternative techniques for carrying quantitative analysis in power systems are based on the computation of market equilibria. For instance the PRIMES model computes a market equilibrium solution in the European Union member states taking into consideration market economics, industry structure, energy/environmental policies and regulation [22].

3.4 Model of the current system

This section presents the current organization of the system. It details the mathematical problems that each stakeholder solves to optimize its decisions. Stakeholders have three decision stages, which are summarized here, and detailed in the next subsections. One day ahead, the energy market, through market operators, collects the offers of the participants (in our case producers and retailers), computes a uniform price for each period, and notifies the participants of the acceptance of their offers. In a second stage, still one day ahead but after the clearing of the energy market, producers can bid in the secondary reserve market. After these two stages, producers and retailers optimize their position according to an estimate of the imbalance tariffs for the next day, and decide the net power they will inject/withdraw from the network in their balancing perimeter. The participants pay or receive an imbalance fee depending on their position at the end of each market period compared to the position they announced one day ahead. The interactions between the actors are illustrated in Figure 3.1.

The main simplifications we make are the following. First, we assume each type of actor makes decisions according to the same mathematical model, but with its own data, and solves these mathematical models to optimality. Second,

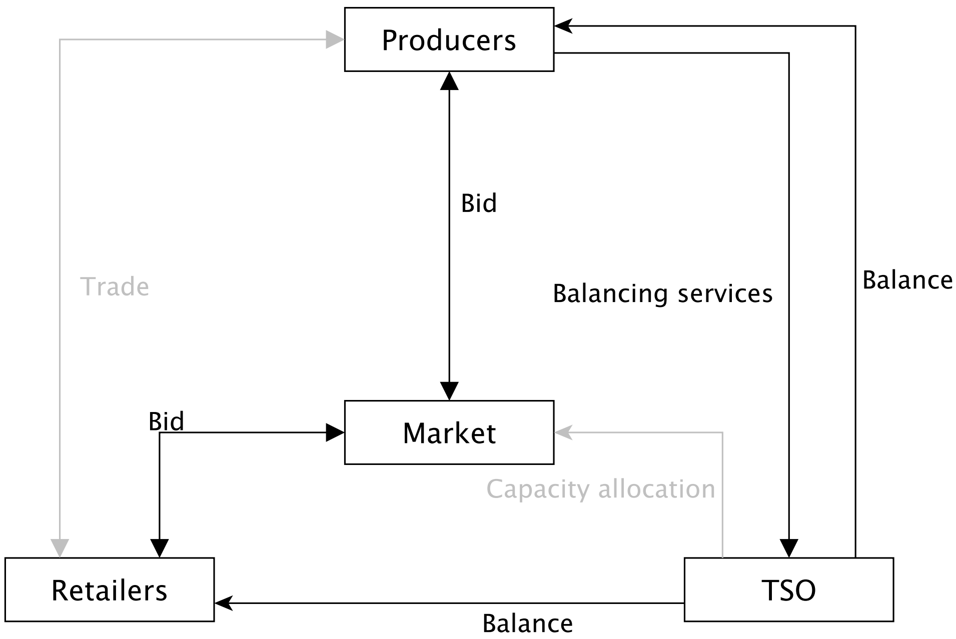


Figure 3.1: Interaction between the actors in the model of the current system.

we use linear programming relaxations of the problems that are actually solved by the stakeholders. Figure 3.2 provides a general view of the simulated agent-based system.

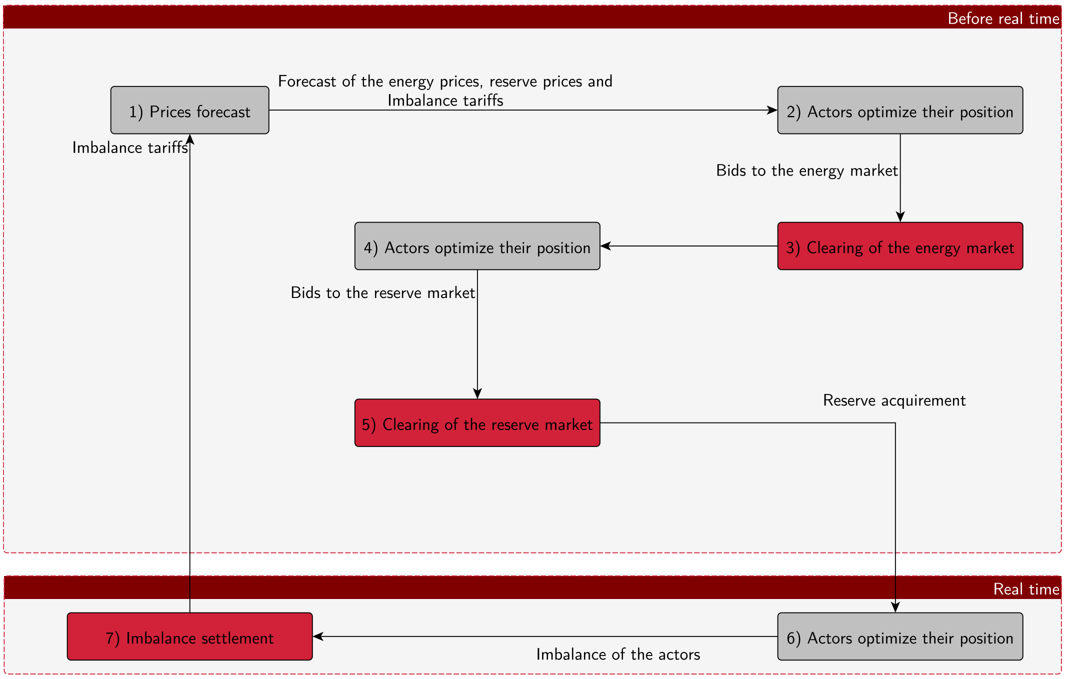


Figure 3.2: General view of the simulated agent-based system.

3.4.1 Energy market

The energy market model takes its inspiration from the Central Western Europe coupled market [32]. Producers and retailers submit offers to the market operator. An offer is defined by a volume and a limit price, and can span only one time period $t \in \mathcal{T}$. A day is divided into T equal periods, usually twenty-four. The proportion of an offer which is accepted is determined by the market clearing price (MCP) computed by the market operator. The MCP is the price at the intersection of the supply and the demand curves, which depends on the period, and is denoted by π_t^E. Supply offers are fully rejected (resp. accepted) when their limit price is greater (resp. smaller) than the MCP, or partially accepted when their limit price equals the MCP. The MCP is bounded by π^{cap}.

For each period t, each actor forecasts the price of the energy market $\hat{\pi}_t^E$. This forecast is obtained by the exponential mean of the prices in the last T rounds. Note that if the price cap π^{cap} is reached for a round in the history, the value is replaced in the mean by the last non-capped one. However, the information that the price cap was reached in a round is directly integrated into the optimization model of the actors.

3.4.2 Reserve market

We assume that the TSO has to procure a quantity of upward and downward reserve for each period, respectively R_t^+ and R_t^-, determined as a fixed percentage of the total consumption in period t. A bid in the reserve market consists of a maximum power (positive for upward reserve, negative otherwise), and an activation price. In case of activation, the TSO pays the activation price times the energy activated. Every bid covers only one period and may be rejected, accepted partially, or totally. The reservation of the capacity of a bid is remunerated at a regulated capacity price π^U for the upward reserve and π^L for the downward reserve. For the Belgian market, the capacity price has been fixed by the regulator to 45 €/MWh in 2013 [6]. In practice, the Belgian TSO contracts obligation to provide reserve on an annual basis. The parties with these contracts are obliged to submit the contracted quantity to a day-ahead reserve market cleared after the day-ahead energy market. In this work we focus only on the short-term aspect of the reserve and consider only the clearing of the day-ahead reserve market.

3.4.3 Imbalance settlement

As our formulation is deterministic, the imbalance of an actor can only be caused either by the technical impossibility of satisfying the outcome of the markets, or by an intentional imbalance. The purpose of this stage is to compute the tariff of upward and downward imbalances, π_t^{I+} and π_t^{I-}. This tariff is the activation price of the most expensive activated bid as it is done by the Belgian TSO [45]. In case of no imbalance, the imbalance tariff is set to 0. If the imbalance is greater than the contracted reserve, we assume that the TSO may use non-contracted reserves to restore balance. This reserve is supposed to be very expensive and drives the imbalance tariff to the price cap π^{nc}.

For each period t, every actor forecasts the imbalance tariffs $\hat{\pi}_t^{I^+}$ and $\hat{\pi}_t^{I^-}$. This forecast is obtained by the exponential mean of the prices in the last T rounds. Note that if the imbalance tariff is equal to either zero or π^{nc} for an iteration in the history, the value is replaced in the mean by the last one which is not zero or π^{nc}. The models of the actors explicitly take into account these cases.

3.4.4 Retailer model

In this setting, retailer a estimates the consumption of its clients and make bids at the upper cap price π^{cap}. We assume that a proportion of the retailer's load is flexible and that the retailer has the power to decide when these loads will consume power. The inelastic part of the demand of the retailer in period t is denoted by $\nu_{a,t}$. The set of flexible loads controlled by retailer a is denoted $\mathcal{M}_a$. We assume that each flexible load $j \in \mathcal{M}_a$ can be accurately represented by a tank model, as in [98]. At each period t, the load consumes power $d_{j,t}$ bounded by (3.1b). The limits on the energy in the tank are given by (3.1c). The state transition is given by (3.1d), where η_i is the efficiency, $\phi_{j,t}$ the losses in one period, and Δt the duration of a period. The total energy consumed in the time horizon is bounded by (3.1f). One day ahead, retailer a optimizes the consumption to minimize its retailing costs:

$$\begin{aligned}\min \sum_{t\,\in \mathcal{T}} [\hat{\pi}_t^E D_{a,t} + (\pi^{cap} - \hat{\pi}_t^E) \max\{0, D_{a,t} - D_{a,t}^{\max}\} + \hat{\pi}_t^{I+} I_{a,t}^+ \\ + (\pi^{nc} - \hat{\pi}^{I+}) \max\{0, I_{a,t}^+ - I_{a,t}^{+\max}\} + \hat{\pi}_t^{I-} I_{a,t}^- \\ + (\pi^{nc} - \hat{\pi}^{I-}) \max\{0, I_{a,t}^- - I_{a,t}^{-\max}\}]\end{aligned} \tag{3.1a}$$

subject to, $\forall j \in \mathcal{M}_a, t \in \mathcal{T}$,

$$d_{j,t}^{\min} \leq d_{j,t}, \leq d_{j,t}^{\max} \tag{3.1b}$$

$$e_{j,t}^{\min} \leq e_{j,t} \leq e_{j,t}^{\max} \tag{3.1c}$$

$$e_{j,t+1} = e_{j,t} - \phi_{j,t} + \eta_j d_{j,t} \Delta t \tag{3.1d}$$

$\forall i \in \mathcal{M}_a$,

$$e_{j,1} = \xi_i^1 \tag{3.1e}$$

$$\xi_j^{\min} \leq \sum_{t \in \mathcal{T}} d_{j,t} \Delta t \leq \xi_j^{\max} \tag{3.1f}$$

$\forall t \in \mathcal{T}$,

$$D_{a,t} - I^+_{a,t} + I^-_{a,t} = \nu_{a,t} + \sum_{j \in \mathcal{M}_a} d_{j,t}. \tag{3.1g}$$

$$I^+_{a,t}, I^-_{a,t} \geq 0 \tag{3.1h}$$

Equation (3.1g) computes the total demand the retailer submits to the energy market, $D_{a,t}$, and the upward (resp. downward) imbalance, $I^+_{a,t}$ (resp. $I^-_{a,t}$). From the history of the previous rounds, the retailer learns a threshold demand $D^{\max}_{a,t}$ above which the clearing of the energy market yields the price cap π^{cap}. If in a previous round $\pi^E_t = \pi^{cap}$, then the retailer sets its threshold demand slightly below the volume submitted, e.g., $D^{\max}_{a,t} = 0.95 D_{a,t}$. The same consideration is applied to imbalance if the tariff of imbalance is either 0 or π^{nc}.

After the clearing of the energy market, the retailer runs again the previous optimization problem with $D_{a,t}$ given as data to optimize its position for the second and the third stages.

3.4.5 Producer model

Producer a optimizes its position using price forecasts and the characteristics of its production units. The output of the following optimization problem is the power to be submitted to the energy market $P_{a,t}$, the upward/downward reserve $U_{a,t}/L_{a,t}$, and its upward/downward imbalance $I^+_{a,t}/I^-_{a,t}$.

$$\begin{aligned}\max \sum_{t \in \mathcal{T}} [&\hat{\pi}^E_t P_{a,t} - (\pi^{cap} + \hat{\pi}^E_t) \max\{0, P^{\min}_{a,t} - P_{a,t}\} - \hat{\pi}^{I+}_t I^+_{a,t} - \hat{\pi}^{I-}_t I^-_{a,t} \\ &- (\pi^{nc} - \hat{\pi}^{I+}) \max\{0, I^+_{a,t} - I^{+\max}_t\} - (\pi^{nc} - \hat{\pi}^{I-}) \max\{0, I^-_{a,t} - I^{-\max}_t\} \\ &+ \gamma_t U_{a,t} + \gamma_t L_{a,t} - \sum_{j \in \mathcal{G}_a} c_{j,t} p_{j,t}]\end{aligned} \tag{3.2a}$$

subject to $\forall t \in \{1, ..., T\}$ and each production unit $j \in \mathcal{G}_a$,

$$p_{j,t} + u_{j,t} \leq p^{\max}_{j,t} \tag{3.2b}$$

$$p^{\min}_{j,t} \leq p_{j,t} - l_{j,t} \tag{3.2c}$$

$$(p_{j,t} + u_{j,t}) - p_{j,t-1} \leq \rho^u_j \tag{3.2d}$$

$$p_{j,t-1} - (p_{j,t} - l_{j,t}) \leq \rho^d_j \tag{3.2e}$$

$$u_{j,t}, l_{j,t} \geq 0 \tag{3.2f}$$

$\forall t \in \mathcal{T}$:

$$P_{a,t} + I^+_{a,t} - I^-_{a,t} = \sum_{j \in \mathcal{G}_a} p_{j,t} \tag{3.2g}$$

$$U_{a,t} = \sum_{j \in \mathcal{G}_a} u_{j,t} \tag{3.2h}$$

$$L_{a,t} = \sum_{j \in \mathcal{G}\prime_a} l_{j,t} \tag{3.2i}$$

$$I^+_{a,t}, I^-_{a,t} \geq 0 \tag{3.2j}$$

The power production of unit i is offered to the energy market ($p_{j,t}$) and the reserve market ($u_{j,t}$ as upward reserve, $l_{j,t}$ as downward reserve) at the price $c_{j,t}$ for every period t. The predicted downward imbalance is submitted as a bid to the energy market at the price $\hat{\pi}^{I-}$.

In the second stage, P_t, the power cleared in the energy market, is a parameter. At the third stage, the accepted upward and downward reserve quantities are also fixed, but the producer may still optimize its imbalance.

Similarly to the retailer model, the producer model uses the data from the rounds where the price cap was reached to learn what minimal quantity to submit to the energy market $P_t^{\min}$. The same mechanism is used for the imbalance volume. The objective function (3.2a) uses a price γ_t to balance the provision of energy and reserve. If we set $\gamma_t = \pi^U$ or π^L, the previous optimization problem assumes that all reserves proposed by the producer will be accepted at the capacity price. To prevent the production of the energy cleared in the energy market at a higher price than the MCP, we put $\gamma_t = 0.005€/\text{MWh}, \forall t$. Other options may be considered, such as taking the product of the capacity prices and the probability of the reserve bid's being accepted.

3.4.6 Results

We simulate a benchmark system for one day, divided into 24 periods. This will be used for comparison with the proposal of Section 3.5. Retailers use Synthetic Load Profiles to estimate their static consumption [136]. Producers own two types of units: (i) slow ramping units with costs randomly generated between 45 and 60€/MWh and (ii) high ramping units with costs between 60 and 80€/MWh. For each period t, the TSO contracts for a quantity of upward and downward reserves (R_t^+ and R_t^-) equal to 2% of the total consumption cleared in MW in the energy market. The capacity price for the reservation of a reserve is set to 45€/MWh. For the Belgian market, 45€/MWh is the price that has been fixed for 2013 [6]. The system evolves until the energy prices and the forecasts converge individually and to the same value, or a cycle is detected, i.e., actors start taking the same set of positions over and over.

We present a typical run for a mean total consumption of 1000 MW with 6% of flexible loads. Figure 3.3 shows for each round a measure of the MCP forecasting error. The system cycles after 113 rounds, repeating rounds 77 to 112. The following results are the mean over these 35 rounds. The mean energy market price is 49.81€/MWh. The TSO cost for reserve procurement is 42683€. This reserve covers a total imbalance of 190 MWh over the day and no non-contracted reserve needs to be used. We observe that this imbalance is caused solely by the producers.

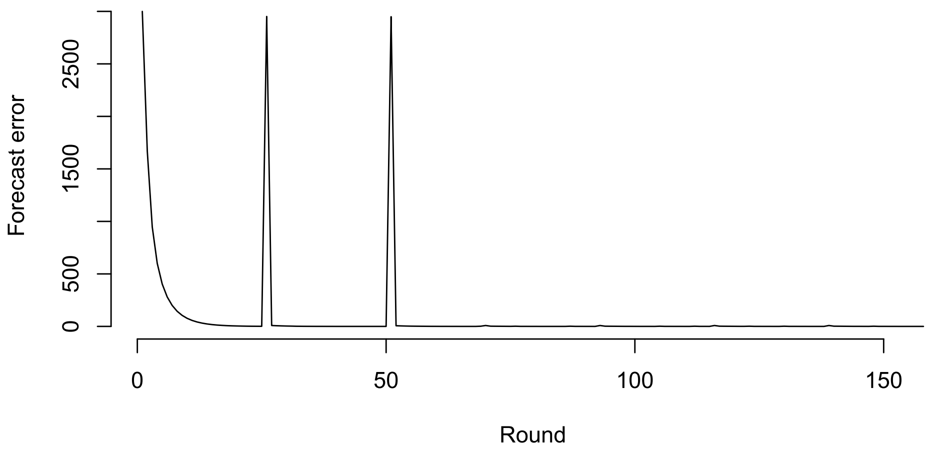


Figure 3.3: Evolution of the forecasting error (€/MWh) for the energy prices ($\|\hat{\pi}^E - \pi^E\|_\infty$) for a system with 6% of flexible consumption.

We now compare the status of the system for a mean total consumption of 1000 MW with 0% to 10% of flexible loads. With our parameters, increasing the flexibility of the demand-side left the mean energy market price and TSO costs for reserve procurement barely unchanged. The variability of the energy price, defined here as the difference between the minimum and maximum price, is given in Figure 3.4. Non-contracted reserve is not needed in any of the cases considered. Figure 3.5 shows that the mean total imbalance of the system increases with the amount of flexible loads. The flexibility of the consumption allows the retailers to earn profits from the variability of the imbalance prices by placing themselves voluntary in imbalance.

3.5 Opening the reserve market to retailers

We focus here on the provision of secondary reserves by retailers using load flexibility. First, we introduce modulation bids for the reserve market that are suitable to demand side management. Sections 3.5.2, 3.5.3 and 3.5.4 propose, respectively, the required modifications to the models of the reserve market, the imbalance settlement, and the retailer. Section 3.5.5 presents the result of opening the reserve market to retailers. The interactions between the actors are illustrated in Figure 3.6.

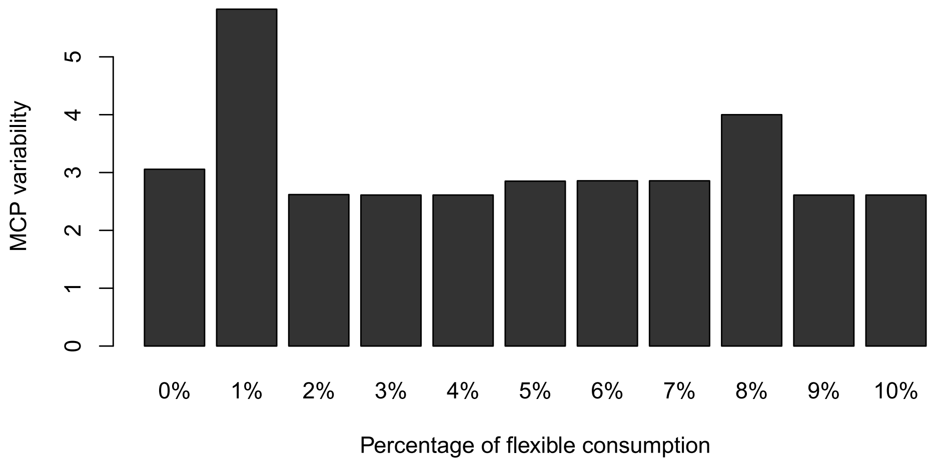


Figure 3.4: Variability of the energy price (€/MWh) as a function of the amount of flexible consumption in the system without the participation of flexible consumption to the reserve market.

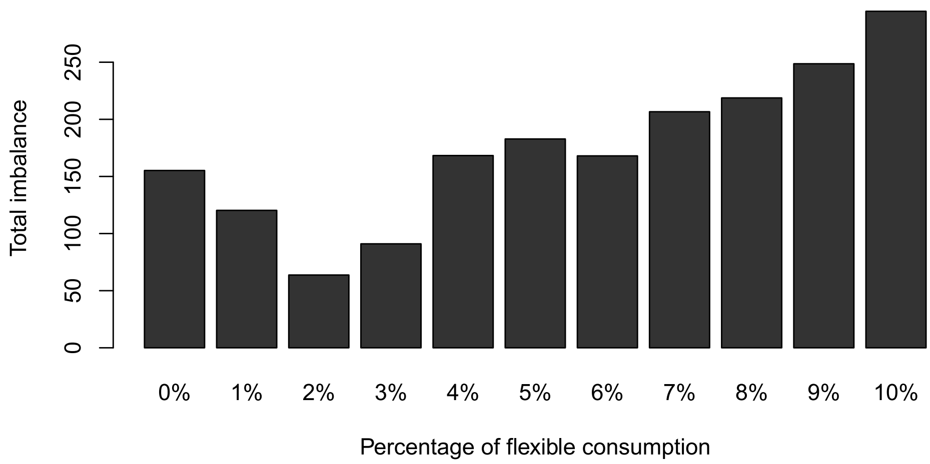


Figure 3.5: Mean total imbalance (MWh) as a function of the amount of flexible consumption in the system without the participation of flexible consumption to the reserve market.

3.5.1 Modulation bids for the reserve market

Unlike production units, the consumption of a load in a period depends on the consumption in the previous periods. Increasing the consumption in one period implies that the consumption will decrease later on. This fact motivates the introduction of bids more adapted to load behavior. A modulation bid i provided by a retailer a consists of a flexibility margin $[D_{a,t} - F_i, D_{a,t} + F_i]$ $\forall t \in \mathcal{N}$ around a baseline consumption $D_{a,t}$ over a set $\mathcal{N}_i$ of consecutive market periods. F_i is the maximum amplitude of the power modulation. The TSO

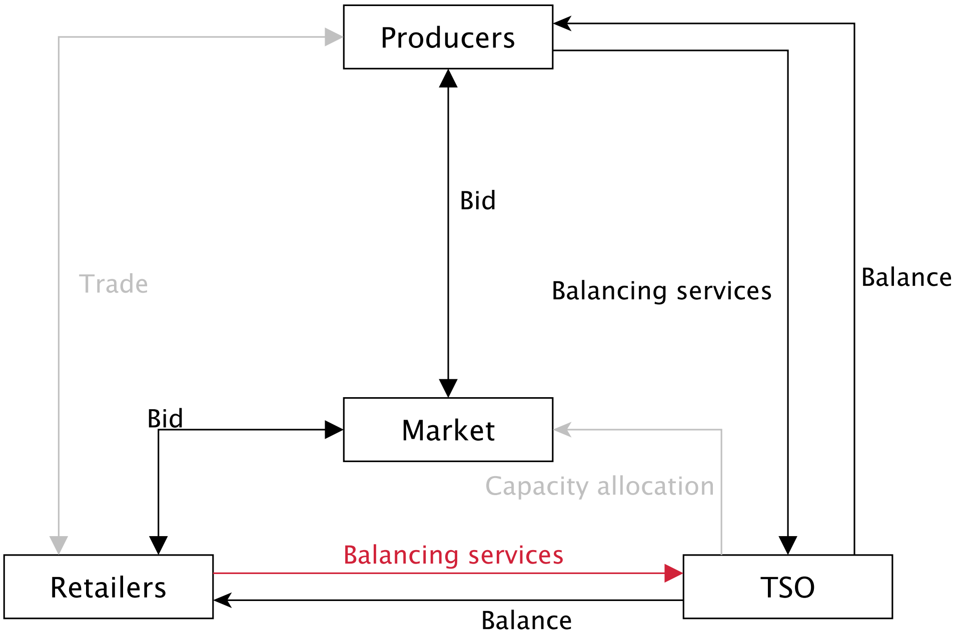


Figure 3.6: Interaction between the actors in the model of the system with the reserve market open to retailers.

can specify the modulation of the consumption of the retailer for every period $t \in \mathcal{N}_i$ under the constraints that these specifications do not violate the margin and that the total energy consumed in the $\mathcal{N}_i$ periods is identical. The total energy consumed must be constant in order to provide the same utility to the load owners. For instance, a heat pump turned off for an hour needs to consume more afterwards to get the temperature back to its set point.

3.5.2 Clearing of the reserve market

The following optimization model considers the cost of reservation as well as the cost of activation c_i^E of each bid. The capacity price of modulation bids is regulated at π^F. As the TSO has no knowledge of the future imbalance, it supposes that it may activate all the contracted reserves. The objective function (3.3a) shows that the TSO receives the activation cost of downward reserve bids. To prevent this model from contracting every downward reserve bid, we introduce an additional modeling parameter c_t^o that penalizes the amount of reserve contracted over the requirements R_t^+ and R_t^- in period t. Our implementation uses $c_t^o = 1.1 \max_{i \in \mathcal{S}^-{}_t} c_i^E$. The efficiency factor ζ of a modulation bid expresses the fact that modulation bids are not equivalent to classical bids.

$$\begin{aligned} \min \sum_{t \in \mathcal{T}} \Bigg[\sum_{i \in \mathcal{S}_t^+} (\pi^U + c_i^E) x_i Q_i + \sum_{i \in \mathcal{S}_t^-} (\pi^L - c_i^E) x_i Q_i \\ + c_t^o (s_t^+ + s_t^-) + \pi^{nc} (n_t^+ + n_t^-) \Bigg] + \sum_{i \in \mathcal{F}} (\pi^F + c_i^E) x_i Q_i \end{aligned} \tag{3.3a}$$

subject to $\forall t \in \mathcal{T}$,

$$\sum_{i \in \mathcal{S}_t^+} Q_i x_i \zeta_i + \sum_{i \in \mathcal{F}: t \in \mathcal{N}_i} Q_i x_i \zeta_i + n_t^+ - s_t^+ = R_t^+ \tag{3.3b}$$

$$\sum_{i \in \mathcal{S}_t^-} Q_i x_i \zeta_i + \sum_{i \in \mathcal{F}: t \in \mathcal{N}_i} Q_i x_i \zeta_i + n_t^- - s_t^- = R_t^- \tag{3.3c}$$

3.5.3 Imbalance settlement

The following model gives the optimal activation scheme a figures/ would use to restore balance in every market period. We assume the figures/ knows exactly what the imbalance of the system, I_t, will be for each period, given the nominations of the actors, i.e. their positions on the day-ahead energy market.

$$\begin{aligned} \min \sum_{i \in \mathcal{F}} c_i^E Q_i \sum_{\tau \in \mathcal{N}_i} (v_{i,\tau} + w_{i,\tau}) \\ + \sum_{t=1}^{T} \Bigg[\sum_{i \in \mathcal{S}^+{}_t} c_i^E x_i Q_i + \sum_{i \in \mathcal{S}^-{}_t} (c_t^o - c_i^E) x_i Q_i + \pi^{nc} (y_t^+ + y_t^-) \Bigg] \end{aligned} \tag{3.4a}$$

subject to $\forall t \in \{1, ..., T\}$,

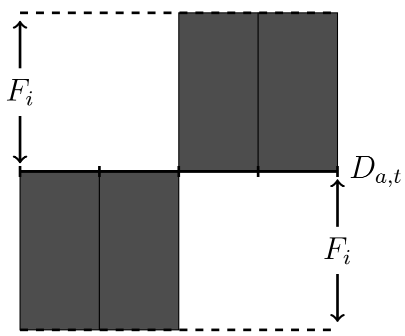
$$\sum_{i \in \mathcal{S}^+{}_t} Q_i x_i - \sum_{i \in \mathcal{S}^-{}_t} Q_i x_i + \sum_{i \in \mathcal{F}: t \in \mathcal{N}_i} Q_i (v_{i,t} - w_{i,t}) + y_t^+ - y_t^- + I_t = 0 \tag{3.4b}$$

$\forall i \in \mathcal{F}$,

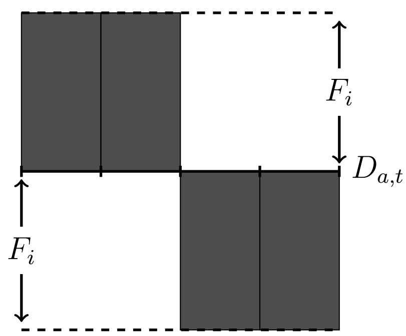
$$\sum_{t \in \mathcal{N}_i} (v_{i,t} - w_{i,t}) = 0 \tag{3.4c}$$

3.5.4 Retailer model to provide secondary reserve

The objective of retailer a is to maximize its profit from the retailing activities and the flexibility services it sells to the TSO. We suppose that the retailer selects a set of modulation bids $\mathcal{F}_a$ to submit to the reserve market, whose quantity is the result of the following optimization problem. These bids are supposed to be non-overlapping. Each bid $i \in \mathcal{F}_a$ starts in period τ_i and lasts N_i periods. For every modulation bid i, we define the two most constraining scenarios for the provision of the modulation range $[D_{a,t}-F_i, D_{a,t}+F_i]$ $\forall t \in \mathcal{N}_i$: $D^+_{a,t}$ and $D^-_{a,t}$ (cf. Figure 3.7) where the notation underline/overline indicates the variables related to these scenarios. In the first one, $D^+_{a,t}$, the TSO asks for a modulation upwards for the $N_i/2$ first periods and ensures an energy balance in the last two periods. In the second one, $D^-_{a,t}$, the TSO asks for modulation downwards for the $N_i/2$ first periods. The two scenarios $D^+_{a,t}$ and $D^-_{a,t}$ may be used to define the range of flexibility described previously. Section 3.7 proves that scenarios $D^+_{a,t}$ and $D^-_{a,t}$ cover every activation scheme that the TSO may ask for from a retailer within the limits $[D_{a,t} - F_i, D_{a,t} + F_i]$ $\forall t \in \mathcal{N}_i$ if loads are modeled by (3.5e)–(3.5g). The cost of activation of the modulation bids of the retailer is considered to be null, as the utility of the load is ensured by the integrality constraint of the bid (3.4c).



(a) $D^-_{a,t}$



(b) $D^+_{a,t}$

Figure 3.7: Illustration of the two most constraining scenarios for the provision of modulation service i by the retailer a.

The optimization problem solved by the retailer is:

$$\min \sum_{t \in \mathcal{T}} [\hat{\pi}^E_t D_{a,t} + (\pi^{cap} - \hat{\pi}^E_t) \max\{0, D_{a,t} + I^-_{a,t} - D^{\max}_{a,t}\} + \hat{\pi}^{I+}_t I^+_{a,t}$$
$$+ (\pi^{nc} - \hat{\pi}^{I+}) \max\{0, I^+_{a,t} - I^{+\max}_{a,t}\} + \hat{\pi}^{I-}_t I^-_{a,t} \quad (3.5a)$$
$$+ (\pi^{nc} - \hat{\pi}^{I-}) \max\{0, I^-_{a,t} - I^{-\max}_{a,t}\} - \sum_{i \in \mathcal{F}_a} \left(\sum_{t \in \mathcal{N}_i} \pi^F_t \right) F_i]$$

subject to $\forall t \in \mathcal{T}$,

$$D_{a,t} - I^+_{a,t} + I^-_{a,t} = \nu_{a,t} + \sum_{j \in \mathcal{M}_a} d_{j,t} \quad (3.5b)$$

$$D^+_{a,t} = \nu_t + \sum_{j \in \mathcal{M}_a} d^+_{j,t} \quad (3.5c)$$

$$D^-_{a,t} = \nu_t + \sum_{j \in \mathcal{M}_a} d^-_{j,t} \tag{3.5d}$$

$\forall j \in \mathcal{M}_a, t \in \mathcal{T},$

$$d^{\min}_{j,t} \leq d_{j,t}, d^+_{j,t}, d^-_{j,t} \leq d^{\max}_{j,t} \tag{3.5e}$$
$$e^{\min}_{j,t} \leq e_{j,t}, e^+_{j,t}, e^-_{j,t} \leq e^{\max}_{j,t} \tag{3.5f}$$
$$e_{j,t+1} = e_{j,t} - \phi_{j,t} + \eta_j d_{j,t} \Delta t \tag{3.5g}$$

$\forall j \in \mathcal{M}_a,$

$$e_{j,1} = \xi^1_i \tag{3.5h}$$
$$\xi^{\min}_j \leq \sum_{t \in \mathcal{T}} d_{j,t} \Delta t \leq \xi^{\max}_j \tag{3.5i}$$

$\forall j \in \mathcal{M}_a, i \in \mathcal{F}_a, t = \tau_i,$

$$e^+_{j,t+1} = e_{j,t} - \phi_{j,t} + \eta_j d^+_{j,t} \Delta t \tag{3.5j}$$
$$e^-_{j,t+1} = e_{j,t} - \phi_{j,t} + \eta_j d^-_{j,t} \Delta t \tag{3.5k}$$

$\forall j \in \mathcal{M}_a, i \in \mathcal{F}_a, t \in \{\tau_i + 1, ..., \tau_i + N_i - 2\},$

$$e^+_{j,t+1} = e^+_{j,t} - \phi_{j,t} + \eta_j d^+_{j,t} \Delta t \tag{3.5l}$$
$$e^-_{j,t+1} = e^-_{j,t} - \phi_{j,t} + \eta_j d^-_{j,t} \Delta t \tag{3.5m}$$

$\forall j \in \mathcal{M}_a, i \in \mathcal{F}_a, t = \tau_i + N_i - 1,$

$$e_{j,t+1} = e^+_{j,t} - \phi_{j,t} + \eta_j d^+_{j,t} \Delta t \tag{3.5n}$$
$$e_{j,t+1} = e^-_{j,t} - \phi_{j,t} + \eta_j d^-_{j,t} \Delta t \tag{3.5o}$$

$\forall i \in \mathcal{F}_a, t \in [\tau_i, \tau_i + N_i/2 - 1],$

$$F_i \leq D^+_{a,t} - (D_{a,t} - I^+_{a,t} + I^-_{a,t}) \tag{3.5p}$$
$$F_i \leq (D_{a,t} - I^+_{a,t} + I^-_{a,t}) - D^-_{a,t} \tag{3.5q}$$

$\forall i \in \mathcal{F}_a, t \in [\tau_i + N_i/2, \tau_i + N_i - 1],$

$$F_i \leq D^-_{a,t} - (D_t - I^+_{a,t} + I^-_{a,t}) \tag{3.5r}$$
$$F_i \leq (D_{a,t} - I^+_t + I^-_{a,t}) - D^+_{a,t} \tag{3.5s}$$

The energy of each load for the modulation scenario D^+, D^- is given by (3.5j) and (3.5k) at the beginning of each bid period, by (3.5l) and (3.5m) in the middle, and by (3.5n) and (3.5o) at the end. The available volumes of modulation are given by (3.5p)–(3.5s).

3.5.5 Results

We now run the system with a reserve market open to modulation bids. The reservation price of these bids is arbitrarily set to 10€/MWh. This value may seems low compared to the 45€/MWh for the single period bid devised to cover the loss of opportunity implied by the capacity reservation. With modulation bids, there is not such a loss as the energy is shifted but still consumed. Note that this figure is coherent with the one given by RTE in a report, making the hypothesis of high fixed cost for load modulation of 14.5k€/MW/year, which roughly corresponds to 1.65€/MWh [131].

Retailers submit a modulation bid for every four hours. We use an arbitrary efficiency factor of 0.5 to express the fact that the quantity brought by modulation bids is worth one-half that of the quantity from a classical reserve bid. The results are reported for flexibility rates between 0 and 10%. Once again, the mean energy market price is left barely unchanged, at around 49.81€/MWh. The price variability is similar to that observed in Figure 3.4. The total imbalance, shown in Figure 3.8, is slightly lower than in Figure 3.5. Figure 3.9 shows that, with flexibility, the TSO reserve procurement costs de-

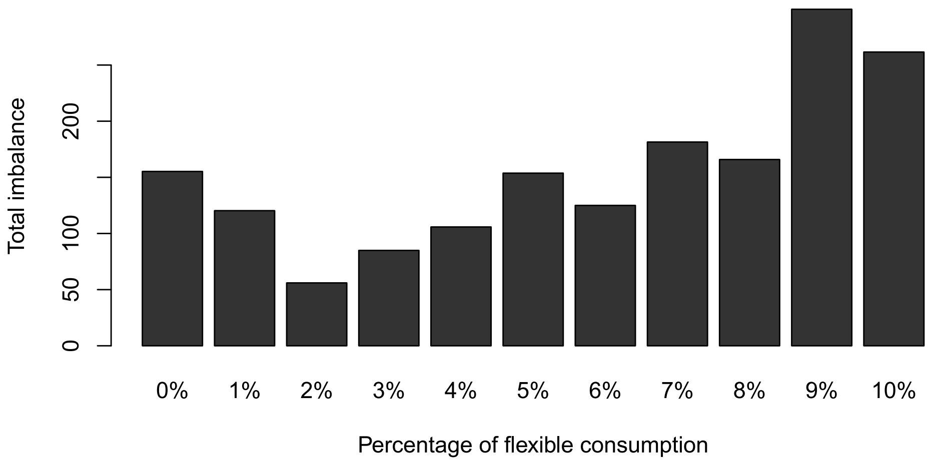


Figure 3.8: Impact of load aggregation included as modulation bids in the reserve market on the total imbalance (MWh) as a function of the flexibility rate.

crease significantly, to only 11% of the initial costs. Nevertheless, the volume of non-contracted reserves that have to be used is increasing in the flexibility rate, as shown by Figure 3.10. This volume of non-contracted reserve is mostly due to the inability of a modulation bid to sustain an imbalance of the same sign for its whole time horizon. One could compute a total reserve cost taking into account the non-contracted reserve. This is presented in Figure 3.11 where the non-contracted reserve cost as been arbitrarily set to 150€/MWh. The minimum total reserve cost is obtained at 4% of flexible consumption.

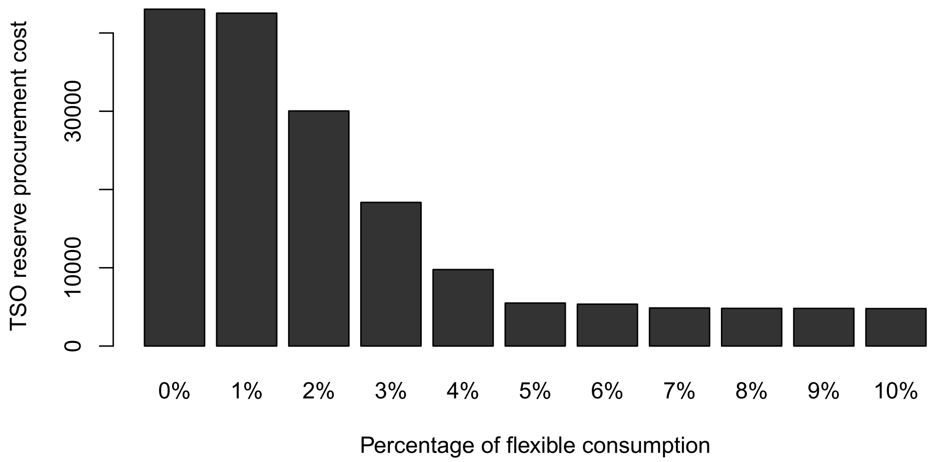


Figure 3.9: Impact of load aggregation included as modulation bids in the reserve market on the SO reserve procurement cost (k€) as a function of the flexibility rate.

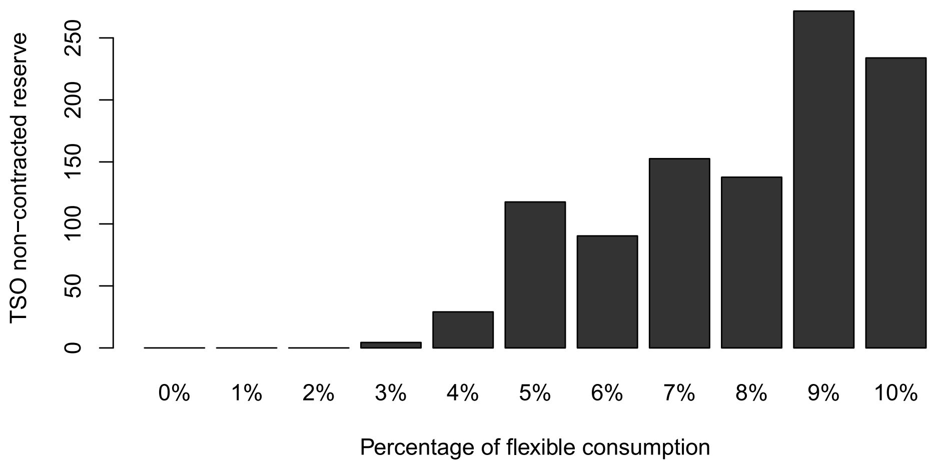


Figure 3.10: Impact of load aggregation included as modulation bids in the reserve market on the volume of non-contracted reserves (MWh) as a function of the flexibility rate.

The total reserve cost for the system with more than 9% of flexibility even increases with respect to the case with no flexibility.

3.6 Conclusion

An agent-based model has been introduced to study the introduction of load aggregation in the secondary reserve market. In this model, each actor maximizes its profit based on a forecast of the prices. Producers and retailers

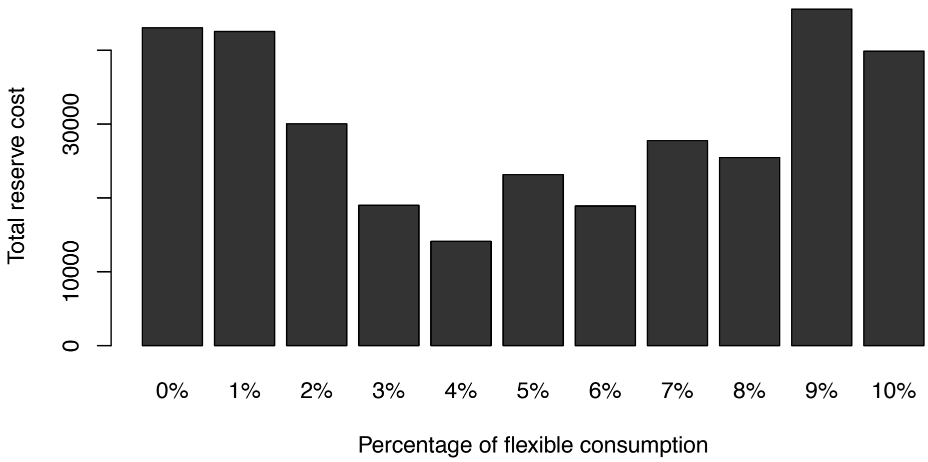


Figure 3.11: Impact of load aggregation included as modulation bids in the reserve market on the total reserve cost (k€) as a function of the flexibility rate.

(which perform load aggregation) optimize their positions in the energy and reserve markets, and in the settlement of imbalances. We propose to add a new product, the modulation bid, to the reserve market, that takes into account the inter-dependency between time periods due to load constraints. The results show that introducing this product decreases drastically the cost for reserve procurement. Unfortunately, modulation bids are not efficient at covering an imbalance of the same sign for multiple periods, which results in the activation of non-contracted reserves. This can be avoided by contracting for more reserves.

The proposed agent-based model could be modified to assess variants of the market model studied in this work. One variant that would be worth exploring is to include energy constrained bids in the day-ahead energy market [135]. One could also consider market-based capacity prices for the reserve. They could decrease the cost of reserve procurement but may, however, lead to gaming. Finally, an extension to the provision of services to solve congestion or over-voltage problems in the distribution network should be investigated.

3.7 Appendix

We prove here that scenarios $D^+_{a,t}$ and $D^-_{a,t}$ cover every activation scheme within the limits $[D_{a,t} - F_i, D_{a,t} + F_i]$ $\forall t \in \mathcal{N}$ for which the TSO may ask if loads are modeled by (3.5e)–(3.5g). For ease of exposition, we develop our argument on the set of periods $\mathcal{N}_i = \{1, ..., N_i\}$. We study the behavior of each load individually and drop the load index for conciseness. Every scenario * must satisfy energy balance:

$$\overset{\star}{e}_2 = e_1 - \phi_1 + \eta \overset{\star}{d}_1 \Delta t \tag{3.6}$$

$$\overset{\star}{e}_{t+1} = \overset{\star}{e}_t - \phi_t + \eta \overset{\star}{d}_t \Delta t \qquad \forall t \in [2, N-1] \tag{3.7}$$

$$e_{N+1} = e_N - \phi_N + \eta \overset{\star}{d}_N \Delta t. \tag{3.8}$$

We use $'$ to refer to a random scenario. d' obeys the constraints (3.6)–(3.8). If we use (3.7) and (3.6) in (3.8), we get

$$e_{N+1} = e_1 - \sum_{t=1}^{N} \phi_t + \eta \Delta t \sum_{t=1}^{N} \overset{\star}{d}_t \tag{3.9}$$

which is true for every $\overset{\star}{d}$ and in particular for $\overset{\star}{d} = d$ and $\overset{\star}{d} = d'$. Therefore, we can identify this equality with the total energy consumed in the bid period:

$$\sum_{t=1}^{N} d'_t = \sum_{t=1}^{N} d_t \tag{3.10}$$

We want to prove that every load scenario $d' : d'_t \in [d^-_t, d^+_t]$ $\forall t$ satisfying (3.6)–(3.8) is feasible if d, d^- and d^+ are feasible. If we use (3.7) and (3.6), we have

$$\overset{\star}{e}_{t+1} = e_1 - \sum_{\tau=1}^{t} \phi_\tau + \eta \Delta t \sum_{\tau=1}^{t} \overset{\star}{d}_\tau \qquad \forall t \in [2, N]. \tag{3.11}$$

As d^- and d^+ are feasible and (3.10) holds, we have for the first one-half of the bid period $[1, N/2]$:

$$d^{\min}_t \leq d^-_t \leq d'_t \leq d^+_t \leq d^{\max}_t \qquad \forall t \in [1, N/2] \tag{3.12}$$

and for the next half,

$$d^{\min}_t \leq d^+_t \leq d'_t \leq d^-_t \leq d^{\max}_t \qquad \forall t \in [N/2+1, N]. \tag{3.13}$$

Employing (3.11) and (3.12) and using the fact that d^- and d^+ are feasible scenarios, we have

$$e^{\min}_t \leq e^-_t \leq e'_t \leq e^+_t \leq e^{\max}_t \qquad \forall t \in [1, N/2] \tag{3.14}$$

and similarly for the remaining periods,

$$e_t^{\min} \leq e_t^+ \leq e_t' \leq e_t^- \leq e_t^{\max} \qquad \forall t \in [N/2+1, N]. \tag{3.15}$$

Now we can see that every scenario $' : D'_{a,t} \in [D^-_{a,t}, D^+_{a,t}]$ which satisfies (3.6)–(3.8) is feasible, by summing over the whole portfolio of loads $\mathcal{M}_a$ of the retailer.

Chapter 4

Flexibility exchange in distribution networks

One step down into the electrical system, this chapter studies how distribution system operators may use flexibility from the grid users connected to the medium voltage distribution network to perform active network management. Active network management consists in anticipating and correcting operational limits violations, in addition to regular control assets, when this is more economical or easier to perform than grid reinforcement. There are many possibilities to organize interactions between the system actors when flexibility comes into play. Therefore, there is a need to clearly state the interaction models that formalize these interactions and to set up a way for comparing their economical and technical performance. This chapter analyzes six interaction models developed with industrial partners by evaluating their impact through a measure of the social welfare, the repartition of the welfare between the actors, and a measure of the service level that is reached. The actors considered are the distribution system operator, the transmission system operator, producers and retailers. The scope of the interaction models covers several stages from day-ahead exchanges of flexibility until settlement. Two methods are used to perform the quantitative analysis of the interaction models: a macroscopic analysis and an agent-based model. The macroscopic representation of the system allows a quick evaluation of the economic efficiency of each model while the agent-based approach is set up to obtain a more detailed information on the behavior of each actors. The interaction models are simulated on 75-bus test system on expected production, consumption and prices in 2025. The results show that a conservative interaction model restricting grid users to an access range computed ahead of time to prevent any congestion avoids shedding distributed generation but restrains considerably the amount of distributed production. A carefully designed interaction model allows to safely increase by 55% the amount of distributed generation in the network and the welfare by 42.5% with respect to the conservative model.

4.1 Nomenclature

In the following of the section the letters a, i, t and n index agents, flexibility services, time periods, and buses, respectively. Superscripts $^+$ and $^-$ depict an upward or a downward modulation.

Sets

$\mathcal{A}$	Set of agents
$\mathcal{T}$	Periods
$\mathcal{N}$	Buses
$\mathcal{N}_a$	Buses of agent a
$\mathcal{N}(n)$	Neighbors of bus n

Parameters

α_n	Flexibility potential indicator
$C_{n,m}$	Capacity of the line (n, m)
$c_{a,n,t}$	Marginal cost of producer units
$V_{a,n}$	Total energy needs over the horizon
$E_{a,t}$	External imbalance to the system
$[g_{a,n}, G_{a,n}]$	Requested access bounds to the network
$[p^{min}_{a,n,t}, p^{\max}_{a,n,t}]$	Minimum and maximum realization
π^E_t	Energy marginal price
π^I_t	Imbalance marginal price
π^S_t	Reservation price of secondary reserve
π^l	Penalty for a local imbalance
π^r_i	Reservation price of a flexibility service
π^b_i	Activation price of a flexibility service
π^f_a	Retailing price
π^{VSP}	Value of shed production
π^{VSC}	Value of shed consumption

Variables

$\mathcal{B}_a$	Set of buses where the agent is responsible for its balance
$\mathcal{E}_n$	Set of energy constrained flexibility offers
$[b_{a,n}, B_{a,n}]$	Safe access bounds to the network
$[d_{a,n,t}, D_{a,n,t}]$	Dynamic access range to the network
δ_g, δ_G	Maximum safe access bound restriction
$\Delta_{a,n,t}$	Total flexibility offered by the FSP
$h_{a,n,t}$	Total requested modulation to the FSP
$I_{a,t}$	Imbalance of an agent
I_t	Total imbalance of the system
$f_{n_1,n_2,t}$	Active power flow in line (n_1, n_2)

$[k_{a,n}, K_{a,n}]$	Flexible access range of a FSP
$[l_{a,n}, L_{a,n}]$	Full access range of a FSP
$[m_{i,t}, M_{i,t}]$	Modulation range of the flexibility service
$p^b_{a,n,t}$	Day-ahead baseline of an agent
$P^b_{n,t}$	Aggregated day-ahead baseline of the agents
$p_{a,n,t}$	Realization of an agent
$P_{n,t}$	Aggregated realization of the agents
$r_{a,n,t}$	Flexibility needs indicator
$\mathcal{S}_n$	Set of single period flexibility offers
R_t	Flexibility activation needs of the TSO
S_t	Flexibility contracted by the TSO
$u_{a,n,t}$	Total requested modulation by the FSU
$v_{a,i}$	Modulation of flexibility service $i \in \mathcal{S}_n$
$[w_{a,i}, W_{a,i}]$	Requested modulation range of flexibility service $i \in \mathcal{E}_n$
$x_{a,i,t}$	Modulation of the flexibility service $i \in \mathcal{E}_n$
$y_{a,i}$	Binary variable for the reservation of service $i \in \mathcal{E}_n$
$z_{n,t}$	Binary variable equal to 1, if the bus is shed

4.2 Introduction

Change of practices in distribution systems call for a revision of the *interaction model*, that is, the set of rules guiding the interactions between all the parties of the system. The main goal of a distribution system operator (DSO) is to maintain the distribution system within operational limits while allowing grid users the largest possible range of behaviors, ensuring the system can accommodate renewable generation, and minimizing the grid operation costs. Distribution systems were typically sized for the empirically observed peak consumption, and to require few preventive or corrective control actions. The peak consumption is in general far lower than the sum of the capacities granted to the end users through their connection contract because, in many distribution systems, most end-users are consumers withdrawing energy asynchronously. However the current trend towards distributed generation, renewable or not, and demand side management makes the "consumption only" and "asynchronous behavior" assumptions on grid users less realistic. These two phenomena are likely to lead to many issues in operation. Two main options are available to overcome these issues. The first one is network reinforcement, which may need few operational expenses (OPEX) but substantial capital expenditures (CAPEX) and is potentially difficult to implement. Network reinforcement requires planning investments over many years and obtaining permits may be tedious. The second is *active network management* (ANM) [64] also called transactive energy techniques in some papers [125]. ANM consists in increasing the efficiency of distribution systems by operating the system using all control means available and the flexibility of the grid users. The ANM option requires low CAPEX but relatively high OPEX compared to network reinforcement, since it implies the remuneration of flexibility services and the cost of control actions. Obviously, a real implementation may be a combination of both options. However, even considering solely the ANM option, it is not trivial first to evaluate the long-term benefit of an ANM strategy, and second to compare alternative interaction models relying on ANM. The literature contains many papers showing the benefits of using such flexibility in distribution networks [4, 111, 112, 134] and the methods to deliver this flexibility [17, 142, 145]. However, few studies have been performed on the way that the flexibility in distribution network can be exchanged as a commodity in an unbundled electric system. Since the DSO does not own the assets able to provide flexibility, it has to procure flexibility by contracting with a Flexibility Services Provider (FSP) which may be producer, retailer, aggregator, etc. One current challenge faced by regulators is to design the legislative framework or interaction model in which actors exchange flexibility services located within a distribution network. An interaction model is a set of rules in a regulatory framework that guides the interactions between all the parties of the system. There are many possibilities to organize interactions between the actors of the system when flexibility comes into play. Therefore, there is a need to clearly state the interaction models that formalize these interactions and to set up a method for comparing their economical and technical performance.

This chapter deals with the second question by evaluating quantitatively a candidate interaction model, through a measure of the global welfare of the system it leads to. This measure accounts for the benefits of all actors in the system. The considered actors are the DSO, the transmission system operator (TSO), the producers and the retailers with the consumers they serve. The distribution network, meshed or radial, is modeled up to its interface to the high voltage network and down to medium voltage-low voltage transformers, i.e. low voltage sub-networks are aggregated. This work focuses on medium-voltage (MV) distribution systems with a substantial amount of renewable generation which may cause a reverse flow coming from the medium-voltage network and going to the high-voltage network. The time frame of this work spans the short term context, from a few days ahead until settlement and, therefore, do not consider structural changes of the system.

Two methods are used to perform the quantitative analysis of the interaction models organizing the exchange of flexibility within the distribution network: a macroscopic analysis and an agent-based model. The macroscopic representation of the system allows a quick evaluation of the economic efficiency of each model. In particular, the analysis uses data that can be easily obtained for each quarter of the year, such as the total production and consumption in the distribution network. The sum of the production and the consumption provides an approximation of the flexibility needs of the DSO for each quarter of the year. Depending on the interaction model, these flexibility needs can be matched either by shedding production or consumption, restraining the grid users or activating flexibility services. Using approximation of the different prices, the macroscopic analysis evaluates the costs on a typical year of the DSO, producers and retailers. The economic efficiency of each interaction model is estimated on the long term by repeating the process for each year of the time horizon using forecasts of the production, consumption, energy prices, etc.

Second, an agent-based simulation framework is set up to obtain more detailed information on the behavior of each actor. The actors of the system are modeled as individual agents which solve an optimization problem at every decision stage in order to maximize their individual objective, and each problem is constrained by the decisions taken at the previous stages, but subsequent recourse possibilities are taken into account. The impact of the agent's decisions are evaluated through a measure of the social welfare, the repartition of the welfare between the actors, and a measure of the service level that is reached. The agent-based model is published as open-source testbed under the name *DSIMA*, standing for *Distribution System Interaction Model Analysis*. This testbed is a framework composed of an instance generator, a simulator and a web-based user interface. These three modules allow various compromises between versatility and ease of modification. For instance, the behavior of an agent can easily be changed by changing its optimization problems written in the user-friendly modeling language ZIMPL; see Figure 4.1 for an example. More flexibility can be obtained by modifying the open-source Python 3 code available at the address http://www.montefiore.ulg.ac.be/~dsima/.

```
# Objective
maximize Profit:
sum <t> in Ts : (piE[t]*Pa[t]-piIP[t]*IP[t]-piIM[t
   ]*IM[t]
+sum <n> in Ns: (piFP[n,t]*fP[n,t]+piFM[n,t]*fM[n,t
   ])
);

# Constraints
subto TotalProductionAnnounced:
forall <t> in Ts : Pa[t] == sum <n> in Ns: pa[n,t];

subto TotalProduction:
forall <t> in Ts : P[t] == sum <n> in Ns: p[n,t];

...
```

Figure 4.1: *ZIMPL* code of one optimization problem solved by a retailer.

Although the methodology applied is general, this chapter defines and studies six interaction models among the numerous possible candidates. In the first two interaction models, the DSO does not use ANM and either restricts or not the access to its network to the grid users. Three additional models are focused on flexibility service usage for ANM. In Model 3, the DSO may restrain the users without providing any financial compensation except for the imbalance created by its request. In Model 4, the grid user is compensated financially for the activation request of the DSO. In Model 5, the DSO places no access restriction on to the grid users and relies on voluntary remunerated flexibility services to operate its network. A last model is based on dynamic access bounds to the network changing throughout the day and computed by the DSO. These bounds are computed using baseline proposals from the grid users at the medium voltage level and prevents the activation of flexibility in directions that could lead to congestions.

This chapter is organized as follows. Section 4.3 reviews the relevant literature. Note that general literature about agent-based modeling is omitted in this chapter and can be found in Section 3.3. The candidate interaction models are presented in Section 4.4. These models are first compared using the macroscopic analysis described in Section 4.6. Section 4.7 gives more insight on how these interaction models could be implemented through a simulation in agent-based model. Finally, Section 4.8 concludes.

4.3 Literature review

In the unbundled electricity sector, regulation of the interactions between the parties is a key factor. Several attempts to define interaction models for the exchange of flexibility within a distribution system have been proposed in the literature. An example of specifications to exchange flexibility services is proposed in [72], which streamlines the relevant business interactions and illustrates the concept on a low-voltage transformer overload case study. Based on these specifications, [150] shows examples of specifications for different flexibility service contracts accommodating the requirements of DSOs. Three potential strategies for congestion management based on the generic interaction model of [72] are presented in [9]: distribution grid capacity market, advance capacity allocation, dynamic grid tariff. The authors provide a qualitative analysis of their models. Their model *advance capacity allocation* is close to the interaction model 6 presented in Section 4.4. Another framework to coordinate the flexibility usage on a low-voltage feeder based on flexibility margins imposed to controllable resources is proposed in [114]. A methodology to evaluate the performance of an ANM strategy is proposed in [63]. The strategies evaluated consider the control of active power injections and the direct-control of loads through flexibility services. Business case from the DSO perspective of dynamic line rating, demand side management and network reinforcement for a MV-grid use case is investigated in [140]. The VirGIL co-simulation platform allows studying interactions between demand response strategies, building comfort, communication networks, and power system operation in smart grids [23]. Its modular architecture allows the platform to integrate complex building models given by differential equations describing the dynamic of the buildings.

To guarantee that distribution networks can be safely operated, the most commonly used method is to restrict the access of distributed generation to the distribution network. DSOs even sometimes tend to deny the connection of new installations to their network. The power systems literature contains many solutions to operate the network considering the curtailment of distributed generation units [33, 78, 79, 81]. Regulated non-firm generation access contracts are introduced in [33] where units are subject to curtailing and/or shedding, depending on the requirements from the network operation. The DSO may define the operating margins of the units with the latter contract to reach a safe operating point. A method to determine these margins is proposed and demonstrated in [33]. Similar methods have been studied from simple heuristic control schemes to computationally intensive optimal power flows [78, 81]. Article [79] reviews and analyses renewable energy curtailment schemes and principles of access contracts.

On the demand side, flexibility is mostly obtained through dynamic pricing [38, 106] or direct control of the loads [66, 98, 132]. Both methods have drawbacks. Dynamic pricing exposes the end-user of the load to volatile prices, which may not be appropriate for retail consumers [117]. Dynamic pricing also leaves uncertainty on the quantity of flexibility that is actually deployed and therefore on the cost of this flexibility. This uncertainty does not arise with the direct control of the load but requires direct access to the controlled loads that implies challenging control issues and intrusive equipments. One alternative to these two methods is to rely on dynamic fuse at the network connection and leave to the connected equipment the optimization of its consumption in order to satisfy the power limit of the fuse [117]. An aggregator may control the fuse limit in case the total net load of the households should be limited to ensure the well functioning of the entire network [93].

Several European projects study the interactions models to integrate flexibility services in the present electric system. The *ADDRESS* project aims at developing a comprehensive commercial and technical framework for the development of active demand in the smart grids of the future [11]. This project focuses on questions such as which information should be available for each actor or which reference consumption to choose. Other projects in Europe rely on the deployment of demand side management such as GREDOR [64], iPower [76], EvolvDSO [126], LINEAR [40], ADINE [2], Local Load Management [84] and Nice Grid [107]. This chapter introduces a testbed that can be seen as a convenient tool to test the ideas proposed in the previous projects.

4.4 Candidate interaction models

An important part of ANM is devoted to the coordination for the usage of flexibility to operate a distribution system. How the flexibility services may be exchanged depends on the interaction model that the agents must follow. In this work, we propose and analyse six interaction models.

Model 1. *The DSO does not use any flexibility service and does not restrict grid users.*

Model 2. *The DSO does not use any flexibility service. To ensure the safety of its system the DSO restricts the users to a safe full access range computed on a yearly basis.*

Model 3. *An access contract specifies a full access range and a wider flexible access range. These access bounds are represented in Figure 4.2. The grid user may produce or consume without any restriction within the full access range,* *which is computed on a yearly basis. The full access range is determined such that no ANM strategies are needed if all grid users are in their full access range. If necessary, the DSO may ask a grid user to restrain its production or consumption in the full access range in critical periods where the agent is in its flexible access range. This restriction does not lead to financial compensation by the DSO except for the imbalance created by the request.*

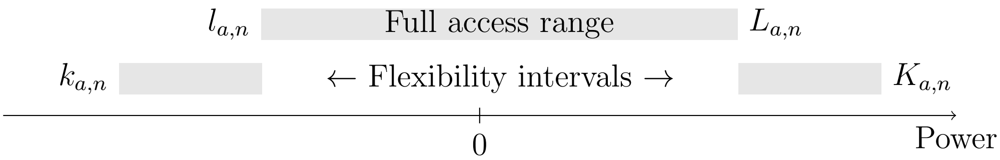


Figure 4.2: Definition of access bounds.

Model 4. *This model is equivalent to Model 3 but the DSO pays for the activation of the flexibility of the grid users. For instance, this case allows producers to recover from the loss of the subsidies for renewable energy generation.*

Model 5. *The DSO acts as a simple flexibility user like the TSO or every BRP. The DSO does not restrict the grid users and relies on the flexibility offered by the other agents to operate its network.*

Model 6. *An access contract specifies a full access range and a wider flexible access range as in Models 3-5. On a daily basis, the DSO defines a dynamic access range, changing in each quarter, computed based on baseline proposals from the BRPs. This dynamic range at least includes the full access range.*

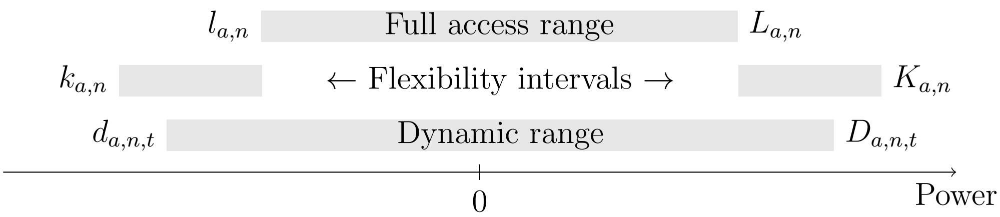


Figure 4.3: Definition of the dynamic access bounds.

A modulation request by the DSO to a grid user is a reduction of its production or consumption to ensure the safe operation of its system. A modulation influences the total balance of the system with respect to the announced total production and consumption. The imbalance created by a modulation is paid by some parties in the system. The choice of the parties responsible for the imbalance depends on the interaction model. In Models 3-5, we consider that the DSO is responsible for the imbalance. The economic performances of the six models are evaluated using the procedures described in Section 4.6 and 4.7.

4.5 Welfare

In the context of this work, we aggregate the large amount of results obtained from simulating a complex system into a single measure. This measure provides a quantitative idea of the health of the system, or welfare. The welfare is a broad term that defines the "health, happiness, prosperity, and well-being in general" [27]. The simple measure of the welfare adopted in this work is obtained by summing up the benefits and costs of each actor with their sign. To this sum, we add the value of lost load [139] in the case of shedding due to the tripping of a protection, e.g. a selector or a circuit breaker resulting in the complete disconnection of the components behind the protection. This last component adds up a notion of quality of service in the welfare which directly impacts the final consumer of electricity. Note that the term shedding is different from the term curtailment which consists in reducing the production of a renewable generation unit and not totally disconnecting it from the network.

With this definition, one way to increase the welfare is to increase the amount of renewable generation that is produced within the distribution network. It directly increases the benefits of the corresponding producers. On the other hand, if the interaction model does not allow managing the network safely, protections are tripped and the welfare decreases. Other changes have no impact on the welfare. For instance, increasing the cost of flexibility services increases the benefits of FSPs but decreases by the same amount the one of FSUs, leaving the sum unchanged. This motivates to not only look at the welfare but also to the various terms composing it.

4.6 Macroscopic analysis

In this section, each model is evaluated using a macroscopic representation of the system, by opposition to more complex techniques such as the one used in Chapter 3 and further in this chapter, in Section 4.7. The purpose of this section is to provide a quick estimation on the long term of the economic efficiency of the candidate interaction models. Section 4.6.1 describes the procedure of the macroscopic analysis. Results on a year representative of the expected trends toward the year 2025 are presented in Section 4.6.2. Results of a complete simulation of the horizon 2015-2030 are presented in the Section 4.6.3.

4.6.1 Evaluation procedure

The evaluation of the interaction models is done by getting an approximation of the actions to take for each quarter of a typical year. To obtain a base scenario for each quarter of a typical year, the simulation is based on historical production and consumption data. The consumption data comes from the total consumption monitored in 2013 by ELIA, the Belgian transmission system operator [42]. The production curve is taken from a real wind production unit in 2013. These curves are scaled to match a given mean and maximum value on the whole year. The resulting production and consumption values for each period t of the simulated year are denoted respectively by P_t^p and P_t^c. The energy and imbalance price curves are processed alike taking as a basis the BELPEX day-ahead energy market prices of 2013 [12]. The net export from the distribution network in a period t is given by $N_t = P_t^p - P_t^c$. If this net export is greater than the capacity of the network C, actions needs to be taken do deal with the exceeding production. For the sake of conciseness, we do not consider the case where $N_t < -C$ as distribution networks are usually designed to satisfy the peak consumption. Note that in the Model 2, P_t^p is capped to C to model the safety restriction of the DSO. In Model 6, P_t^p is capped to $C + P_t^c$ so that the exceeding production is curtailed without activating flexibility.

The next step decides for each period t what are the quantities of flexibility activated on the production side and on the demand side; and the production quantities curtailed, R_t^p. For Model 1, if $N_t > C$, then $R_t^p = N_t - C$. For the other interaction models R_t^p are the remaining exceeding quantity that could not be handled using flexibility services.

In Models 3 to 5, the DSO uses flexibility to deal with the exceeding energy $N_t - C$. Regarding the flexibility on the production side, the maximum quantity available is bounded by the total production. The cost for the DSO to use this production flexibility is equal to the imbalance price plus the flexibility cost depending on the interaction model. In Model 3, there is no flexibility cost. In Model 4, the DSO pays only for the activation and in Model 5 the DSO pays for the reservation and the activation of the flexibility. The DSO may also use flexibility from the demand side. The total amount available is computed by βP_t^c where β is the flexibility ratio of the demand side. We consider that a modulation of the demand side in one period induces an equal modulation in the opposite direction in the next period. The costs for the DSO to use the flexibility spans on two periods, the period t and the following one, $t+1$, to consider this payback effect. Therefore a modulation F_t^c modifies the consumption in the next period such that $N_{t+1} := P_{t+1}^p - P_{t+1}^c + F_t^c$. As a result, we consider that the DSO pays the imbalance created in period t and $t+1$. However, the reservation and activation is paid only for the period t. A last criterion to check before using demand side flexibility is that the payback effect does not cause problems in the period $t+1$. To remove a congestion using flexibility, the DSO chooses the cheapest option between the production and the demand side if the payback effect allows its use.

The final step is the computation for the period t of the surpluses and the costs of each actors. These surpluses and costs are computed following the interaction models directives using if needed a reservation and/or an activation price for the flexibility provided. Producers earn surpluses from selling electricity at the day-ahead energy market price and must cover their marginal production costs. Retailers buy energy to the day-ahead market and earn money from the retailing activity whose tariff is fixed for the whole year. The network management costs imputable to the DSO is given by the cost of activating flexibility. A welfare is computed as the sum of the surpluses minus the costs of each actor. A penalty term is added for the production and the consumption shed assuming a value of lost production and a value of lost load.

The whole evaluation procedure was implemented in Python and run on several scenarios as detailed in the next sections.

4.6.2 Focus on 2025

This section shows results for a one year simulation corresponding to the expectation of 2025. The maximum capacity of the network is assumed to be 40MW. The marginal production costs of producers is fixed to -45€/MWh. This cost is negative to take into account the subsidies for renewable production. The value of lost load is arbitrarily set to 1000€/MWh and the one of lost production to 500€/MWh. The flexibility prices of the producers are 20€/MW for the reservation and 45€/MWh at the activation. For the retailers, the flexibility has a reservation cost of 5€/MW and no activation cost. Remaining parameters values can be found in Table 4.5 for the year 2025.

The mean expected production and consumption traded for the simulated year are respectively 93805 and 140525MWh or an average by day of 257 and 385MWh. Note that the real total production, without curtailment or shedding, is given by Model 6 with 363MWh on average per day. The maximum production for one hour is 76.4MWh traded, 61.7MWh produced, and the maximum consumption 25.6MWh. The energy production exceeding the network capacity for each day is shown in Figure 4.4 and is handled as explained in the previous section.

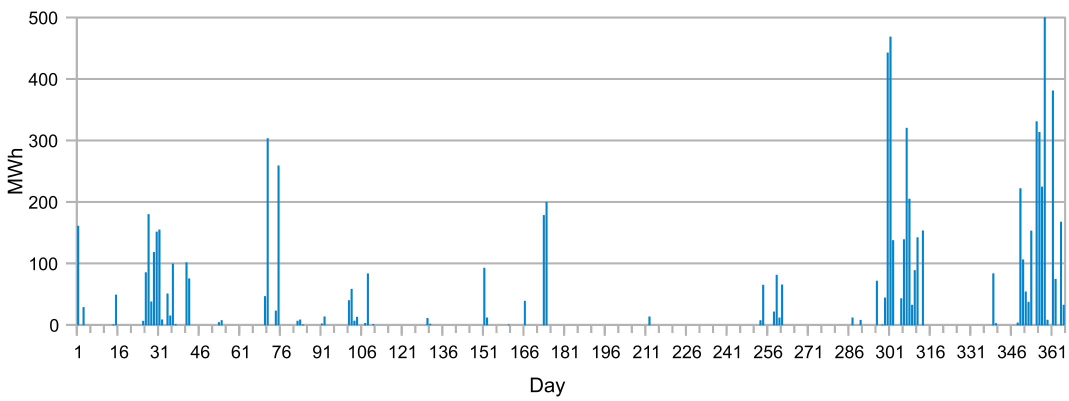


Figure 4.4: Simulation of the expected production exceeding the network capacity for each day in 2025.

A comparison of the models is provided in Table 4.4 where the figures are aggregated by day. Over the year, the mean lack of capacity is 21.9MWh which represents 5.69% of the total production. Models $3-5$ obtain identical welfares as their only difference is the remuneration of the flexibility from the DSO to the producers. The flexibility from the demand side is not used by the DSO due to the imbalance costs. This is the consequence of having imbalance prices greater than prices of the production side flexibility services. In Model 5, the flexibility of the demand side is cheaper for some hours but the payback effects prevents using it. Model 6 obtains a slightly lower welfare even though no imbalance occurs due to activation of flexibility. However, less production is sold on the day ahead energy market which reduces the surplus of the producers on average by 741€ per day. This loss of surplus does not compensate the imbalance costs of 388€ on average per day that the DSO may have to pay.

4.6.3 Trends to 2030

The expected evolution of the macroscopic parameters can be obtained from [22]. This report provides the development of the EU energy system under current trends and policies in the EU27 and its Member States. An evolution of the gross wind onshore generation is forecasted for 2015, 2020, 2025 and 2030 of respectively 72, 146, 204 and 276TWh. The after tax energy prices for industry should be approximately of 92, 101, 104 and 98€/MWh. The expected electricity consumption is about 3000, 3194, 3370 and 3515TWh. These expected evolutions are used to obtain the parameters of our network in the horizon 2015-2030 based on the 2015 figures. The obtained expected parameters are summarized in Table 4.5.

Figure 4.5 shows the expected evolution of a system welfare for the six interaction models and the 15 years horizon. The first three years, all the models perform similarly with a difference of welfare of less than 2%. The welfare of Model 1 increases until 2021 then decreases constantly due to the high necessity of production shedding. By restricting the production, Model 2 achieves reasonable welfares at about 10% of the one of Models 3 to 5 for the five last years. The cumulative welfare of Model 1 and 2 are respectively 24% and 7% smaller than the one of Models 3 to 5. Model 6 is slightly less efficient than Models 3 to 5, with a difference smaller than 0.05%.

Since the welfare of Models 3 to 5 are identical, selecting one of these models should be based on other criteria. The main difference is the amount that the DSO pays to the producers. These models impacts the actors costs and revenues favoring either the producers or the DSO. The active network management costs paid by the DSO for the Model 3, 4 and 5 are represented in Figure 4.6. Model 3 achieves the minimum cost for the DSO and comes from the payment for the imbalance created by the activation of flexibility services.

Table 4.4: General daily results of the macroscopic analysis for the six interaction models for the expected 2025 year.

	Model 1			Model 2			Model 3			Model 4			Model 5			Model 6			
	min	*mean*	*max*	*min*	*mean*	*max*	*min*	*mean*	*max*	*min*	*mean*	*max*	*min*	*mean*	*max*	*min*	*mean*	*max*	
Welfare	-168431	26450	127255	-24668	34059	112788	-24668	37217	156499	-24668	37217	156499	-24668	37217	156499	-246678	36865	146614	€
DSO costs	0	0	0	0	0	0	-606	389	8540	0	1358	26839	0	1789	36127	0	0	0	€
Prod. surplus	-15455	35346	157779	-154534	32188	118334	-15454	35735	165598	-15454	36704	179052.5	-15454	37135	185032	-15454	34994	147894	€
Ret. surplus	-13986	1871	11407	-13986	1871	11407	-13986	1871	11407	-13986	1871	11407	-13986	1871	11407	-13986	1871	11407	€
Production	-135	385	1654	-135	335	960	-135	385	1654	-135	385	1654	-135	385	1654	-135	363	1343	MWh
Consumption	38	262	498	38	262	498	38	262	498	38	262	498	38	262	498	38	262	498	MWh
Net export	-619	124	1424	-619	124	1424	-619	124	1424	-619	124	1424	-619	124	1424	-619	124	1424	MWh
Over-capacity	0	21.5	498	0	0	0.7	0	21.5	498	0	21.5	498	0	21.5	498	0	0	0	MWh
Prod. shedding	0	21.5	498	0	0	0.7	0	0	0	0	0	0	0	0	0	0	0	0	MWh
Prod. flexibility	0	0	0	0	0	0	0	21.5	498	0	21.5	498	0	21.5	498	0	0	0	MWh

Table 4.5: Expected evolution of the parameters of the evaluation procedure.

	2015	2020	2025	2030	
Demand					
mean	9.70	10.33	10.90	11.37	MW
max	19.0	20.23	21.34	22.26	MW
Production					
mean	10.0	12.84	16.04	18.68	MW
max	40.0	51.37	64.16	74.72	MW
Electricity price					
mean	47.45	52.09	53.64	50.54	€/MWh
max	82.3	90.35	93.03	87.67	€/MWh
Retailing price	60.5	66.42	68.39	64.45	€/MWh

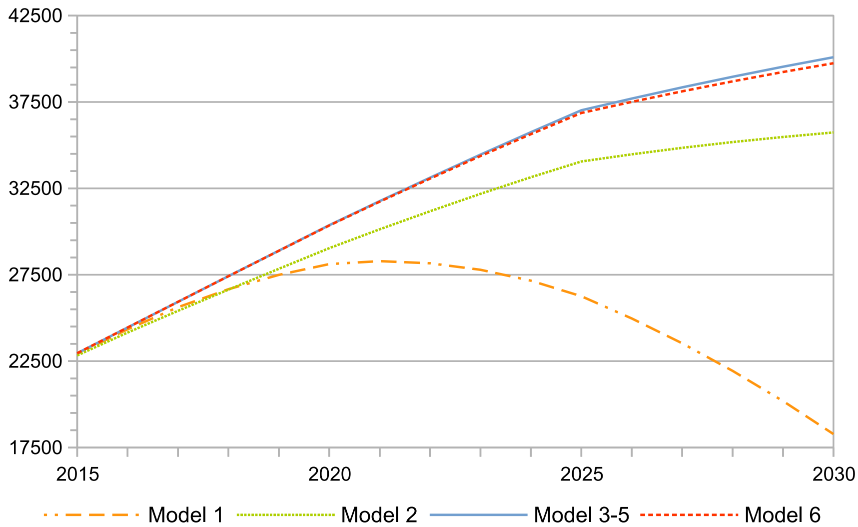


Figure 4.5: Mean daily welfare of the six interaction models in € for the 15 years horizon.

On the last five years, Model 4 and 5 cost respectively around 3.5 and 4.5 times more than Model 3 to the DSO. The cumulative costs of Model 3 and 4 on the whole horizon are respectively 21% and 76% of the cumulative costs of Model 5.

4.7 Agent-based analysis

This section presents an agent-based model devised to evaluate in more detailed the interaction models briefly described in Section 4.4. Section 4.7.1 presents the model of the system and the roles that each agent can take. The method used to integrate the six interaction models into this agent-based model is described in Section 4.7.2. The agents of the system are described in Sec-

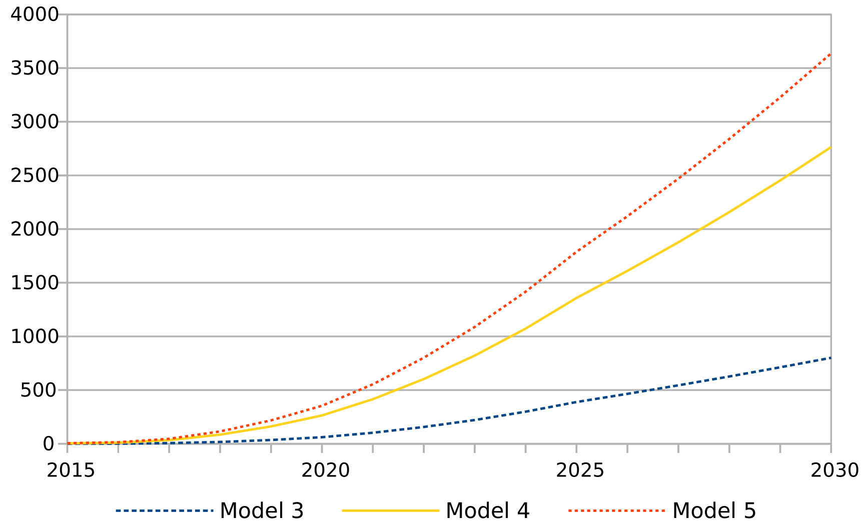


Figure 4.6: Mean active network management daily costs of Models 3, 4 and 5 in € for the 15 years horizon.

tion 4.7.3 which refers extensively to Section 4.9 containing the optimization problems characterizing the behavior of the agents. Section 4.7.4 describes the implementation of the testbed as a simulator, an instance generator and a user interface. Finally, results with the agent-based model are given in Section 4.7.5.

4.7.1 General view of the system

In the sequel of this section, we make the distinction between the roles and the actors. The roles considered in this work are balancing responsible parties (BRP), flexibility services providers (FSP) and flexibility services users (FSU). The actors simulated in this work are the DSO, the TSO, producers and retailers, and may fulfill more than one role, depending on the interaction model. To operate its network, the DSO may act as a FSU. The TSO is, from the point of view of this simulation, a FSU with given needs of flexibility services. Producers and retailers both act simultaneously as BRPs, FSPs and FSUs.

All of the interaction models we consider follow the procedure below.

1. DSO analysis and TSO imbalance settlement are based on reference baselines for every bus given by the BRPs.
2. On the day ahead, after the clearing of day-ahead energy market, BRPs submit their baselines to the DSO and the TSO.
3. The DSO assesses the state of the system and announces its flexibility needs to the FSPs. Other FSUs also announce their needs to the FSPs.
4. FSPs provide flexibility offers sequentially to each potential FSU, the first FSU being the DSO.

5. FSUs request and/or buy some proposed flexibility offers for their needs.
6. Closer to real-time, FSUs request the activation of their flexibility services.
7. Right before real-time, each FSP optimizes its realization taking into account the request of its FSUs, the penalty incurred if a flexibility service is not provided and its imbalance with respect to the BRPs of the loads it controls.
8. The distribution network is operated using these realizations taking last resort actions if necessary, such as shedding buses. If such actions are needed and FSPs did not provide their service to the DSO, the FSPs are penalized at a regulated price.

These interactions between the actors of the system are summarized in Figure 4.7. The course of events is described from left to right going from short-term interactions to settlement. Each type of actor is represented by a horizontal line. Each vertical arrow represents an interaction between two types of actors. The actor on the heads of the arrow receives the information from the ones at the tails of the arrow indicated by the circles.

In Model 6, the first step of the previous procedure is replaced by the following.

1. BRPs provide baseline proposals that may belong to the full access range or the flexibility intervals.
2. Based on these proposals, the DSO computes the dynamic ranges so that its network is secure and communicates them to the BRPs.
3. BRPs submit new baselines, constrained to lie within the dynamic ranges, to the DSO and the TSO.
4. These baselines are used as reference for the provision of flexibility services. If the realization of a BRP violates the dynamic ranges, the BRP is penalized at a regulated tariff higher but of the same order of magnitude than the imbalance price.

These additional steps are depicted by the dashed lines in Figure 4.7.

We assume that there is an access contract between a BRP a and the DSO for each bus n that the BRP has access to. A contract specifies a full access range $[l_{a,n}, L_{a,n}]$ and a wider flexible access range $[k_{a,n}, K_{a,n}]$, cf. Figure 4.2. The BRP can produce or consume without any restriction within the full access range $[l_{a,n}, L_{a,n}]$, but in the flexibility intervals $[k_{a,n}, l_{a,n}]$ and $[L_{a,n}, K_{a,n}]$ the DSO may order a restriction of the production or consumption, if necessary. In general, stakeholders are free to exchange flexibility services among them, but if the profile of a BRP is within the flexibility intervals, it has to propose flexibility offers to the DSO via FSPs. In Model 6, the DSO defines for each

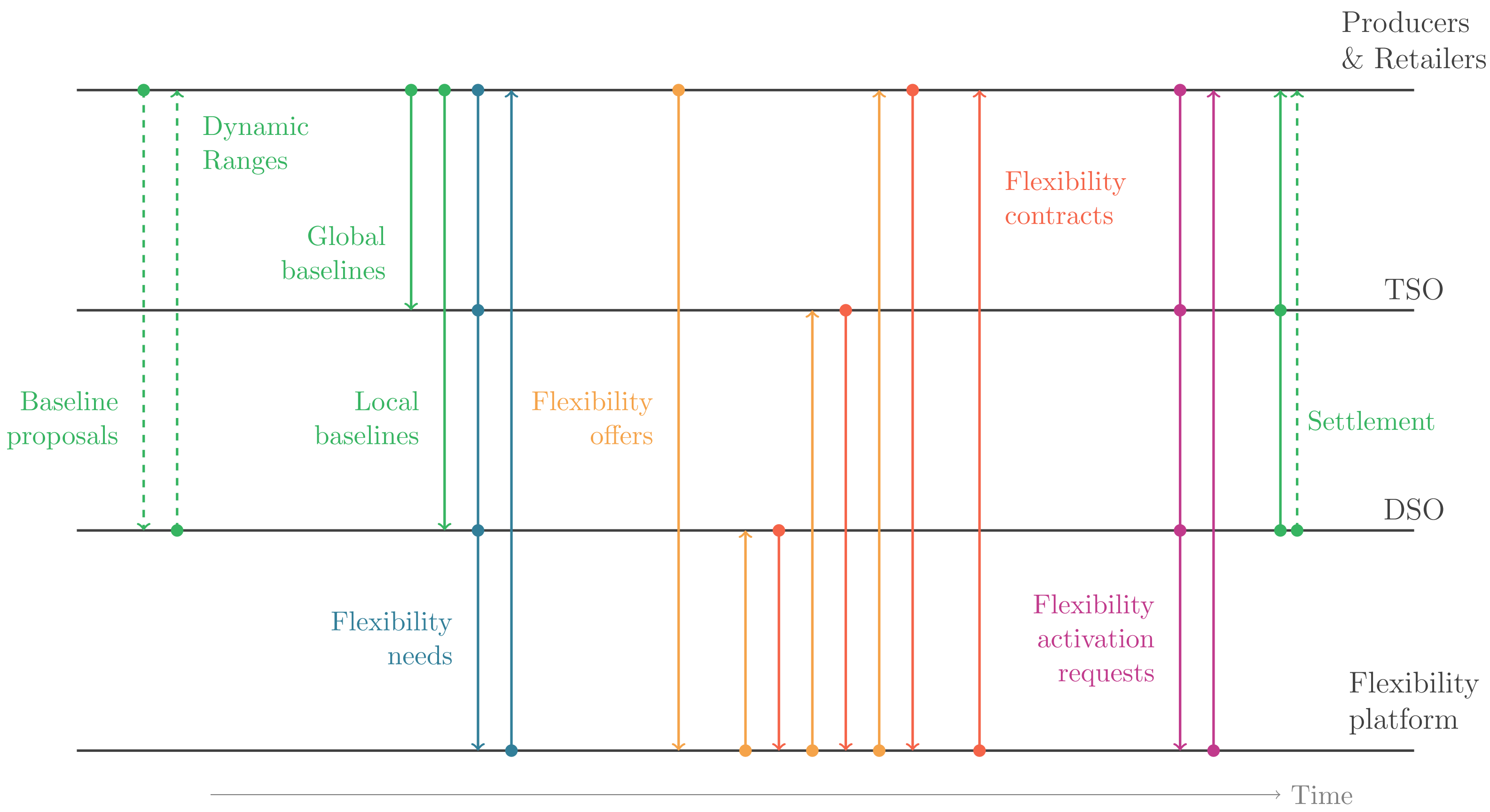


Figure 4.7: Timeline of the interaction between the agents. To each arrow corresponds an exchange of information whose source is given by the dot and the destination by the arrow head. The dotted lines correspond to additional interactions in Model 6.

quarter the dynamic ranges $[d_{a,n,t}, D_{a,n,t}]$, constrained to be larger than the full access range as shown in Figure 4.3.

Note that baseline proposal of Model 6 are mandatory to prevent a grid user to have incentive to produce or consume outside of its dynamic range established on a definitive baseline. Assume a producer has a flexible range of $[6, 10]$MW in one bus and decides to produce 10MW, so that its baseline its 10MW. After its computations, the DSO grants the dynamic range $[0, 8]$MW. The producer has to choose between paying an imbalance of 2MW to the TSO or paying a penalty to the DSO. Now assume that the producer may sell a flexibility service to another BRP for a downward modulation of 1MW, modulation for which the producer is remunerated. The producer has incentive to produce 9MW, provide the flexibility service and pay only for 1MW of penalty to the DSO. Note that the producer is not responsible for any imbalance with this decision.

The following subsections details roles that agents can take and the consequences on the information they exchanged and actions they take. The index a is used for an agent, n for a bus and t for a period. The set of buses considered by an agent a is denoted $\mathcal{N}_a$ and is a subset of the total set of buses $\mathcal{N}$.

Balancing Responsible Party

Each BRP, in the interaction model proposed, is responsible for its local and global baseline and performs the following actions:

1. Submits its global baseline to the TSO.
2. Submits its local baselines to the DSO.
3. Notifies FSPs of the obligation to provide flexibility services, if the local baseline is not in the full access range.
4. Pays the TSO for its global imbalance.
5. Pays the DSO for the local imbalance, if a FSP for which the BRP is responsible did not provide a flexibility service.

In Model 6, there is an additional step at the beginning of this procedure to submit a baseline proposal to the DSO, denoted $p^p_{a,n,t}$.

The local baseline, $p^b_{a,n,t}$, that is positive for a production and negative for a consumption, is provided on a day-ahead basis by the BRP a for each MV bus n and each period t to the DSO. The global baseline $P^b_{a,t}$ is provided by the BRP to the TSO. For the sake of clarity and conciseness, we consider that the BRP only has assets in the simulated system such that $P^b_{a,t} = \sum_{n\in\mathcal{N}_a} p^b_{a,n,t}$.

The global imbalance of the BRP, $I_{a,t}$, is computed considering the actual realization $P_{a,t}$, the total amount of flexibility activated $U_{a,t}$ and the flexibility provided $H_{a,t}$:

$$I_{a,t} = P_{a,t} - (P^b_{a,t} + U_{a,t} + H_{a,t}). \tag{4.1}$$

The imbalance cost to the BRP is given by

$$\sum_{t\in\mathcal{T}}(\pi_t^{I^+} I_{a,t}^+ + \pi_t^{I^-} I_{a,t}^-). \tag{4.2}$$

where $I_{a,t}^+$ and $I_{a,t}^-$ are equal, respectively, to the positive and negative part of the imbalance in period t, $I_{a,t}$.

If the local baseline of the BRP, $p_{a,n,t}^b$ is within a flexibility interval such that $k_{a,n} \leq p_{a,n,t}^b < l_{a,n}$ or $L_{a,n} < p_{a,n,t}^b \leq K_{a,n}$, the BRP notifies the concerned FSPs that they must provide flexibility services such that the realization $p_{a,n,t}$ can be included in the full access range $[l_{a,n}, L_{a,n}]$. Note that in order to ensure that the FSP can propose such a service, it probably has to optimize by itself the whole baseline $\{p_{a,n,t}|\forall t \in \mathcal{T}\}$.

The local information of the day-ahead baseline $p_{a,n,t}^b$ is used by the DSO to operate its system. If requests to activate flexibility are not satisfied, the BRP responsible for these FSPs is penalized at a regulated price proportional to the volume π^n:

$$\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}_a} \pi^n|(p_{a,n,t}^b + u_{a,n,t} + h_{a,n,t}) - p_{a,n,t}|. \tag{4.3}$$

where $p_{a,n,t}$ is the local realization, $u_{a,n,t}$ the local flexibility activated in period t and $h_{a,n,t}$ the local flexibility provided.

Flexibility Services Provider and User

The main interactions that the FSP conducts are:

1. Obtains the flexibility needs of the FSUs, the baselines and the obligations from the BRPs of the assets it controls.
2. Proposes flexibility services.
3. Activates flexibility upon the request of the FSUs.

Flexibility services need a reference in order to be quantified. In this paper, we chose a baseline $p_{a,n,t}$ provided by the BRP. Note that if the FSP is its own BRP, the agent with the combined roles can optimize its baseline to maximize the flexibility it provides. The other data at the FSP's disposal is an indicator of the flexibility needs in each bus and each period upward and downward. The latter indicator is provided by the FSUs to help the FSP to quantify the action it is able to perform.

A FSP offers a service i at a reservation price π_i^r and an activation price π_i^b. We consider two types of flexibility services which are represented in Figure 4.8: a single period flexibility service, tailored, for instance, for curtailment offers of wind generation units, which defines a modulation range $[m_{i,\tau_i}, M_{i,\tau_i}]$ available for a given period τ_i; and energy constrained flexibility service which defines for each period t, a modulation range $[m_{i,t}, M_{i,t}]$. A user of the latter service may choose the modulation for each period under the constraint that the average modulation is zero. This service is well tailored for load modulation because the energy procured to the load is preserved over the time horizon with respect to the baseline consumption.

In the last part, FSUs that have contracted some services provide their activation requests to the FSPs. The total desired modulation at bus n and period t is denoted by $h_{a,n,t}$. The FSP controls its assets to provide the desired modulation with respect to the precomputed baseline.

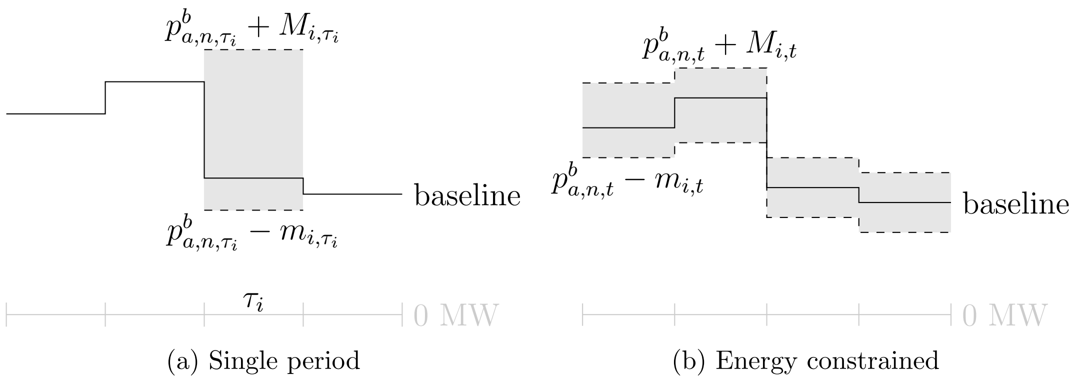


Figure 4.8: Flexibility services implemented.

A FSU has three main interactions with the system.

1. Computes and communicates an indicator of its upward and downward flexibility needs in adjustable power for each bus and each period.
2. Evaluates the flexibility offers from the FSP and selects the services to contract.
3. Requests the activation of the contracted flexibility services.

How the needs are defined, the offers are evaluated and the activation requested depends on the agent acting as FSU, cf. Section 4.7.3. For instance, the DSO computes its flexibility needs to overcome anticipated congestions.

4.7.2 Formalization of the interaction models

The differences between interaction models originate from the usage the stakeholders make of flexibility services, from the remuneration and financial compensations associated with the flexibility services, and from the terms of access contracts delivered to the grid users. Table 4.6 summarize the parameters of the interaction models. In the interaction models 1, 2 and 6, the DSO does not use flexibility services while in Models 3, 4 and 5, the DSO recourse to flexibility services to operate its network. In the latter case, one can distinguish

Table 4.6: Summary of the parameters differentiating the interaction models.

Interaction model	**Access type**	**Financial compensation**
Model 1	unrestricted	/
Model 2	restricted	/
Model 3	flexible	imbalance
Model 4	flexible	full
Model 5	unrestricted	full
Model 6	dynamic	/

different cases of remuneration for flexibility. The flexibility service may be remunerated for its reservation, its activation, both or none. In all considered interaction models, the DSO pays the imbalance caused by the activation of the flexibility services it requests.

Access contracts define whether and to which extent the DSO could require flexibility from its grid users. We consider that a FSP first makes a request to the DSO to get access to a range $[g_{a,n}, G_{a,n}] : g_{a,n} \leq 0 \leq G_{a,n}$. Based on these requested access bounds, the DSO computes safe access bounds $[b_{a,n}, B_{a,n}] : b_{a,n} \leq 0 \leq B_{a,n}$. The safe access range ensures that no congestion occurs if every grid user accesses the network in the limits $[b_{a,n}, B_{a,n}]$. A more comprehensive definition of this procedure is outside the scope of this work. Section 4.7.3 presents one simple method adopted for the simulation. The link between $[b_{a,n}, B_{a,n}]$, $[l_{a,n}, L_{a,n}]$ and $[k_{a,n}, K_{a,n}]$ is particular to each interaction model. The access bounds in function of the interaction model are represented in Figure 4.9.

4.7.3 Agents

The agents simulated in this work are the DSO, TSO, producers and retailers. They may fulfill more than one role depending on the interaction model. To operate its network, the DSO may act as a FSU, as in Model 5 for instance. The TSO is, from the point of view of this simulation, a FSU with given needs of flexibility services. Producers and retailers both act simultaneously as BRPs, FSPs and FSUs. The complete optimization problems solved by the agents are provided in the Appendix.

Distribution System Operator

In our simulations, the DSO acts with the objective to reach the technical optimum of the system. In this testbed, the network is modeled as a network flow model. This model considers only active power flows and therefore cannot grasp voltage issues. The extension to an optimal power flow model over

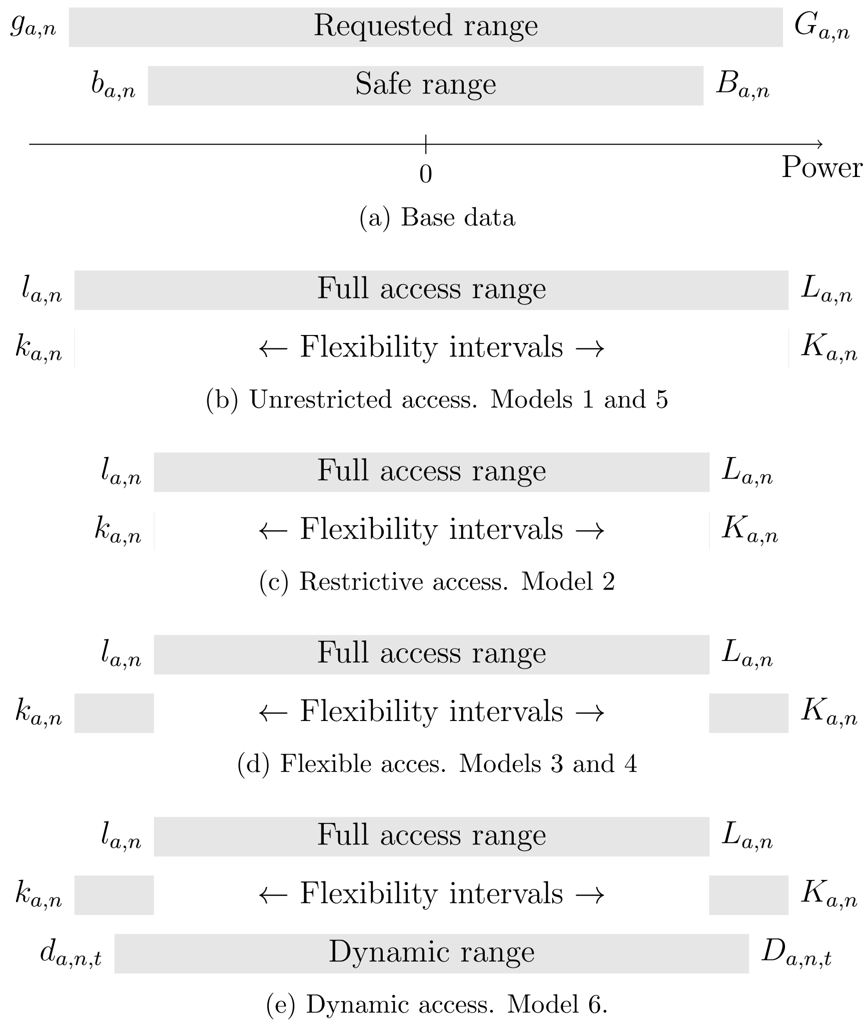


Figure 4.9: Definition of the access bounds for each interaction model.

multiple time periods leads to non-linear and non convex problems. These problems are computationally challenging, see [68] for a review of the current techniques to solve this kind of dynamic optimal power flows. Integrating OPF would make the testbed more complex to use, increase the computational power needed to perform the simulations, and further increase the quantity of input data.

Prior to the day by day simulation of the system, the DSO computes the access bounds $[b_{a,n}, B_{a_n}]$ to its system. In a real system, this procedure would not be performed in one round, since access contracts are delivered within a certain delay after they were requested. The aggregation of contracts of the low voltage contributors should also be considered. A more realistic simulation would, therefore, consider the legacy of access contracts. Instead, we opt for the following procedure, that is performed only once for the whole simulation range, e.g. one year. The bounds for each bus and each agent $[b_{a,n}, B_{a,n}]$ are computed by solving the problem (4.5), that provides bounds that are as close a possible from the bounds that were requested by minimizing the e, but taking into account the operational limits of the system, here the line capacities only.

The next procedure applies only for Model 6. The DSO takes as input the baseline proposals provided by the BRP and compute the dynamic ranges of each agents for the whole day. These dynamic ranges are obtained by solving the problem (4.8).

Next, the DSO acts as a FSU. Taking as input the baselines provided by the BRPs and the characteristics of the network, the DSO provides an indicator of its flexibility needs in MW for each bus and each period in order to alleviate congestion in the lines. These needs are obtained by solving problem (4.10).

For the flexibility procurement, the DSO solves problem (4.11). Note that even though the DSO is not a BRP, it pays imbalance fees caused by its usage of flexibility. This incentivizes the DSO to activate, when it uses one flexibility service downward to alleviate a congestion, to use another one upward to restore the balance. The flexibility activation optimization problem is identical to (4.11) except that it considers only the contracted flexibility services and their activation costs. Note that as we use a deterministic simulation, nothing changes between the reservation phase and the activation phase. As a result, the decisions taken in the activation phase are identical to the ones foreseen in the reservation phase.

Finally, we opt for a simple real-time system control strategy based on the realizations of the agents, which mimics a protection scheme. If the flow in some lines is over their thermal capacity, the DSO sheds production or consumption according to the optimization problem (4.12).

Producer and retailer

A producer is a BRP, FSU and FSP. In order, a producer announces its baselines, requests flexibility services, offers flexibility services, buys and activates flexibility services, and finally regulates its balance. A retailer is an actor that retails energy to its customers and manages aggregated loads at the MV level. It fulfills the same roles as the producer and, therefore, follows exactly the same steps. For the sake of conciseness, we only detail the producer. The specific optimization problems of the retailer (4.17), (4.18), (4.20) and (4.21), are given in Section 4.9.

The baselines of the producer are obtained by solving the optimization problem (4.13). In this stage, the producer already considers the possibility to be in imbalance considering the expected imbalance price. The problem takes as parameters the total upward and downward flexibility needs for each bus and each period. These parameters could come from forecasts of the producer or FSUs. The first approach is utilized in this paper, where the forecast is based on the flexibility needs communicated by the FSU in the previous days.

Note that the real needs of the flexibility users are communicated after the baselines are obtained according to the interaction models. In Model 6, the producer submits first a baseline proposal obtained by solving the optimization problem (4.13). Once the producer receives its dynamic ranges, it solves again the same optimization problems adding the constraints given by the dynamic range to its expected realization.

Once the producer is notified of the acceptance of its offers on the day-ahead energy market, it optimizes the flexibility it is able to provide with respect to the announced baseline. This step is modeled by the optimization problem (4.14). The producer procures and activates flexibility services based on the solution of problem (4.15).

Finally, the producer decides on the actual realizations for each bus and each period by solving problem (4.16) considering as a parameter the flexibility activation requests as a FSP and its flexibility requests as a FSU. The producer minimizes its imbalance in each period and also ensures that the realization in the bus in which it should deliver a flexibility service is equal to the expected value. The local imbalance is penalized at a regulated price.

Transmission System Operator

The TSO is a pure FSU in this simulation. Its flexibility needs are taken as data and are not localized. Any FSP may, in principle, provide flexibility offers to the TSO. The flexibility contracted by the TSO is computed by optimization problem (4.22). Since the TSO can procure flexibility outside the distribution system, the surplus of this stage is taken as the amount of flexibility collected in this network weighted by a price representing the system benefits of contracting reserve into the considered distribution network.

When the TSO decides to activate flexibility services, the TSO solves an imbalance external to the considered distribution system. Note that the imbalance coming from the considered distribution system should not be solved by local flexibility services to avoid counterbalancing DSO actions. The benefits of the TSO coming from the activation of flexibility services in this part of the network are computed by optimization problem (4.23).

Flexibility platform

The flexibility platform is an intermediary between FSUs and FSPs. This platform could be operated either by a DSO, a TSO or an independent party. One could implement interaction models without a flexibility platform where FSUs and FSPs directly exchange their services. The flexibility platform facilitates the interaction and can anonymize the flexibility exchanges. Note that a pure market based platform would probably not be a good alternative due to the lack of liquidity as these services are highly dependent on the location.

In our settings, the flexibility platform collects the flexibility needs of the FSUs and communicates them to the FSPs. FSPs submit their offers to the platform which sequentially proposes them to the DSO, TSO and finally to other FSUs as depicted in Figure 4.7. As the DSO solves local problems such as congestions, its needs can be satisfied by fewer services than the imbalance problems faced by the TSO or BRPs. This motivates the priority given to the DSO, but other rankings could be investigated.

4.7.4 Implementation

The testbed is divided into three parts: the simulator, an instance generator and a user interface. The first two are implemented in Python 3. The user interface is composed of a client in HTML5-Javascript and a server in Python 3 that facilitates the use of the generator and the simulator, and provides an interface to visualize the results.

Simulator

The simulator takes as input the parameters of all agents on the simulated horizon, i.e. one day. A one year simulation can be performed by simulating each day independently. All of the parameters of the optimization problems given in Section 4.9 should be provided by CSV files. A self detailed example is given in the implementation. Based on all of the parameters, the simulator performs the simulation of one time horizon, e.g. one day. The simulation requires solving the optimization problems of the actors, that are encoded in ZIMPL and solved using the SCIP solver [1]. To conduct the simulation, some agents need predictions of some parameters such as the flexibility needs of the FSU (see Section 4.7.3 for more details). The simulation is initiated with these predictions set to zero. If the realizations do not match the predictions after one run, the simulation is reinitialized with the new realization. This process is repeated until the realizations match the predictions or after a predetermined number of iterations.

Instance generator

The instance generator allows a user to create an instance from high-level parameters instead of having to specify manually the assets of each agent. The reduced set of parameters is given in Table 4.7. The user must also define the parameters of the interaction model it wants to evaluate or choose one of the models described in this chapter.

The instance generator takes as its basis, a network. The networks already integrated are given in Figure 4.10. The ratio of installed production and consumption with respect to the one of the whole network is given along with

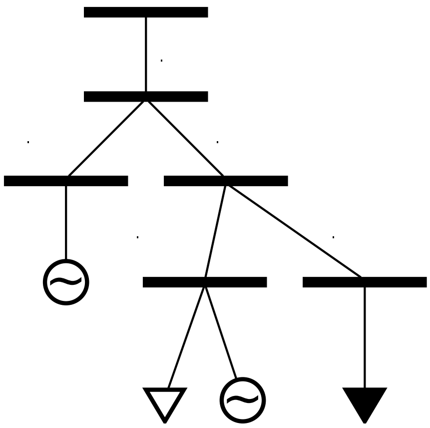
Table 4.7: Parameters of the instance generator.

Parameter	Type
Title of the instance	`String`
Network	Network name, see Figure 4.10
Total production	Mean and maximum value in MW
Production cost	Price in €/MWh
Total consumption	Mean and maximum value in MW
Retailing price	Price in €/MWh
Energy price	Mean and maximum value in €/MWh
Days of simulation	Start and end day in $\{1, \ldots, 365\}$
Number of periods by day	`Integer`
Number of producers and retailers	`Integer`
External imbalances of the producers and retailers	Percentage of their total volume
Retailers flexibility reservation cost	Price in €/MWh
TSO flexibility needs	Volume in MW
TSO reservation price	Price in €/MWh

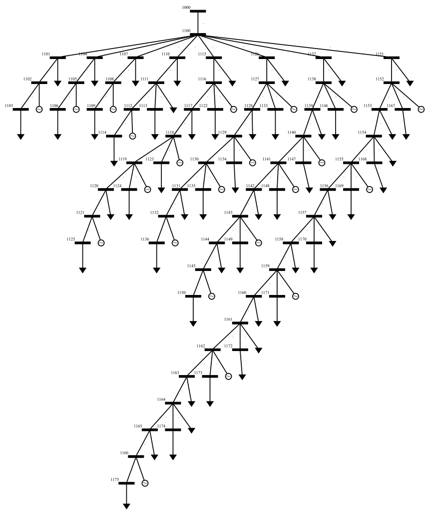
their topology. We now detail the generation of the CSV parameter files for the simulator.

Generation of the producers The producers are generated from the data of a wind farm in 2013 scaled to the input mean and maximum production in the network. Each bus with production in the selected network is assigned randomly to one producer. The maximum production for each period is set to the scaled baseline value times the contribution of the bus with respect to the total production in the network. The minimum production is the latter value minus the flexibility of the producer. The external imbalance of the producer is generated uniformly between plus and minus its mean production.

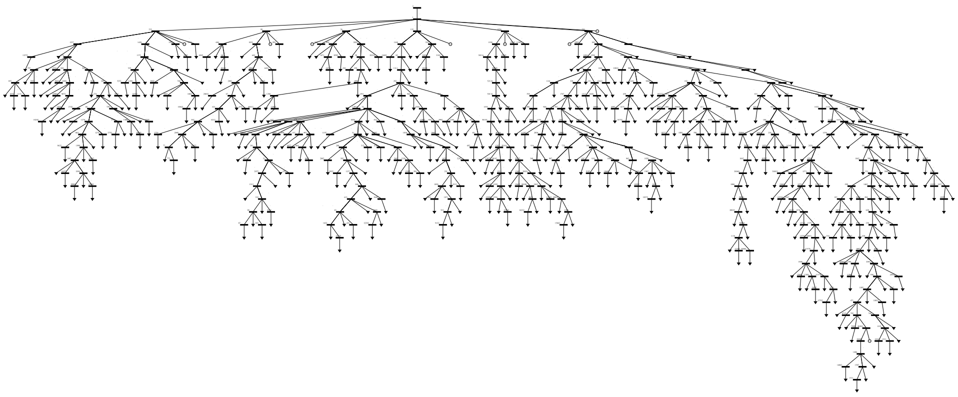
Generation of the retailers The consumption curve is built from the Belgian total load consumption of 2013 [42]. The share of each retailer is randomly generated so that the shares sum up to one. The mean baseline consumption of a retailer in one bus is the total consumption curve times the retailer's share and the ratio of the consumption with respect to the one of the total network. The sum in each period of the horizon of mean consumption provides the total energy to consume $V_{a,n}$. The minimum and maximum realization bounds, $p^{min}_{a,n,t}$ and $p^{max}_{a,n,t}$, are given by the mean consumption plus and minus the consumption



(a) Six buses example network.



(b) 75 bus network from [133].



(c) Ylpic network.

Figure 4.10: Networks available.

flexibility.

Generation of the price curves The prices are generated by scaling concurrently the Belpex day-ahead energy market prices [12] and the Belgian imbalance tariffs of 2013 [42]. To ensure the coherence of the input parameters of the simulator, the generator sets the minimum imbalance tariff to the energy prices. If it was not done, an agent could sell energy to the market without producing and still get benefits from this controversial action.

Generation of the TSO The flexibility needs of the TSO, R_t^- and R_t^+, are constant values given as input parameters. The external imbalance that the TSO faces in each period is randomly generated following a uniform distribution between $-R_t^-$ and R_t^+.

Generation of the flexibility potential indicators To operate its system, the DSO needs an indication on where flexibility can be obtained in its network. This information could, for instance, come from access contracts or notification from the FSUs willing to sell their services to the DSO. In this testbed, the knowledge of potential availability of flexibility services is taken as a parameter in optimization problem (4.10) which allows the DSO to communicate its flexibility needs to the FSU based on the baselines of the BRPs. The indicators of upward and downward flexibility availability for each bus, respectively α_n^+ and $\alpha_n^- \in [0, +\infty[$ are dimensionless parameters. A 0 indicator indicates that no flexibility is available while a large value indicates that a large amount of flexibility can potentially be contracted in the bus with respect to the others.

In the instance generator, these indicators are arbitrarily computed by the following procedure. First, the generator computes the installed flexible production and consumption in each bus in terms of power. We denote them respectively $\kappa^+ \in [0, +\infty[$ MW and $\kappa^- \in]-\infty, 0]$ MW. Second, these data are used to define the indicators:

$$\alpha_n^+ = \frac{1}{\kappa^+/10 - \kappa^-/10} \tag{4.4a}$$

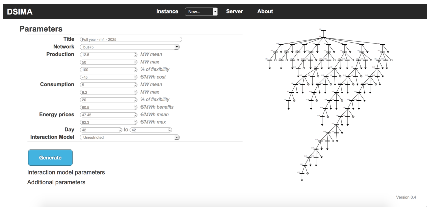
$$\alpha_n^- = \frac{1}{\kappa^+/1 - \kappa^-/10} \tag{4.4b}$$

where the numeric coefficient gives more importance to downward flexibility by the production side.

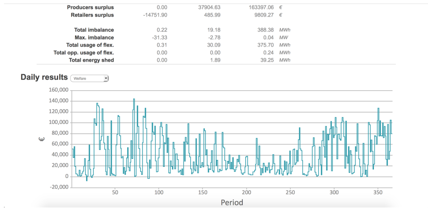
User Interface

The user interface simplifies the access to the instance generator described in Section 4.7.4 and the simulator of Section 4.7.4. Screenshots of the interface are given in Figure 4.11. The home screen, shown in Figure 4.11a, allows the creation of an instance via the generator giving it parameters such as the interaction model or the total production in the network. The global results screen, seen in Figure 4.11b, provides a summary of the simulation results of an instance and graphs to see the evolution of the welfare or DSO costs day by day. Daily results can be seen in the screen of Figure 4.11c. The state of the network in each period is represented and specific results to each bus and line can be obtained by clicking on it in the network picture.

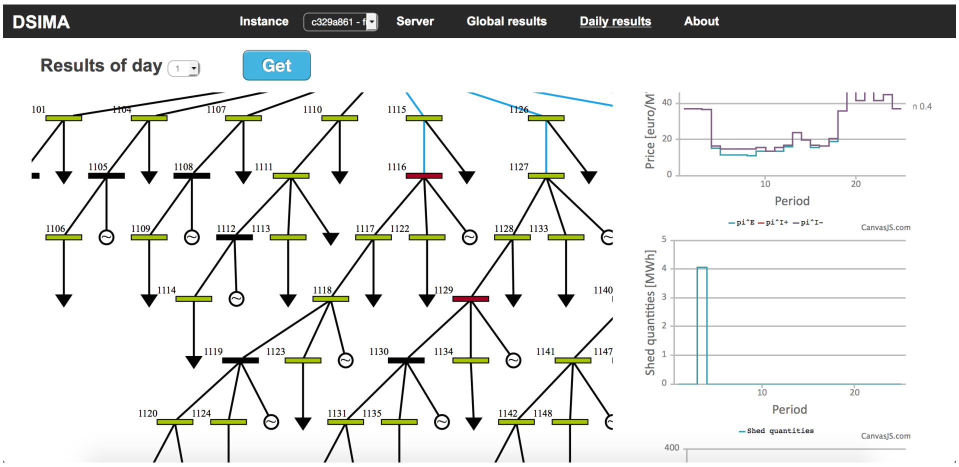
The webpage includes a Javascript client communicating with a Python server which can either be on the same computer or a different one. The server handles the requests of the user and executes the instance generator or simulator. Note that the server can handle multiple clients at the same time, queuing the simulation requests if needed. The simulations of multiple days is performed efficiently by assigning the simulation of each day to a different thread.



(a) Homepage with the parameters of the instance.



(b) Global results for the simulated horizon.



(c) Daily results. Each bus and line can be clicked on to display its specific information.

Figure 4.11: Screenshots of the user interface.

4.7.5 Results

The test case for the results is described in Section 4.7.5. A typical one-day run is presented for one interaction model in Section 4.7.5. Finally, the comparison of the six interaction models over one year is presented in Section 4.7.5. The experiments are carried out on a computer equipped with an Intel Core i7-3770 CPU at 3.40GHz with 32GB of RAM. The optimization problems are solved with SCIP [1].

Test case

The interaction models are tested on a generic 11kV distribution network composed of 75 buses and hosting 22 distributed generation units [133]. The network topology is fixed. The simulation is run for a time horizon of one day divided into 24 periods. Production data of the 22 generation units are taken from a production curve of 2013 scaled such that the maximum production reaches 64.2MW and the mean production is 16MW. Distributed generation units are clustered into three portfolios, each managed by a different producer.

The producers only ask for a remuneration of the activation of their flexibility services. The consumption of the 53 connected loads is built from the Belgian total load consumption of 2013 scaled to a mean of 10.8MW and a maximum of 21MW [42]. This consumption is divided into three parts, each belonging to one retailer. The consumption of a retailer is divided into a static part and a flexible part respectively accounting for 80% and 20% of the total consumption of the load. The retailer proposes its flexibility with no activation fee but requires a reservation fee. The reservation fee of a demand side flexibility offer is assumed to be 5€/MWh. The TSO flexibility aims to contract a volume of flexibility equal to 2% of the total installed production capacity of the system in each period. We use the following reservation prices of secondary reserve: $\pi_t^{S^+} = -\pi_t^{S^-} = 45$€/MWh. The total activation request of the TSO is drawn using a zero mean Gaussian distribution with a variance equal to the target flexibility volume of the TSO. Energy prices are taken from the clearing of 2013 of the Belpex day-ahead energy market [12] scaled to a mean of 53.64€/MWh and a maximum of 93.03€/MWh, excluding the extreme 2.5% of the original data. The value of lost load taken into account in the case of shedding due to the tripping of a protection is set to 500€/MWh for the production and 1000€/MWh for the consumption. The imbalance prices come from the Belgian TSO [42]. A FSP not providing a contracted flexibility service or violating its dynamic range is penalized at 150% of the maximum imbalance price.

Typical run

We first provide illustrative results for Model 4 applied to the test system based on the data of July 10, 2013. Figure 4.12 illustrates the events happening in the course of the simulation.

Shedding an MV bus disconnects only the devices connected to that bus, and does not impact the surrounding MV buses of the system. The maximum flow injected in the transmission network is 44.1MW at period 20. The DSO sheds a total of 12.6 of the 913.44MWh of generation potential, and activates 11.57MWh of flexibility, on a total of 17.8MWh, which causes an equal imbalance volume. The total imbalance, i.e. the sum of the imbalances over all agents in the system, is 25.8MWh. The total flexibility activated as well as the imbalance of all agents in the system is given in Figure 4.13. The total welfare of the system is 32107€. The total DSO cost is 823€ and does not consider remuneration for the shed quantities. However, the value of lost load and production are taken into account in the welfare value. The total benefit of all retailers is 3340€. Benefits earned by the producers sum up to 29455€. The TSO obtains a welfare of 1400€ from the services acquired in the system. A one day simulation lasts five minutes.

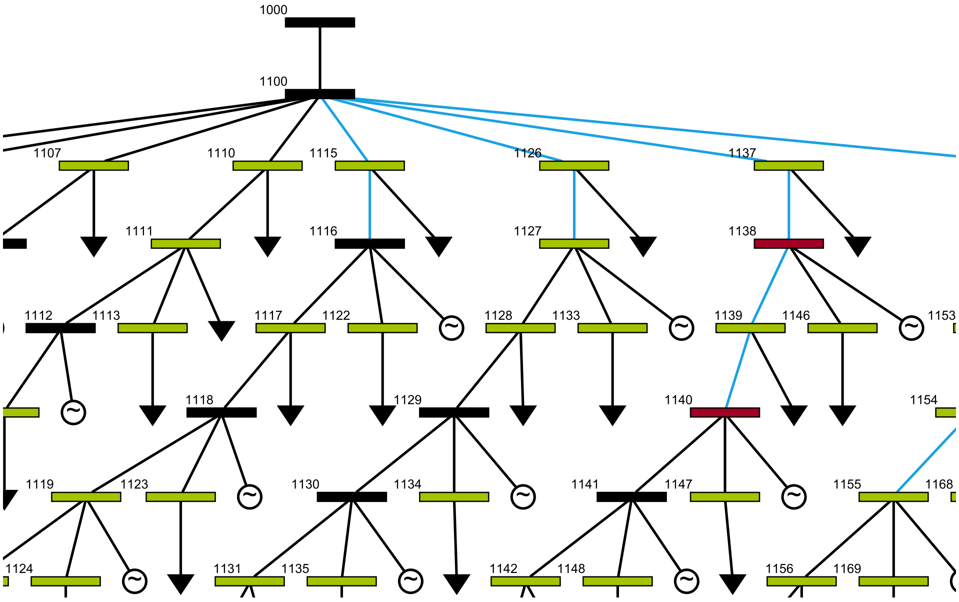


Figure 4.12: A bus is colored in green if flexibility is used locally for at least one period of the simulation horizon. Similarly, a bus is colored in red when shedding occurs, and a line is colored in blue when a thermal limit is reached.

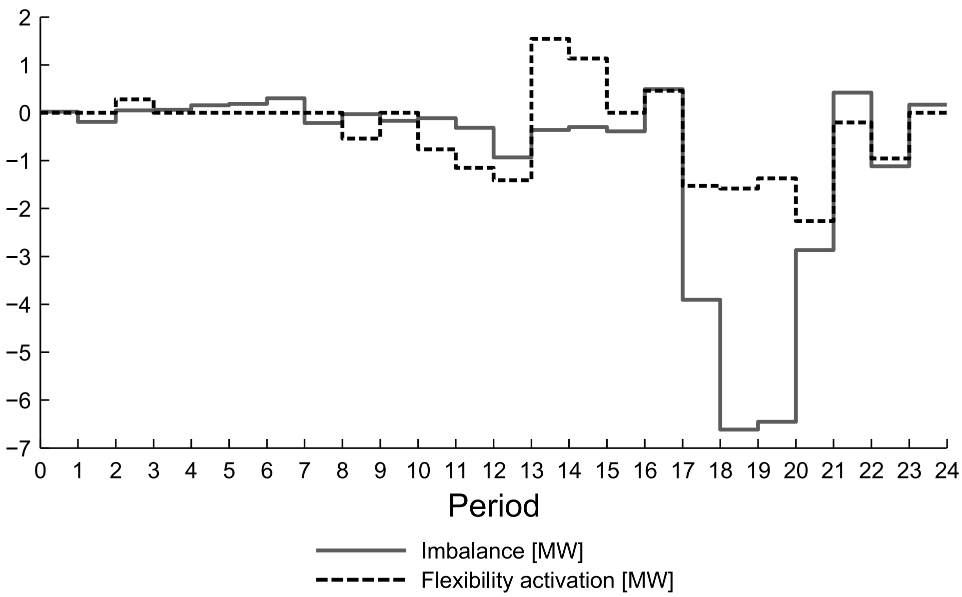


Figure 4.13: Total flexibility activated and imbalance for all agents in the system.

Comparison of the interaction models

All of the models are simulated under the same conditions for 365 days individually, which amounts to 32 hours of simulation by model. The main results are reported in Table 4.9. This Table could be compared to Table 4.4 with the results from the macroscopic analysis. Both table provides similar figures, the major differences coming from the more detailed modeling of the system.

Model 1 highlights that leaving the network with no control leads to high shedding and impacts the welfare with a penalty of about 12000€ by day. Only Model 2 yields no shedding but a lower welfare due to the conservative actions of the DSO to restrict the access of production units. This conservative strategy penalizes the producers that are not allowed to produce in situations where the network can handle higher injections. Using ANM strategies as it is proposed in Models 3, 4 and 5, leads to equivalent and higher welfares even with average shedding penalties of 1000€ per day. These models would be more efficient if shedding could be avoided. This necessity is caused by the activation of flexibility services by the TSO in an opposite direction to the directives of the DSO. In Models 3-5, stakeholders are free to exchange flexibility services. However, if the profile of a BRP is within the flexibility intervals, it has to propose flexibility offers to the DSO via FSPs. The DSO acts as a standard FSU to contract and to activate the flexibility services. Since every FSU activates flexibility simultaneously, some actions may counterbalance the action of the DSO. Fig. 4.14 illustrates this issue on a small example where an action of the DSO which was supposed to solve a congestion counterbalances and action of the TSO. Assume that the flow exceeds the capacity of the line 3 by 1MW. To solve this issue, the DSO curtails a wind turbine by 1MW. In the same time, assume that the TSO asks a storage unit to increase its production by 0.4MW. The simultaneous activation of these flexibility services leads to a remaining congestion of 0.4MW even though every agent stays within its access bounds and satisfies its flexibility request.

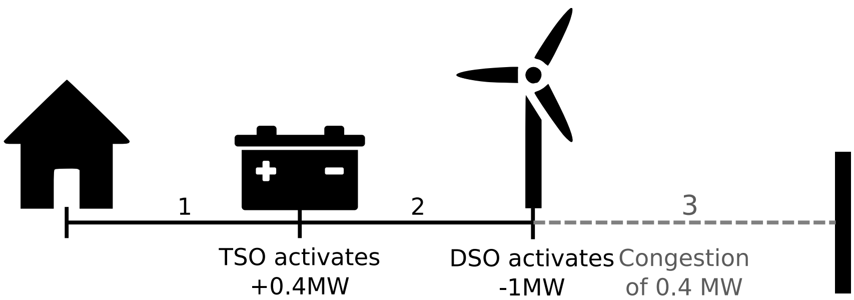


Figure 4.14: Illustration of a flexibility activation coordination issue between the DSO and the TSO.

In Model 6, this coordination problem does not occurs and the interaction model provides better welfare than the others. Coming back to our example, assume that the wind turbine has the safe range$[0, 5]$ and provide the baseline proposal of 6MW. The storage unit has a safe range of $[-1, 1]$ and a baseline proposal of 0MW. To solve the congestion problem, the DSO may grant the

Table 4.9: General daily results of the agent-based model for the six interaction models for the expected 2025 year.

	Model 1			**Model 2**			**Model 3**			**Model 4**			**Model 5**			**Model 6**			
	min	*mean*	*max*	*min*	*mean*	*max*	*min*	*mean*	*max*	*min*	*mean*	*max*	*min*	*mean*	*max*	*min*	*mean*	*max*	
Welfare	-162550	29077	121971	-7727	27411	68434	-7727	39868	145420	-7727	39692	146987	-7727	39665	146793	-7727	40281	146492	€
Shedding costs	0	12071	248409	0	0	0	0	914	19340	0	1077	23969	0	1103	23382	0	0	0	€
DSOs costs	0	0	0	0	0	0	0	444	10936	0	656	16494	0	656	16494	0	0	0	€
TSOs surplus	832	2878	3850	832	2878	3850	832	2874	3850	832	2873	3850	832	2873	3850	832	2878	3850	€
Prod. surplus	0	37743	162939	0	24005	68265	0	37825	164104	0	38024	164160	0	38023	164117	0	36876	146322	€
Ret. surplus	-14750	527	9852	-14750	527	9852	-14750	528	9852	-14750	528	9852	-14750	528	9852	-14750	527	9852	€
Production	0	379	1295	0	247	622	0	387	1372	0	387	1372	0	387	1372	0	382	1287	MWh
Consumption	40	257	490	40	257	490	40	257	490	40	257	490	40	257	490	40	257	490	MWh
Total imbalance	0.15	25.56	501.51	0.15	1.4	5.11	0.15	17.51	381.82	0.15	17.83	402.43	0.15	17.88	409.18	0.15	1.4	5.14	MWh
Max. imbalance	-34.82	-3.12	0.3	-0.6	-0.13	0.3	-24.14	-2.44	0.3	-33.42	-2.52	0.3	-32.65	-2.53	0.3	-0.6	-0.13	0.3	MW
Usage of flex.	0	14.93	65.1	0	14.94	65.1	0	29.44	380.42	0	29.39	380.42	0	29.4	380.42	0	14.92	65.1	MWh
Energy shed	0	24.14	496.82	0	0	0	0	1.83	38.68	0	2.15	47.94	0	2.21	46.76	0	0	0	MWh

dynamic range $[0, 6]$MW to the wind turbine and $[-1, 0]$MW to the storage unit. As a result, the TSO cannot ask the storage unit to increase its production and the DSO prevents the congestion of line 3. In Table 4.9, the parameters $\beta_{a,n,t}$, defining the relative deviation from the baseline proposals, is fixed to 40%. This parameter is designed to take into account that actors are free to deviate from their baselines if they are within the safe access range. To take into account these additional deviations, the DSO can easily use some margins and increase the relative deviation it considers. Figure 4.15 shows that taking values greater than 30% avoid the shedding issues. Figure 4.16 depicts the impact of the relative maximal deviation parameter on the welfare and the total production with respect to the restricted model. Even though the total production increases as $\beta_{a,n,t}$ decreases, the welfare reaches its maximum, 40450€, around $\beta_{a,n,t} = 25\%$. Figure 4.16 shows that the welfare is not as sensitive with respect to $\beta_{a,n,t}$ as we would expect. Therefore, a practical choice for this application would be $\beta_{a,n,t} = 40\%$ to avoid the shedding issues without notably impacting the welfare.

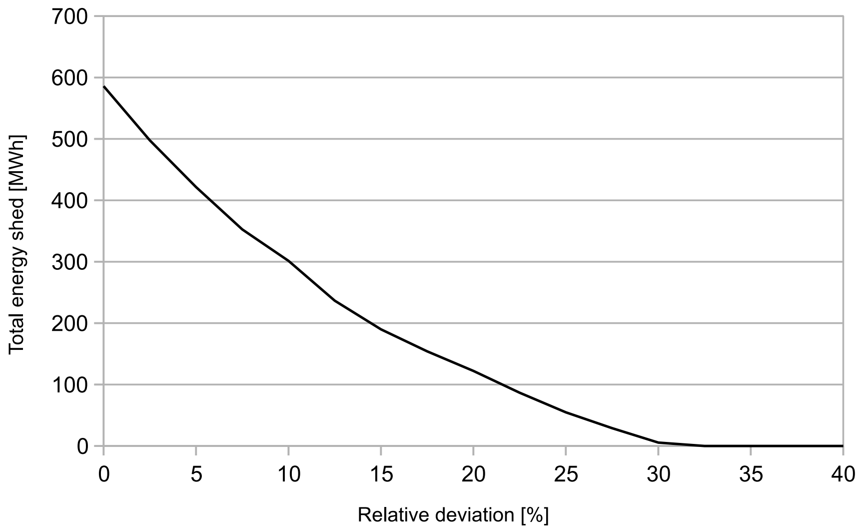


Figure 4.15: Evolution of the yearly shed production with the relative maximal deviation parameter considered by the DSO.

In this simulation, the TSO is the only agent to use demand side flexibility. Even though flexibility services from the demand side are cheaper, their usage is expensive for the DSO which must compensate the imbalance created to solve a congestion problem. In addition, an activation of an energy constrained flexibility offer in one period requires another activation in a different period and, therefore, up to a double imbalance compensation. A variant of these interaction models could allow a discount for the DSO on the imbalance tariff.

Table 4.10 compares the surpluses of the producers in the last four interaction models with respect to the first one. These results show that Model 2 significantly decreases the surplus of each producer. The smallest producer is

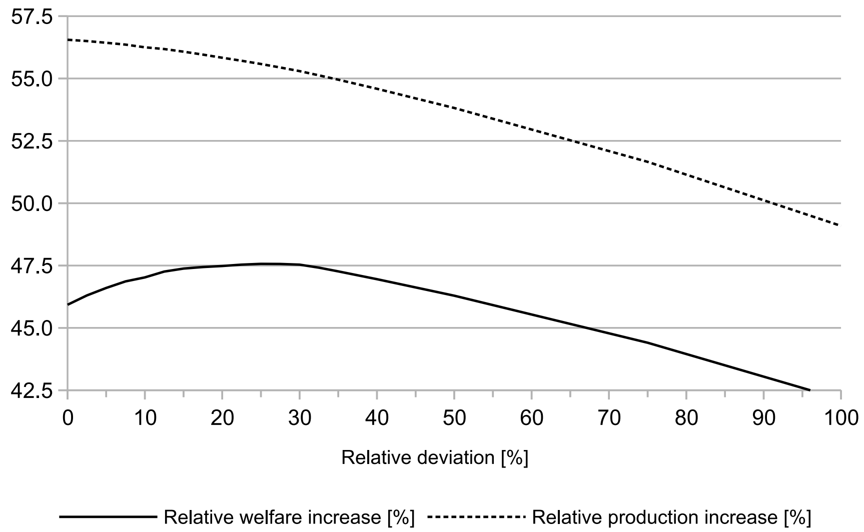


Figure 4.16: Welfare and total production increase with respect to Model 2 as a function of the relative maximal deviation parameter considered by the DSO.

Table 4.10: Comparison of the annual surpluses of the producers with respect to Model 1.

Producer	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	
1	20647€	-34.97	-0.09	0.43	0.52	-1.44	%
2	3287€	-44.70	3.06	4.26	3.66	-2.82	%
3	13809€	-36.56	-0.01	0.38	0.37	-3.46	%

the most sensitive to the choice of the interaction model but still the difference between Models 3, 4 and 5 on its surplus is of the order of 1%. The two other producers are less impacted by the ANM interaction models. This result may motivate producers to bargain their flexibility for free, as long as the imbalance is paid, in order to obtain an increased access to the distribution network. Model 6 slightly decreases the surplus of the producers as the DSO takes some margins to avoid shedding actions that results in slightly less production than the maximum that the distribution network is able to handle. The value observed here remains small but the procedure to obtain the dynamic ranges could be improved to maximize the fairness of the dynamic ranges.

4.8 Conclusion

This chapter proposes six interaction models governing exchanges of flexibility services within a distribution system. These six models are first evaluated quantitatively in a macroscopic study using expected data for a typical network in 2025 according to the expected energy trends to 2030 [22]. The economic efficiency of each interaction model is estimated on the long term by repeating the process for each year of the horizon 2015-2030 using forecasts of the evolution of the parameters of the simulation. Results show that for the first five years, the six models provide similar economic efficiency. For the remaining ten years, active network management interaction models clearly provide higher economic efficiency. These results are confirmed by a simulation with an agent-based system completely formalizing the interactions between the stakeholders of a distribution system and all the interaction models. However, some of these models, in their current version, cannot guarantee the operation of the network without shedding due to the lack of coordination between the DSO and the TSO. Out of the proposed models, only the ones restricting grid users to access ranges computed ahead of time to prevent any congestion are able to avoid shedding. Dynamically defining these ranges for each quarter is the best option out of the proposed model to avoid shedding while increasing the amount of renewable generation within the system. In our results, the interaction model using dynamic range allows to safely increase by 55% the amount of distributed generation in the network and the welfare by 42.5% with respect to a restrictive model representing the currently applied interaction model. The interaction models are shown to penalize some producers more than others, raising the question of fairness of access range allocation and flexibility activation. Except for the most conservative interaction model, these penalizations are rather small and therefore each actor of the system would have incentive to accept a more flexible interaction model. Producers may even have incentive to bargain their flexibility for free, as long as the imbalance is paid, in order to obtain an increased access to the distribution network; even a dynamic one. However, not remunerating the producer for the subsidies' loss due to the curtailment of a renewable generation may discourage future investments in renewable generation in distribution networks.

The testbed used for these simulations is released as open source code under the name *DSIMA* [37]. This testbed is composed of an instance generator, a simulator and a web-based user interface written in Python and HTML5/Javascript. These three modules allow various compromises between flexibility and ease of modification.

The work presented in this chapter could be extended along several lines. For instance, one could seek a method to ensure fairness of access ranges to the network taking into account the current system situation and future evolution. In the proposed interaction models, the BRP is responsible for providing baselines for every medium-voltage bus. However, several stakeholders may forecast these references: the DSO, FSPs, BRPs, etc. and these choices should be further investigated by implementing them in this testbed. One could also refine the modeling level by considering more detailed agents, the analysis in a stochastic environment and better network model such as in [14] or alternating current power flow equations; study dynamics of the system, such as the entry or exit of new players or production units. Finally, the proposed interaction models should be compared to network reinforcement decisions without change of interaction model. This kind of methodology has already been applied to the transmission operator network, see for instance [131]. The open-source code needed to continue this work can be found at the address http://www.montefiore.ulg.ac.be/~dsima/.

4.9 Appendix

This appendix contains the optimization problems defining the behavior of the agents in the system described in Section 4.7.

4.9.1 Optimization problems of the distribution system operator

Access agreement Optimization problem solved to obtain the safe access interval $[b_{a,n}, B_{a,n}]$ from the access request $[g_{a,n}, G_{a,n}]$. The objective minimizes the maximum relative access restriction downward and upward, δ_g and δ_G, subject to the operational limits of the system, here the line capacities only. Worst case conditions are considered through the auxiliary variables $\underline{f}_{n,m}$ and $\overline{f}_{n,m}$. The first case corresponds to no production and the consumption at its maximal allowed value for each agent and each bus, $b_{a,n}$. The second is equivalent to a case where there is no consumption and all production is at its maximum bound $B_{a,n}$.

$$\min \delta_g + \delta_G \tag{4.5a}$$

subject to, $\forall n, m \in \mathcal{N}^2$,

$$-C_{n,m} \leq \underline{f}_{n,m}, \overline{f}_{n,m} \leq C_{n,m} \tag{4.5b}$$

$\forall n \in \mathcal{N}$

$$\sum_{a \in \mathcal{A}(n)} b_{a,n} = \sum_{m \in \mathcal{N}(n)} \underline{f}_{n,m} \tag{4.5c}$$

$$\sum_{a \in \mathcal{A}(n)} B_{a,n} = \sum_{m \in \mathcal{N}(n)} \overline{f}_{n,m} \tag{4.5d}$$

$\forall n \in \mathcal{N}, a \in \mathcal{A}(n)$

$$\delta_g \geq (g_{a,n} - b_{a,n})/g_{a,n} \tag{4.5e}$$

$$\delta_G \geq (G_{a,n} - B_{a,n})/G_{a,n} \tag{4.5f}$$

$$g_{a,n} \leq b_{a,n} \leq 0 \leq B_{a,n} \leq G_{a,n} \tag{4.5g}$$

Dynamic ranges computation In the day by day simulation of the system, the DSO uses the local baseline proposals to compute the dynamic access ranges $[d_{a,n,t}, D_{a,n,t}]$ of each agent a for each period t and bus n. To compute the dynamic ranges, the DSO first determines credible scenarios of deviation from the baseline proposal $p^p_{a,n,t}$, denoted $\underline{p}_{a,n,t}$ and $\overline{p}_{a,n,t}$. If $p^p_{a,n,t}$ is in the safe access range,

$$\underline{p}_{a,n,t} = \max\{l_{a,n,t}, p^p_{a,n,t} - \beta_{a,n,t}|p^b_{a,n,t}|\} \tag{4.6a}$$

$$\overline{p}_{a,n,t} = \min\{L_{a,n,t}, p^p_{a,n,t} + \beta_{a,n,t}|p^b_{a,n,t}|\} \tag{4.6b}$$

where $\beta_{a,n,t}$ is a parameter quantifying the credible relative deviation. If $p^p_{a,n,t}$ is in the flexible access range, we set $\underline{p}_{a,n,t} = \overline{p}_{a,n,t} = p^p_{a,n,t}$. Two additional parameters, $\Delta d^{max}_{a,n,t}$ and $\Delta D^{max}_{a,n,t}$ give amount of curtailable power by the DSO and are computed by

$$\Delta d^{max}_{a,n,t} = \max\{0, l_{a,n} - p^p_{a,n,t}\} \tag{4.7a}$$

$$\Delta D^{max}_{a,n,t} = \max\{0, p^p_{a,n,t} - L_{a,n}\} \tag{4.7b}$$

Together with the dynamic ranges, the DSO computes its upward and downward flexibility needs, $r^+_{n,t}$ and $r^-_{n,t}$ by solving the following optimization problem. The positive parameters α^+_n and α^-_n characterize the flexibility potential at each bus and are obtained from the DSO knowledge of the system (this may for instance be part of the access contract procedure).

$$\min \sum_{n\in\mathcal{N}} \left(\alpha^+_n r^+_{\text{DSO},n,t} + \alpha^-_n r^-_{\text{DSO},n,t}\right) \tag{4.8a}$$

subject to, $\forall n, n' \in \mathcal{N}^2, t \in \mathcal{T}$,

$$-C_{n,n'} \leq \underline{f}_{n,n',t}, \overline{f}_{n,n',t} \leq C_{n,n'} \tag{4.8b}$$

$\forall n \in \mathcal{N}, t \in \mathcal{T}$,

$$\sum_{a\in\mathcal{A}} (\underline{p}_{a,n,t} - \Delta D_{a,n,t} + \Delta d_{a,n,t}) - \underline{r}^-_{n,t} + \underline{r}^+_{n,t} = \sum_{n'\in\mathcal{N}(n)} \underline{f}_{n,n',t} \tag{4.8c}$$

$$\sum_{a\in\mathcal{A}} (\overline{p}_{a,n,t} - \Delta D_{a,n,t} + \Delta d_{a,n,t}) - \overline{r}^-_{n,t} + \overline{r}^+_{n,t} = \sum_{n'\in\mathcal{N}(n)} \overline{f}_{n,n',t} \tag{4.8d}$$

$$r^-_{n,t} \geq \underline{r}^-_{n,t}, \overline{r}^-_{n,t} \tag{4.8e}$$

$$r^+_{n,t} \geq \underline{r}^+_{n,t}, \overline{r}^+_{n,t} \tag{4.8f}$$

$\forall a \in \mathcal{A}, n \in \mathcal{N}, t \in \mathcal{T}$,

$$\Delta d_{a,n,t} \leq \Delta d^{max}_{a,n,t} \tag{4.8g}$$

$$\Delta D_{a,n,t} \leq \Delta D^{max}_{a,n,t} \tag{4.8h}$$

The dynamic ranges are built using the solution of optimization problem (4.8) such that

$$d_{a,n,t} = \min\{l_{a,n}, p^p_{a,n,t} + \Delta d_{a,n,t}\} \tag{4.9a}$$

$$D_{a,n,t} = \max\{L_{a,n}, p^p_{a,n,t} - \Delta D_{a,n,t}\}. \tag{4.9b}$$

Announcement of the flexibility needs The flexibility needs of the DSO in MW, $r^{+}_{\mathrm{DSO},n,t}$ and $r^{-}_{\mathrm{DSO},n,t}$ are obtained by solving

$$\min \sum_{n\in\mathcal{N}} \left(\alpha^{+}_{n,t} r^{+}_{\mathrm{DSO},n,t} + \alpha^{-}_{n,t} r^{-}_{\mathrm{DSO},n,t}\right) \tag{4.10a}$$

subject to,

$$-C_{n,m} \le f_{n,m,t} \le C_{n,m} \qquad \forall n,m \in \mathcal{N}^2 \tag{4.10b}$$

$$\sum_{a\in\mathcal{A}} p^{b}_{a,n,t} + r^{+}_{\mathrm{DSO},n,t} - r^{-}_{\mathrm{DSO},n,t} = \sum_{m\in\mathcal{N}(n)} f_{n,m,t} \qquad \forall n \in \mathcal{N} \tag{4.10c}$$

$$r^{+}_{\mathrm{DSO},n,t}, r^{-}_{\mathrm{DSO},n,t} \ge 0 \qquad \forall n \in \mathcal{N} \tag{4.10d}$$

The positive parameters α^{+}_{n} and α^{-}_{n} characterize the flexibility potential at each bus and are obtained from the DSO knowledge of the system. In the implementation, these parameters are provided by the instance generator.

Flexibility procurement and activation The DSO can procure a subset of the proposed single period flexibility offers, $\mathcal{S}_n$, and energy constrained flexibility offers $\mathcal{E}_n$. Let $v_{a,i}$ be the variable for the modulation of the single period flexibility service i and $[w_{a,i}, W_{a,i}]$ the requested reservation range. Let $x^{+}_{a,i,t}, x^{-}_{a,i,t} \ge 0$ be the variables for the upward and downward activation of the energy constrained flexibility service $i \in \mathcal{E}$. Note that an energy constraint bid cannot be partially contracted which is ensured by the binary variable $y_{a,i}$.

$$\begin{aligned}\min \; & \sum_{i\,\in\mathcal{S}_n} \left(\pi^{r}_{i}(W_{DSO,i} - w_{DSO,i}) + \pi^{b}_{i} v_{DSO,i}\right) \\ & + \sum_{i\,\in\mathcal{E}_n} \left(\pi^{r}_{i} y_{DSO,i} + \pi^{b}_{i}(x^{+}_{DSO,i,t} + x^{-}_{DSO,i,t})\right) \\ & + \sum_{t\,\in\mathcal{T}} \sum_{n\,\in\mathcal{N}} z_{n,t}\left(\pi^{VSP}\max\{0, p^{b}_{a,n,t}\} + \pi^{VSC}\min\{0, p^{b}_{a,n,t}\}\right) \\ & + \sum_{t\,\in\mathcal{T}} (\pi^{I^+}_{t} I^{+}_{DSO,t} + \pi^{I^-}_{t} I^{-}_{DSO,t})\end{aligned} \tag{4.11a}$$

subject to

$$-C_{n,m} \le f_{n,m,t} \le C_{n,m} \qquad \forall n,m \in \mathcal{N}^2 \tag{4.11b}$$

$$(1 - z_{n,t})\sum_{a\in\mathcal{A}} p^{b}_{a,n,t} + u_{\mathrm{DSO},n,t} = \sum_{m\in\mathcal{N}(n)} f_{n,m,t} \qquad \forall n \in \mathcal{N}, t \in \mathcal{T} \tag{4.11c}$$

$$u_{DSO,n,t} = \sum_{i \in \mathcal{S}_n : \tau_i = t} v_{DSO,i} + \sum_{i \in \mathcal{E}_n} x^+_{DSO,i,t} - x^-_{DSO,i,t} \qquad \forall n \in \mathcal{N}, t \in \mathcal{T} \tag{4.11d}$$

$$m_i \leq w_{DSO,i} \leq v_{DSO,i} \leq W_{DSO,i} \leq M_i \qquad \forall n \in \mathcal{N}, i \in \mathcal{S}_n \tag{4.11e}$$

$$m_{i,t} y_{DSO,i} \leq x^+_{DSO,i,t} - x^-_{DSO,i,t} \leq M_{i,t} y_{DSO,i} \qquad \forall n \in \mathcal{N}, i \in \mathcal{E}_n, t \in \mathcal{T} \tag{4.11f}$$

$$\sum_{t \in \mathcal{T}} (x^+_{DSO,i,t} - x^-_{DSO,i,t}) = 0 \qquad \forall n \in \mathcal{N}, i \in \mathcal{E}_n \tag{4.11g}$$

$$I^+_{\mathrm{DSO},t} - I^-_{\mathrm{DSO},t} = \sum_{n \in \mathcal{N}_{\mathrm{DSO}}} u_{\mathrm{DSO},n,t} \qquad \forall t \in \mathcal{T} \tag{4.11h}$$

$$I^+_{\mathrm{DSO},t}, I^-_{\mathrm{DSO},t} \geq 0 \qquad \forall t \in \mathcal{T} \tag{4.11i}$$

The DSO minimizes the cost of flexibility procurement, the estimated cost of shedding production or consumption encoded by the variable $z_{n,t}$ and the imbalance cost caused by the activation of flexibility services. The result of the activation of flexibility services is computed in the variable $u_{DSO,n,t}$ by (4.11d). Constraint (4.11g) ensures that the energy constrained bids are used neutrally in energy. Note that the problem should always be feasible since the DSO could shed every bus of the network i.e. all the production and consumption attached to the bus. This shedding is penalized at a virtual cost π^{VLP} for the production and π^{VLC} for the consumption.

Real-time operation A simple protection scheme is represented by the following optimization problem where the only decision variables left to optimize handle the shedding of buses $z_{n,t}$.

$$\min \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{N}} z_{n,t} \left(\pi^{VSP} \max\{0, p_{a,n,t}\} + \pi^{VSC} \min\{0, p_{a,n,t}\} \right) \tag{4.12a}$$

subject to

$$-C_{n,m} \leq f_{n,m,t} \leq C_{n,m} \qquad \forall n, m \in \mathcal{N}^2 \tag{4.12b}$$

$$(1 - z_{n,t}) \sum_{a \in \mathcal{A}} p_{a,n,t} = \sum_{m \in \mathcal{N}(n)} f_{n,m,t} \qquad \forall n \in \mathcal{N}. \tag{4.12c}$$

4.9.2 Optimization problems of the producer

Optimization of the baselines With the following optimization problem, the producer obtains a baseline for each of its assets considering the following decision stages. We consider that the producer has only one asset by bus. Note that this is not the restrictive assumption since a bus with two assets can be represented as two buses with one asset linked by a line of infinite capacity.

The flexibility needs of the FSUs cannot be expressed as constraints, since the producer may not be willing nor be able to satisfy them, a simple way to translate these needs in the optimization model of the FSP is to define fictive prices which incentivize offering flexibility in the most valuable periods: $\pi^{\Delta^+}_{n,t} = \frac{\pi^R_n R^+_{n,t}}{\sum_{t\in\mathcal{T}} R^+_{n,t}}$ and $\pi^{\Delta^-}_{n,t} = \frac{\pi^R_n R^-_{n,t}}{\sum_{t\in\mathcal{T}} R^-_{n,t}}$, where π^R_n is a reference flexibility price. This price is a scale factor to quantify the importance of the expected revenue from flexibility with respect to the cost of energy. The flexibility that could be offered upward and downward is denoted $\Delta^+_{a,n,t}$ and $\Delta^-_{a,n,t}$.

$$\begin{aligned}\max \sum_{t\in\mathcal{T}} &\left(\pi^E_t \sum_{n\in\mathcal{N}_a} p^b_{a,n,t} - \pi^{I^+}_t I^+_{a,t} - \pi^{I^-}_t I^-_{a,t}\right) \\ &- \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}_a} c_{a,n,t} p_{a,n,t} + \sum_{t\in\mathcal{T}} \left(\pi^{\Delta^+}_{n,t} \Delta^+_{a,n,t} + \pi^{\Delta^-}_{n,t} \Delta^-_{a,n,t}\right)\end{aligned} \tag{4.13a}$$

subject to,

$$k_{a,n} \le p^b_{a,n,t}, p_{a,n,t}, p^+_{a,n,t}, p^-_{a,n,t} \le K_{n,t} \qquad \forall n \in \mathcal{N}_a, t \in \mathcal{T} \tag{4.13b}$$

$$I^+_{a,t} - I^-_{a,t} = \sum_{n\in\mathcal{N}_a} p_{a,n,t} - \sum_{n\in\mathcal{N}_a} p^b_{a,n,t} \qquad \forall t \in \mathcal{T} \tag{4.13c}$$

$$\Delta^+_{a,n,t} \le p^+_{a,n,t} - p^b_{a,n,t} \qquad \forall n \in \mathcal{N}_a, t \in \mathcal{T} \tag{4.13d}$$

$$\Delta^-_{a,n,t} \le p^b_{a,n,t} - p^-_{a,n,t} \qquad \forall n \in \mathcal{N}_a, t \in \mathcal{T} \tag{4.13e}$$

$$p^{\min}_{a,n,t} \le p^b_{a,n,t}, p_{a,n,t}, p^+_{a,n,t}, p^-_{a,n,t} \le p^{\max}_{a,n,t} \qquad \forall n \in \mathcal{N}_a, t \in \mathcal{T} \tag{4.13f}$$

$$p^-_{a,n,t} \le L_{a,n} \qquad \forall n \in \mathcal{N}_a, t \in \mathcal{T} \tag{4.13g}$$

$$l_{a,n} \le p^+_{a,n,t} \qquad \forall n \in \mathcal{N}_a, t \in \mathcal{T} \tag{4.13h}$$

$$I^+_{a,t}, I^-_{a,t} \ge 0 \qquad \forall t \in \mathcal{T} \tag{4.13i}$$

Optimization of the flexibility offers The baseline of the producer, $p^b_{a,n,t}$ is now fixed. The flexibility available from the producer portfolio is computed using the following optimization problem. If all the baselines of the producer are unchanged with respect to the one computed in the baseline optimization step, the flexibility available is equal to the one predicted in the previous stage.

$$\max \sum_{t\in\mathcal{T}} \left(\pi^{\Delta^+}_{n,t} \Delta^+_{a,n,t} + \pi^{\Delta^-}_{n,t} \Delta^-_{a,n,t}\right) - \sum_{t\in\mathcal{T}} \left(\pi^{I^+}_t I^+_{a,t} + \pi^{I^-}_t I^-_{a,t}\right) - \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}_a} c_{a,n,t} p_{a,n,t} \tag{4.14a}$$

subject to,

$$p^{\min}_{a,n,t} \le p_{a,n,t}, p^+_{a,n,t}, p^-_{a,n,t} \le p^{\max}_{a,n,t} \qquad \forall n \in \mathcal{N}_a, t \in \mathcal{T} \tag{4.14b}$$

$$
\begin{aligned}
& I^+_{a,t} - I^-_{a,t} = \sum_{n\in\mathcal{N}_a} p_{a,n,t} - \sum_{n\in\mathcal{N}_a} p^b_{a,n,t} && \forall t\in\mathcal{T} && (4.14c)\\
& \Delta^+_{a,n,t} \le p^+_{a,n,t} - p^b_{a,n,t} && \forall n\in\mathcal{N}_a, t\in\mathcal{T} && (4.14d)\\
& \Delta^-_{a,n,t} \le p^b_{a,n,t} - p^-_{a,n,t} && \forall n\in\mathcal{N}_a, t\in\mathcal{T} && (4.14e)\\
& k_{a,n} \le p_{a,n,t}, p^+_{a,n,t}, p^-_{a,n,t} \le K_{n,t} && \forall n\in\mathcal{N}_a, t\in\mathcal{T} && (4.14f)\\
& p^-_{a,n,t} \le L_{a,n} && \forall n\in\mathcal{N}_a, t\in\mathcal{T} && (4.14g)\\
& l_{a,n} \le p^+_{a,n,t} && \forall n\in\mathcal{N}_a, t\in\mathcal{T} && (4.14h)
\end{aligned}
$$

For each period t and each bus n, the producer makes one single period flexibility offer upward of $\Delta^+_{a,n,t}$ at an activation price $c_{a,n,t}$ and one downward of $\Delta^-_{a,n,t}$ at an activation price $\pi^E_t - c_{a,n,t}$. The reservation price is a parameter of the simulated instance.

Flexibility services procurement and activation The producer flexibility procurement optimization problem is similar to the one of the DSO (4.11) adapted to its specific constraints.

$$
\begin{aligned}
\min \sum_{i\,\in\mathcal{S}_n} \left(\pi^r_i (W_{a,i} - w_{a,i}) + \pi^b_i v_{a,i}\right) + \sum_{i\,\in\mathcal{E}_n} \left(\pi^r_i y_{a,i} + \pi^b_i (x^+_{a,i,t} + x^-_{a,i,t})\right)& \\
+ \sum_{t\,\in\mathcal{T}} \left(\pi^{I^+}_t I^+_{a,t} + \pi^{I^-}_t I^-_{a,t}\right) + \sum_{t\,\in\mathcal{T}} \sum_{n\,\in\mathcal{N}_a} c_{a,n,t} p_{a,n,t}& \qquad (4.15a)
\end{aligned}
$$

subject to

$$
\begin{aligned}
& p^{\min}_{a,n,t} \le p_{a,n,t} \le p^{\max}_{a,n,t} && \forall n\in\mathcal{N}_a, t\in\mathcal{T} && (4.15b)\\
& k_{a,n} \le p_{a,n,t} \le K_{n,t} && \forall n\in\mathcal{N}_a, t\in\mathcal{T} && (4.15c)\\
& u_{a,n,t} = \sum_{i\in\mathcal{S}_n:\tau_i=t} v_{a,i} + \sum_{i\in\mathcal{E}_n} x^+_{a,i,t} - x^-_{a,i,t} && \forall n\in\mathcal{N}, t\in\mathcal{T} && (4.15d)\\
& m_i \le w_{a,i} \le v_{a,i} \le W_{a,i} \le M_i && \forall n\in\mathcal{N}, i\in\mathcal{S}_n && (4.15e)\\
& m_{i,t} y_{a,i} \le x^+_{a,i,t} - x^-_{a,i,t} \le M_{i,t} y_{a,i} && \forall n\in\mathcal{N}, i\in\mathcal{E}_n, t\in\mathcal{T} && (4.15f)\\
& \sum_{t\in\mathcal{T}} (x^+_{a,i,t} - x^-_{a,i,t}) = 0 && \forall n\in\mathcal{N}, i\in\mathcal{E}_n && (4.15g)\\
& I^+_{a,t} - I^-_{a,t} = \sum_{n\in\mathcal{N}_a} p_{a,n,t} - \sum_{n\in\mathcal{N}_a} (p^b_{a,n,t} + u_{a,n,t}) && \forall t\in\mathcal{T} && (4.15h)\\
& I^+_{a,t}, I^-_{a,t} \ge 0 && \forall t\in\mathcal{T} && (4.15i)
\end{aligned}
$$

Optimization of the realization The producer chooses the realization considering as a parameter the flexibility activation requests $h_{a,n,t}$ as a FSP and its flexibility requests as a FSU $u_{a,n,t}$. Note the local imbalance penalty for the buses $n \in \mathcal{B}_a$ in which the producer is providing flexibility services.

$$\min \sum_{t \in \mathcal{T}} \left(\pi_t^{I^+} I_{a,t}^+ + \pi_t^{I^-} I_{a,t}^- \right) + \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{N}_a} c_{a,n,t} p_{a,n,t} + \pi^l \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{B}_a} (i_{a,n,t}^+ + i_{a,n,t}^-) \tag{4.16a}$$

subject to

$$p_{a,n,t}^{\min} \leq p_{a,n,t} \leq p_{a,n,t}^{\max} \qquad \forall n \in \mathcal{N}_a, t \in \mathcal{T} \tag{4.16b}$$
$$k_{a,n} \leq p_{a,n,t} \leq K_{n,t} \qquad \forall n \in \mathcal{N}_a, t \in \mathcal{T} \tag{4.16c}$$
$$I_{a,t}^+ - I_{a,t}^- = \sum_{n \in \mathcal{N}_a} p_{a,n,t} - \sum_{n \in \mathcal{N}_a} (p_{a,n,t}^b + u_{a,n,t} + h_{a,n,t}) \qquad \forall t \in \mathcal{T} + E_{a,t} \tag{4.16d}$$
$$I_{a,t}^+, I_{a,t}^- \geq 0 \qquad \forall t \in \mathcal{T} \tag{4.16e}$$
$$i_{a,n,t}^+ - i_{a,n,t}^- = p_{a,n,t} - (p_{a,n,t}^b + u_{a,n,t} + h_{a,n,t}) \qquad \forall t \in \mathcal{T}, n \in \mathcal{N}_a \tag{4.16f}$$
$$i_{a,n,t}^+, i_{a,n,t}^- \geq 0 \qquad \forall t \in \mathcal{T}, n \in \mathcal{N}_a \tag{4.16g}$$

4.9.3 Optimization problems of the retailer

Optimization of the baselines The retailer baseline optimization problem is similar to the one of the producer (4.13) adapted to its specific constraints.

$$\begin{aligned} \max \sum_{t \in \mathcal{T}} & \left(\pi_t^E \sum_{n \in \mathcal{N}_a} p_{a,n,t}^b - \pi_t^{I^+} I_{a,t}^+ - \pi_t^{I^-} I_{a,t}^- \right) \\ & - \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{N}_a} c_{a,n,t} p_{a,n,t} + \sum_{t \in \mathcal{T}} \left(\pi_{n,t}^{\Delta^+} \Delta_{a,n,t}^+ + \pi_{n,t}^{\Delta^-} \Delta_{a,n,t}^- \right) \end{aligned} \tag{4.17a}$$

subject to,

$$p_{a,n,t}^{\min} \leq p_{a,n,t}^b, p_{a,n,t}, p_{a,n,t}^+, p_{a,n,t}^- \leq p_{a,n,t}^{\max} \qquad \forall n \in \mathcal{N}_a, t \in \mathcal{T} \tag{4.17b}$$
$$\sum_{t \in \mathcal{T}} p_{a,n,t} = V_{a,n} \qquad \forall n \in \mathcal{N}_a \tag{4.17c}$$
$$I_{a,t}^+ - I_{a,t}^- = \sum_{n \in \mathcal{N}_a} p_{a,n,t} - \sum_{n \in \mathcal{N}_a} p_{a,n,t}^b \qquad \forall t \in \mathcal{T} \tag{4.17d}$$
$$\Delta_{a,n,t}^+ \leq p_{a,n,t}^+ - p_{a,n,t}^b \qquad \forall n \in \mathcal{N}_a, t \in \mathcal{T} \tag{4.17e}$$
$$\Delta_{a,n,t}^- \leq p_{a,n,t}^b - p_{a,n,t}^- \qquad \forall n \in \mathcal{N}_a, t \in \mathcal{T} \tag{4.17f}$$
$$k_{a,n} \leq p_{a,n,t}^b, p_{a,n,t}, p_{a,n,t}^+, p_{a,n,t}^- \leq K_{n,t} \qquad \forall n \in \mathcal{N}_a, t \in \mathcal{T} \tag{4.17g}$$
$$p_{a,n,t}^- \leq L_{a,n} \qquad \forall n \in \mathcal{N}_a, t \in \mathcal{T} \tag{4.17h}$$
$$l_{a,n} \leq p_{a,n,t}^+ \qquad \forall n \in \mathcal{N}_a, t \in \mathcal{T} \tag{4.17i}$$
$$I_{a,t}^+, I_{a,t}^- \geq 0 \qquad \forall t \in \mathcal{T} \tag{4.17j}$$

The main difference with the producer model is equation (4.17c) which enforce that flexible loads connected to bus n are supplied an amount of energy $V_{a,n}$. This kind of model of the flexible loads have already been used for instance in [120] or in [98].

Optimization of the flexibility offers The baseline of the retailer, $p^b_{a,n,t}$ is now fixed. The flexibility available from the producer portfolio is computed using the following optimization problem. If all the baselines of the retailer are unchanged with respect to the one computed in the baseline optimization step, the flexibility available is equal to the one predicted in the previous stage.

$$\max \sum_{t\in\mathcal{T}} \left(\pi^{\Delta^+}_{n,t}\Delta^+_{a,n,t} + \pi^{\Delta^-}_{n,t}\Delta^-_{a,n,t}\right) - \sum_{t\in\mathcal{T}}\left(\pi^{I^+}_t I^+_{a,t} + \pi^{I^-}_t I^-_{a,t}\right) - \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}_a} c_{a,n,t}p_{a,n,t} \tag{4.18a}$$

subject to,

$$k_{a,n} \le p_{a,n,t}, p^+_{a,n,t}, p^-_{a,n,t} \le K_{n,t} \qquad \forall n \in \mathcal{N}_a, t\in\mathcal{T} \tag{4.18b}$$

$$\sum_{t\in\mathcal{T}} p_{a,n,t} = V_{a,n} \qquad \forall n \in \mathcal{N}_a \tag{4.18c}$$

$$I^+_{a,t} - I^-_{a,t} = \sum_{n\in\mathcal{N}_a} p_{a,n,t} - \sum_{n\in\mathcal{N}_a} p^b_{a,n,t} \qquad \forall t\in\mathcal{T} \tag{4.18d}$$

$$\Delta^+_{a,n,t} \le p^+_{a,n,t} - p^b_{a,n,t} \qquad \forall n \in \mathcal{N}_a, t\in\mathcal{T} \tag{4.18e}$$

$$\Delta^-_{a,n,t} \le p^b_{a,n,t} - p^-_{a,n,t} \qquad \forall n \in \mathcal{N}_a, t\in\mathcal{T} \tag{4.18f}$$

$$p^{\min}_{a,n,t} \le p_{a,n,t}, p^+_{a,n,t}, p^-_{a,n,t} \le p^{\max}_{a,n,t} \qquad \forall n \in \mathcal{N}_a, t\in\mathcal{T} \tag{4.18g}$$

$$p^-_{a,n,t} \le L_{a,n} \qquad \forall n \in \mathcal{N}_a, t\in\mathcal{T} \tag{4.18h}$$

$$l_{a,n} \le p^+_{a,n,t} \qquad \forall n \in \mathcal{N}_a, t\in\mathcal{T} \tag{4.18i}$$

$$I^+_{a,t}, I^-_{a,t} \ge 0 \qquad \forall t\in\mathcal{T} \tag{4.18j}$$

For each bus n, the retailer makes one energy constrained flexibility offer for the whole time horizon where the maximum and minimum modulation amplitudes are given by $\Delta^+_{a,n,t}$ and $\Delta^-_{a,n,t}$. No activation fee is required by the retailer for its service. The reservation fee is given by

$$\pi^R.\frac{\sum_{t\in\mathcal{T}}(\Delta^+_{a,n,t} + \Delta^-_{a,n,t})}{2} + \sum_{t\in\mathcal{T}} \pi^E_t (p^b_{a,n,nt} - p^0_{a,n,t}) \tag{4.19}$$

where the second part is the cost of using the baseline $p^b_{a,n,t}$ to increase the available flexibility with respect to a baseline $p^0_{a,n,t}$ which is obtained by solving problem (4.17) without considering flexibility (by setting $\pi^{\Delta^+}_{n,t} = \pi^{\Delta^-}_{n,t} = 0$).

Flexibility services procurement and activation The retailer flexibility procurement optimization problem is similar to the one of the DSO (4.11) adapted to its specific constraints.

$$\begin{aligned}\min \sum_{i\in\mathcal{S}_n}\left(\pi_i^r(W_{a,i}-w_{a,i})+\pi_i^b v_{a,i}\right)+\sum_{i\in\mathcal{E}_n}\left(\pi_i^r y_{a,i}+\pi_i^b(x_{a,i,t}^+ + x_{a,i,t}^-)\right)\\ +\sum_{t\in\mathcal{T}}\left(\pi_t^{I^+}I_{a,t}^+ + \pi_t^{I^-}I_{a,t}^-\right)+\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}_a}c_{a,n,t}p_{a,n,t}\end{aligned} \tag{4.20a}$$

subject to

$$p_{a,n,t}^{\min} \le p_{a,n,t} \le p_{a,n,t}^{\max} \qquad \forall n\in\mathcal{N}_a, t\in\mathcal{T} \tag{4.20b}$$

$$\sum_{t\in\mathcal{T}} p_{a,n,t} = V_{a,n} \qquad \forall n\in\mathcal{N}_a \tag{4.20c}$$

$$k_{a,n} \le p_{a,n,t} \le K_{n,t} \qquad \forall n\in\mathcal{N}_a, t\in\mathcal{T} \tag{4.20d}$$

$$u_{a,n,t} = \sum_{i\in\mathcal{S}_n:\tau_i=t} v_{a,i} + \sum_{i\in\mathcal{E}_n} x_{a,i,t}^+ - x_{a,i,t}^- \qquad \forall n\in\mathcal{N}, t\in\mathcal{T} \tag{4.20e}$$

$$m_i \le w_{a,i} \le v_{a,i} \le W_{a,i} \le M_i \qquad \forall n\in\mathcal{N}, i\in\mathcal{S}_n \tag{4.20f}$$

$$m_{i,t}y_{a,i} \le x_{a,i,t}^+ - x_{a,i,t}^- \le M_{i,t}y_{a,i} \qquad \forall n\in\mathcal{N}, i\in\mathcal{E}_n, t\in\mathcal{T} \tag{4.20g}$$

$$\sum_{t\in\mathcal{T}}(x_{a,i,t}^+ - x_{a,i,t}^-) = 0 \qquad \forall n\in\mathcal{N}, i\in\mathcal{E}_n \tag{4.20h}$$

$$I_{a,t}^+ - I_{a,t}^- = \sum_{n\in\mathcal{N}_a} p_{a,n,t} - \sum_{n\in\mathcal{N}_a}(p_{a,n,t}^b + u_{a,n,t}) \qquad \forall t\in\mathcal{T} \tag{4.20i}$$

$$I_{a,t}^+, I_{a,t}^- \ge 0 \qquad \forall t\in\mathcal{T} \tag{4.20j}$$

Optimization of the realization The retailer chooses the realization considering as parameter flexibility activation requests $h_{a,n,t}$ as a FSP and its flexibility requests as a FSU $u_{a,n,t}$. Note the local imbalance penalty for the buses $n\in\mathcal{B}_a$ in which the retailer is providing flexibility services.

$$\min \sum_{t\in\mathcal{T}}\left(\pi_t^{I^+}I_{a,t}^+ + \pi_t^{I^-}I_{a,t}^-\right)+\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}_a}c_{a,n,t}p_{a,n,t}+\pi^l\sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{B}_a}(i_{a,n,t}^+ + i_{a,n,t}^-) \tag{4.21a}$$

subject to

$$p_{a,n,t}^{\min} \le p_{a,n,t} \le p_{a,n,t}^{\max} \qquad \forall n\in\mathcal{N}_a, t\in\mathcal{T} \tag{4.21b}$$

$$\sum_{t\in\mathcal{T}} p_{a,n,t} = V_{a,n} \qquad \forall n\in\mathcal{N}_a \tag{4.21c}$$

$$k_{a,n} \leq p_{a,n,t} \leq K_{n,t} \qquad \forall n \in \mathcal{N}_a, t \in \mathcal{T} \tag{4.21d}$$

$$I^+_{a,t} - I^-_{a,t} = \sum_{n \in \mathcal{N}_a} p_{a,n,t} - \sum_{n \in \mathcal{N}_a} (p^b_{a,n,t} + u_{a,n,t} + h_{a,n,t}) + E_{a,t} \qquad \forall t \in \mathcal{T} \tag{4.21e}$$

$$I^+_{a,t}, I^-_{a,t} \geq 0 \qquad \forall t \in \mathcal{T} \tag{4.21f}$$

$$i^+_{a,n,t} - i^-_{a,n,t} = p_{a,n,t} - (p^b_{a,n,t} + u_{a,n,t} + h_{a,n,t}) \qquad \forall t \in \mathcal{T}, n \in \mathcal{N}_a \tag{4.21g}$$

$$i^+_{a,n,t}, i^-_{a,n,t} \geq 0 \qquad \forall t \in \mathcal{T}, n \in \mathcal{N}_a \tag{4.21h}$$

4.9.4 Optimization problems of the transmission system operator

Flexibility procurement The TSO is a pure FSU in which flexibility needs R^+_t and R^-_t are taken as data and are not localized. Any FSP may, in principle, offer up to these amounts to the TSO. The flexibility contracted by the TSO, S^+_t and S^-_t, is obtained by solving (4.22). Since the TSO can procure flexibility outside of the distribution system, the amount of flexibility collected in this network weighted by the price difference enters the objective function as a surplus.

$$\begin{aligned} \max \sum_{t \in \mathcal{T}} & \left(\pi^{S+}_t \min\{S^+_t, R^+_t\} + \pi^{S-}_t \max\{S^-_t, R^-_t\}\right) \\ & - \sum_{i \in \mathcal{S}_n} \left(\pi^r_i (W_{TSO,i} - w_{TSO,i}) + \pi^b_i v_{a,TSO,i}\right) \\ & - \sum_{i \in \mathcal{E}_n} \left(\pi^r_i y_{TSO,i} + \pi^b_i (x^+_{TSO,i,t} + x^-_{TSO,i,t})\right) \end{aligned} \tag{4.22a}$$

subject to

$$S^+_t = \sum_{n \in \mathcal{N}} \sum_{i \in \mathcal{S}(n): \tau_i = t} W_{TSO,i} + \sum_{n \in \mathcal{N}} \sum_{i \in \mathcal{E}(n)} M_{i,t} y_{TSO,i} \qquad \forall t \in \mathcal{T} \tag{4.22b}$$

$$S^-_t = \sum_{n \in \mathcal{N}} \sum_{i \in \mathcal{S}(n): \tau_i = t} w_{TSO,i} + \sum_{n \in \mathcal{N}} \sum_{i \in \mathcal{E}(n)} m_{i,t} y_{TSO,i} \qquad \forall t \in \mathcal{T} \tag{4.22c}$$

$$m_i \leq w_{TSO,i} \leq v_{TSO,i} \leq W_{TSO,i} \leq M_i \qquad \forall n \in \mathcal{N}, i \in \mathcal{S}_n \tag{4.22d}$$

$$m_{i,t} y_{TSO,i} \leq x^+_{TSO,i,t} - x^-_{TSO,i,t} \leq M_{i,t} y_{TSO,i} \qquad \forall n \in \mathcal{N}, i \in \mathcal{E}_n, t \in \mathcal{T} \tag{4.22e}$$

$$\sum_{t\in\mathcal{T}}(x^+_{TSO,i,t} - x^-_{TSO,i,t}) = 0 \qquad \forall n \in \mathcal{N}, i \in \mathcal{E}_n \tag{4.22f}$$

Flexibility activation When the TSO decides to activate flexibility services, the TSO solves an imbalance E_t which is external to the considered distribution system.

$$\min \sum_{t\in\mathcal{T}}\left(\pi^{I^+}_t I^+_{TSO,t} + \pi^{I^-}_t I^-_{TSO,t}\right) - \sum_{i\in\mathcal{S}_n}\pi^b_i v_{TSO,i} - \sum_{i\in\mathcal{E}_n}\pi^b_i(x^+_{TSO,i,t} + x^-_{TSO,i,t}) \tag{4.23a}$$

subject to

$$E_{TSO,t} + I^+_t - I^-_t + \sum_{n\in\mathcal{N}} u_{TSO,n,t} = 0 \qquad \forall t \in \mathcal{T} \tag{4.23b}$$

$$u_{TSO,n,t} = \sum_{i\in\mathcal{S}_n:\tau_i=t} v_{TSO,i} + \sum_{i\in\mathcal{E}_n} x^+_{TSO,i,t} - x^-_{TSO,i,t} \qquad \forall n \in \mathcal{N}, t \in \mathcal{T} \tag{4.23c}$$

$$I^+_{TSO,t}, I^-_{TSO,t} \geq 0 \qquad \forall t \in \mathcal{T} \tag{4.23d}$$

$$w_{TSO,i} \leq v_{TSO,i} \leq W_{TSO,i} \qquad \forall n \in \mathcal{N}, i \in \mathcal{S}_n \tag{4.23e}$$

$$m_{i,t}y_{TSO,i} \leq x^+_{TSO,i,t} - x^-_{TSO,i,t} \leq M_{i,t}y_{TSO,i} \qquad \forall n \in \mathcal{N}, i \in \mathcal{E}_n, t \in \mathcal{T} \tag{4.23f}$$

$$\sum_{t\in\mathcal{T}}(x^+_{TSO,i,t} - x^-_{TSO,i,t}) = 0 \qquad \forall n \in \mathcal{N}, i \in \mathcal{E}_n \tag{4.23g}$$

Chapter 5

Price signal in distribution networks

The work of this chapter is based on a price signal with two settings: off-peak tariff and on-peak tariff. Some loads are connected to specific electricity meters which allow consumption only in off-peak periods. Historically, off-peak periods were located during the night and on-peak periods during the day. Changing the assignment of off-peak periods is an easy method for distribution system operators to access to the flexibility of small consumers. This solution can be implemented quickly as the infrastructure needed already exists in some countries.

This chapter proposes a mixed-integer linear model to assign optimally the off-peak hours in order to minimize a societal cost. This cost gathers together the cost of electricity, the financial losses due the shedding of photovoltaic installations and the loads' wellbeing. The model considers automatic shedding of inverters and constraints of the electrical distribution networks. Simulation results show that the new disposition of off-peak hours could reduce significantly the photovoltaic energy curtailed in the summer.

5.1 Nomenclature

This section defines the main symbols used in this chapter.

Indexes

d	Day
g	Group
i	Flexible load
n	Bus
t	Period

Sets

$\mathcal{T}(d)$	Periods of day d
$\mathcal{T}$	Periods, $\mathcal{T} = \{0, \ldots, T\}$
$\mathcal{T}_0$	$\mathcal{T} \setminus \{0\}$
$\mathcal{N}$	Buses
$\mathcal{N}_h$	Buses to which houses are connected
$\mathcal{M}$	Flexible loads
$\mathcal{G}$	Distinct groups
$g(k)$	Group $g \in \mathcal{G}$ to which load j belongs
$\mathcal{M}(n)$	Flexible loads connected to bus n
$\mathcal{D}$	Days of the time horizon

Parameters

$C_{n,n',t}$	Capacity of link (n, n') in period t
Δt	Length of a period
π_t^E	Energy price in period t
π^p	Solar production surplus
π^{VSC}	Value of shed consumption
$\eta_{n,t}$	Maximum power injected at bus n in period t
$\rho_{n,t}$	Curtailable power production at bus n in period t
$\zeta_{n,t}$	Static load power consumption at bus n in period t
$\beta_{j,t}$	Amount of off-peak periods added to load j in period $t \geq 0$
$\tau_{j,t}$	Lower bound on $s_{j,t}$, $t \geq 0$
$\gamma_{j,t}$	Upper bound on $s_{j,t}$, $t \geq 0$
M_j	Upper bound on $s_{j,t-1} + \beta_{j,t}$
θ_j	Nominal power of load j
$\xi_{g,d}$	Number of off-peak periods for group g during day d
μ	Daily maximum number of rate switchings

Variables

$l_{n,n',t}$	Power going from bus n to bus n' in period t

$q_{n,t}$	Power balance at bus n in period t
$r_{n,t}$	Curtailed quantity at bus n in period t
$s_{j,t}$	State of load j at the end of period $t \geq 0$
$x_{j,t}$	Fraction of period during which load j consumes power in period t
$u_{g,t}$	Equals 1 if consumer group g is off-peak in period t, 0 otherwise
$w_{j,t}$	Auxiliary variable needed in order to define $x_{j,t}$
$p_{n,t}$	Variable power consumed at bus n in period t
$z_{g,t}$	Equals 1 if there is a rate switching between periods $t-1$ and t

5.2 Introduction

Chapter 4 is dedicated to increasing the amount of distributed generation into the medium voltage network using flexibility services. The practical implementation of the solutions proposed in Chapter 4 is not straightforward as it requires modifying the legislation and, for the DSO, to develop the complex tools needed to perform active network management. One could therefore need a simpler solution, even if this solution is less efficient, in order to increase the amount of renewable energy that can be connected to the distribution network on the short term. The low voltage network is, in some places, already often saturated since the connected photovoltaic (PV) installations create overvoltages. Therefore, the PV installations are now equipped with a mandatory protection which monitors the voltage and switch off the production if the voltage is too high. This typically occurs around noon when the sun is shining but there is few residential consumption since people are working away from home. As a result, the green energy that would have been produced if not curtailed is completely lost.

To solve this issue, this chapter focuses on a mechanism with two types of tariffs: a cheap one called off-peak tariff, and a more expensive one referred to as on-peak tariff. Historically, off-peak periods were located during the night and on-peak periods during the day. Changing the off-peak hours is an easy method to access to the flexibility of small consumers since the infrastructure needed already exists in some countries. The off-peak signal received by the consumers is broadcasted through the distribution network by a relay located at the HV/MV transformer. This relay broadcasts a signal in the network which indicates the starting or the ending of the off-peak period. The signal is received by the specific electricity meters which are programmed to consume power only if the tariff is off-peak. These meters are called “night-only” meters. Typical loads connected to night-only meters are electric storage heaters and electric boilers. These loads are externally turned on when the meter receives the off-peak tariff signal and switched off when the tariff becomes on-peak. This chapter proposes to determine and quantify the impact of an optimal assignment of off-peak periods during the day in order to optimize the flexibility of the consumption of loads consuming only in off-peak periods.

This chapter is structured as follows. Section 5.3 reviews the relevant literature. The problem is stated in Section 5.4. The practical considerations are discussed in Section 5.5. Section 5.6 describes how loads can be modeled. The mathematical formulation of the complete problem is given in Section 5.7. Results on a typical distribution network are provided in Section 5.8. Finally, Section 5.9 concludes. The work presented in this chapter is the result of a collaboration with the master student Luca Merciadri in the context of his master thesis [105].

5.3 Literature review

Several methods have been proposed in order to deal with overvoltages in distribution networks. As a mean of protection, last-resort automatic security solutions such as automatic shedding of the distributed generation have been implemented. Curtailing is the shedding of an inverter of a production unit when the voltage exceeds a given threshold. This phenomenon occurs more and more with the increasing penetration of photovoltaic installations. Various solutions exist to further increase the injection from renewable energy sources. One of these solutions is to modify the PV's inverters controller to provide reactive power control [36]. Batteries combined with decentralized storage strategy to provide voltage control in low-voltage (LV) feeders also help to reduce overvoltages [20, 94]. A centralized controller could also regulate distribution network voltages by adjusting the output of distributed generations [138]. Operating these kinds of centralized controls would be the DSO's responsibility.

With the growth of PV production, DSOs are more and more considering alternatives to expensive investments in network components (*i.e.*, lines, cables, transformers, *etc.*). To this end, DSOs could exploit the flexibility from consumers connected to their distribution network. This flexibility can be provided by market-based mechanism as done in Chapter 4 and be directly controlled in short-term operation of distribution networks [64]. An alternative is to modify the consumer's electricity tariff depending on the time of the day to incite consumption shifting. A comparison at the market level of these incentive-based mechanisms for load curve improvement is given in [109]. From a local perspective, this indirect mechanism has been shown to avoid congestions in the distribution network if coupled with a smart electric vehicle charging algorithm [113]. In the Nice Grid project, EDF identified 40 solar days of summer 2014 and summer 2015, and notified the day before to shift their electricity consumption between 12 pm and 4 pm. At the end of each summer, EDF paid a voucher enabling the customer to benefit from a price equivalent to the Off-peak Hours for their electricity consumption during the Solar Hours [124]. One could also modify the tariff depending not only on the time but also on the location. This leads to the notion of nodal pricing. These prices can be built up to meet global objectives such as avoiding congestion [77] or maximizing the network performance and the global welfare of all the flexible consumers [15].

5.4 Problem statement

Consider a time horizon divided into T periods, and a timestep Δt, for instance one day divided into 24 periods. Loads responsive to off-peak patterns are divided in groups responding to the same off-peak pattern. The aim of this work is to determine off-peak tariff patterns, i.e to obtain vectors such that

$$u_{g,t} = \begin{cases} 1, & \text{if off-peak tariff in period } t \text{ for group } g \\ 0, & \text{if on-peak tariff in period } t \text{ for group } g \end{cases}. \tag{5.1}$$

An example of such pattern is given in Figure 5.1.

Hour	$0-6$ h	$6-12$ h	$12-15$ h	$15-24$ h
Off-peak signal	**X**		**X**	

Figure 5.1: Example of output off-peak pattern for one group of loads.

Problem's inputs are network data, power productions, loads' power consumptions, and loads' utilities. The set of feasible off-peak tariff patterns is restricted by some constraints. First, the pattern must assign a given number of off-peak hours, e.g. 9h of off-peak hours and 15h of on-peak hours. Second, the number of tariff switchings is bounded, because the relay sending signals to every meter quickly heats up. We also consider the impact of the pattern on the distribution network at the medium voltage level, the power capacities of the lines and of the HV/MV transformer.

5.5 Practical implementation

This section discusses the interest and the technical requirements needed to change the assignment of off-peak hours. The figures that are present in this section are realistic figures for Belgium.

In summer, the only flexible loads are electric boilers. In winter, electric heaters increase consequently the flexibility of the consumption. It is legitimate to ask ourself if there is enough flexibility available in a sunny summer hour to substantially reduce sheddings. Let us assume that 70 houses are connected behind an MV/LV transformer. If 8% of houses are equipped with 6kW of PV panels, a maximum of 33.6kW of solar production can be reached during peak sunny hours in summer. If 5% of houses are equipped with a night-only meter connected to a 3kW load such as an electric boiler, one gets around 10.5kW of maximal variable power consumption by MV bus. It can be reasonably assumed to have, each day, at least two hours of variable power consumption from electric boilers in summer. A minimum static power consumption by MV bus in a peak sunny hour is estimated to be 20kW. Shifting the consumption of the boilers to these sunny peak hours can reduce the infeed to the MV network from 13.6kW to 3.1kW, which causes neither overvoltages nor sheddings.

Is it technically feasible to switch from a predetermined off-peak periods pattern to a dynamic one? In Belgium [34], remote controls are already able to send off-peak signals to every belgian household. The main modification to the existing implementation is the fact that off-peak hours' repartition is different than the classic off-peak pattern. This has technical implications, which are now considered briefly. There is a need for a computation platform to assign the off-peak hours optimally. Following the results of Section 5.8.3, little computation power is needed. Changing the off-peak hours may change the configuration of electric heaters which were previously configured to consume power during the classical off-peak hours. Performing this configuration requires to send a technician to houses with old electric heaters connected to night-only meters which may not be responsive to the modification of the off-peak hours. Despite this additional cost for a minority of households, this solution can be seen as a quick and easy solution that can be used for night-only meters and smart meters.

5.6 Load modeling

This section proposes to model, at the medium voltage level, the behavior of the loads consuming only in off-peak periods. These loads are typically electric boilers and heaters. The model proposed in this chapter is a tank model similar to the ones proposed in [73, 98]. Let us consider a flexible load j. This load can only consume power during off-peak periods. We assume that this load consumes either zero power or a nominal power θ_j. Flexible loads are assumed to have no starting or ending phase. The energy needs in period t are given by $\beta_{j,t}$, a number of periods during which nominal power needs to be consumed. The state of the load is denoted by $s_{j,t}$. It represents the number of periods during which load j needs to consume at nominal power to reach its maximal storage capacity. For instance, if load j is a boiler, $s_{j,t}$ is the number of off-peak periods necessary to reach its set-point temperature in period t.

Note that one load might consume less energy than a full period of consumption at nominal power. This is modeled by a variable $x_{j,t} \in [0; 1]$ defined as the fraction of period during which load j consumes power in period t. For example, if $x_{j,t} = \frac{1}{2}$, the power consumed by the load is defined as $x_{j,t}\theta_j$. Therefore, this case is modeled as consuming half power during one period. $x_{j,t}$ is mathematically defined by

$$x_{j,t} = \min\left\{u_{g(k),t}; s_{j,t-1} + \beta_{j,t}\right\}, \quad \forall t \in \mathcal{T}_0, k \in \mathcal{M}. \tag{5.2}$$

The variable $x_{j,t}$ can be expressed by linear constraints using Observation 2.

Observation 2. *The affectation $c = \min\{a; b\}$ can be replaced by the following constraints, introducing an auxiliary binary variable w:*

$$c \geq a - \overline{a}w \tag{5.3}$$

$$c \geq b - \overline{b}(1 - w) \tag{5.4}$$

$$a \leq b + \overline{a}w \tag{5.5}$$

$$b \leq a + \overline{b}(1-w) \tag{5.6}$$
$$c \leq a + \overline{a}w \tag{5.7}$$
$$c \leq b + \overline{b}(1-w) \tag{5.8}$$

where $\overline{a}$ and $\overline{b}$ are upper bounds on a and b.

For $s_{j,t-1} + \beta_{j,t}$, the following upper bound can be used:

$$M_j := \overline{s_{j,t-1} + \beta_{j,t}} = \sum_{t=0}^{T} \beta_{j,t}. \tag{5.9}$$

The state of load j at the end of period t, $s_{j,t}$, is given by:

$$s_{j,t} = s_{j,t-1} + \beta_{j,t} - x_{j,t}, \qquad \forall t \in \mathcal{T}_0, k \in \mathcal{M}, \tag{5.10}$$

with $\beta_{j,t} \in \mathbb{R}^+ \quad \forall k \in \mathcal{M}, t \in \mathcal{T}$. The load's state is bounded by the following inequations:

$$\tau_{j,t} \leq s_{j,t} \leq \gamma_{j,t}, \quad \forall k \in \mathcal{M}, t \in \mathcal{T}. \tag{5.11}$$

In order to ensure that the load's state at the end of the time window does not hinder the flexibility for the following day, the following constraint is added:

$$s_{j,T} \leq \beta_{j,0} \quad \forall k \in \mathcal{M}. \tag{5.12}$$

This constraint impose to the state of the load at the end of the simulation horizon to require no more consumption that the one needed at the first period of the horizon. The variable power consumption at bus n in period t is the sum of the consumptions of the flexible loads connected to bus n:

$$p_{n,t} = \sum_{j \in \mathcal{M}(n)} \theta_j x_{j,t}, \quad \forall n \in \mathcal{N}_h, t \in \mathcal{T}_0. \tag{5.13}$$

Table 5.5 provides an example of load state transition for a boiler whose consumption is dependent of the number of showers taken by house occupants.

Table 5.5: Example of load state transition.

t	0	1	2	3	4	5	6	7	8
off-peak				X		X	X		X
Showers			2						1
$\beta_{k,t}$	0	0	2	0	0	0	0	0	1
$x_{k,t}$			0	1	0	1	0		1
$s_{k,t}$	0	0	2	1	1	0	0	0	0

5.7 Mathematical formulation

This section describes the mathematical formulation of the problem. First, a model of inverters' shedding at the medium voltage level is established. Second, the terms constituting the objective function are detailed. Third, the complete mixed-integer linear program is defined.

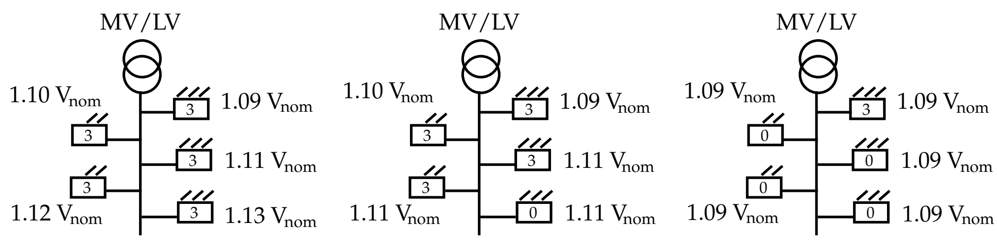
5.7.1 Inverters shedding model

This subsection models the shedding of PV inverters at the medium voltage level. An inverter trips when it detects a voltage ten percent higher than the nominal voltage. At that moment, it stops injecting power into the grid. The maximum power injected in a MV/LV transformer can be approximated with the following algorithm:

Algorithm 1 Algorithm computing an approximation of the maximum power that can be injected in a MV/LV transformer.

while there exists a bus with PV that has a voltage $\geq 1.1\text{V}_{\text{nom}}$ **do**
  Curtail the production at the highest voltage point
  Compute bus voltages using a load flow
end while
return Net power injected in the slack bus.

Figure 5.2 illustrates the algorithm on a simplified example. This maximum power injected is denoted by $\eta_{n,t}$ for bus n in period t. Applying this algorithm on a feeder with 70 houses consuming 0.5kW and considering one quarter of the houses with 6kW of PV panels leads to a maximum injected power of 79kW.



(a) Initial situation. (b) One producer is curtailed. (c) $\eta_{n,t} = 3$ kW.

Figure 5.2: Example showing the application of Algorithm 1.

The maximum power injected $\eta_{n,t}$, determined considering active and reactive power flows in the low voltage network, is used as parameter in the model of the medium voltage network. The medium voltage network is modeled with a network flow considering only active power flows. The power balance at bus

$n \in \mathcal{N}_h$ in period t is denoted by $q_{n,t}$, positive for a production. The low voltage part under each medium voltage bus is aggregated by three quantities: the static consumption $\zeta_{n,t}$, the variable consumption $p_{n,t}$, and the curtailable production $\rho_{n,t}$. The shedding of PV panels due to overvoltage is modeled using $\eta_{n,t}$ to compute the power balance $q_{n,t}$:

$$q_{n,t} = \min \left\{ \eta_{n,t}, \rho_{n,t} - \zeta_{n,t} - p_{n,t} \right\}, \quad \forall n \in \mathcal{N}_h, t \in \mathcal{T}_0. \tag{5.14}$$

The minimum function models the sheddings when $\rho_{n,t} - \zeta_{n,t} - p_{n,t} > \eta_{n,t}$. In this case, sheddings happen roughly from the end of the feeders to their beginnings until a value equal or just a little bit smaller than $\eta_{n,t}$. The curtailed quantity is given by

$$r_{n,t} = (\rho_{n,t} - \zeta_{n,t} - p_{n,t}) - q_{n,t}, \quad \forall n \in \mathcal{N}_h, t \in \mathcal{T}_0. \tag{5.15}$$

5.7.2 Objective function

The objective function represents a societal cost to minimize. This cost is divided into three parts. The first cost is an estimation of the total money loss due to sheddings. If π^p is the surplus produced with PV panels for a consumer, this can be expressed as

$$\pi^p \sum_{n \in \mathcal{N}_h, t \in \mathcal{T}_0} r_{n,t}. \tag{5.16}$$

The value of π^p is typically the sum of the energy price and the surplus due to green certificates.

The cost of buying or selling energy using the energy price π_t^E in period t is defined by:

$$\sum_{n \in \mathcal{N}_h, t \in \mathcal{T}_0} \pi_t^E (-q_{n,t}). \tag{5.17}$$

If $q_{n,t} \geq 0$ and $\pi_t^E > 0$, this term is the surplus due to energy being sold.

The third cost is the cost of loads' wellbeing. This part of the objective function represents the comfort of loads' users. If $s_{j,t} = 0$, the state of load j is at its minimum bound at the end of period t. If e.g. a boiler is such that its $s_{j,t}$ is maximal, that implies that the boiler is cold. It is interesting to have $s_{j,t}$ as low as possible for every $t \in \mathcal{T}_0$ in order for load j to be able to deal with a larger demand than foreseen. If a cost for loads' wellbeing π^{VSC} is defined, the following quantity should be minimized:

$$\pi^{VSC} \sum_{j \in \mathcal{M}, t \in \mathcal{T}_0} s_{j,t} \theta_j \tag{5.18}$$

5.7.3 Optimization problem

The determination of the optimal off-peak pattern for each consumer group is given by solving the following mixed-integer linear program:

$$\min \sum_{t \in \mathcal{T}_0} \left(\pi^p \sum_{n \in \mathcal{N}_h} r_{n,t} - \pi_t^E \sum_{n \in \mathcal{N}_h} q_{n,t} + \pi^{VSC} \sum_{j \in \mathcal{M}} s_{j,t} \theta_j \right) \tag{5.19a}$$

subject to
$\forall n \in \mathcal{N}, j \in \mathcal{N}, t \in \mathcal{T}_0$:

$$l_{n,n',t} \leq C_{n,n',t} \tag{5.19b}$$
$$l_{n,n',t} + l_{ji,t} = 0 \tag{5.19c}$$

$\forall n \in \mathcal{N}, t \in \mathcal{T}_0$:

$$\sum_{j \in \mathcal{N}} l_{n,n',t} = q_{n,t} \tag{5.19d}$$

$\forall n \in \mathcal{N}_h, t \in \mathcal{T}_0$:

$$q_{n,t} \leq \eta_{n,t} \tag{5.19e}$$
$$r_{n,t} = (\rho_{n,t} - \zeta_{n,t} - p_{n,t}) - q_{n,t} \tag{5.19f}$$
$$p_{n,t} = \sum_{j \in \mathcal{M}(n)} \theta_j x_{j,t} \tag{5.19g}$$

$\forall n \in \mathcal{N}_0 \setminus \mathcal{N}_h, t \in \mathcal{T}_0$:

$$q_{n,t} = 0 \tag{5.19h}$$

$\forall k \in \mathcal{M}$:

$$s_{j,0} = \beta_{j,0} \tag{5.19i}$$
$$s_{j,T} \leq \beta_{j,0} \tag{5.19j}$$

$\forall k \in \mathcal{M}, t \in \mathcal{T}_0$:

$$s_{j,t} = s_{j,t-1} + \beta_{j,t} - x_{j,t} \tag{5.19k}$$
$$x_{j,t} \geq u_{g(k),t} - w_{j,t} \tag{5.19l}$$
$$x_{j,t} \geq (s_{j,t-1} + \beta_{j,t}) - M_j(1 - w_{j,t}) \tag{5.19m}$$
$$u_{g(k),t} \leq (s_{j,t-1} + \beta_{j,t}) + w_{j,t} \tag{5.19n}$$
$$s_{j,t-1} + \beta_{j,t} \leq u_{g(k),t} + M_j(1 - w_{j,t}) \tag{5.19o}$$
$$x_{j,t} \leq u_{g(k),t} + w_{j,t} \tag{5.19p}$$
$$x_{j,t} \leq (s_{j,t-1} + \beta_{j,t}) + M_j(1 - w_{j,t}) \tag{5.19q}$$

$\forall k \in \mathcal{M}, t \in \mathcal{T}$:

$$\tau_{j,t} \leq s_{j,t} \leq \gamma_{j,t} \tag{5.19r}$$

$\forall g \in \mathcal{G}, d \in \mathcal{D}$:

$$\sum_{t \in \mathcal{T}(d)} u_{g,t} = \xi_{g,d} \tag{5.19s}$$

$\forall d \in \mathcal{D}$:

$$\sum_{t \in \mathcal{T}(d)} \sum_{g \in \mathcal{G}} z_{g,t} \leq \mu \tag{5.19t}$$

$\forall t \in \mathcal{T} \setminus \{0, 1\}, g \in \mathcal{G}$:

$$z_{g,t} \geq u_{g,t-1} - u_{g,t} \tag{5.19u}$$
$$z_{g,t} \geq u_{g,t} - u_{g,t-1} \tag{5.19v}$$
$$z_{g,t} \leq u_{g,t-1} + u_{g,t} \tag{5.19w}$$
$$z_{g,t} \leq 2 - u_{g,t-1} - u_{g,t}. \tag{5.19x}$$

This optimization problem contains a total number of $|\mathcal{T}|(2|\mathcal{G}|+|\mathcal{K}|)$ binary variables. Constraint (5.19b) limits the power going from bus n to bus n'. (5.19c)–(5.19d) are the network flow equations for bus n. (5.19e)–(5.19f) define the power balance and the curtailed quantity at bus n in period t. The variable power consumed at bus n in period t is defined by (5.19g). (5.19h) accounts for buses under which no houses are present. (5.19i) sets the initial value of the load j's state. (5.19j) ensures that the load's state at the end of the time window does not hinder the flexibility for the following day. (5.19k) defines the state of load j at the end of period t. (5.19l)–(5.19q) define the fraction of period during which load j consumes power in period t. The load's state is bounded by (5.19r). (5.19s) ensures a given number of off-peak periods for every day. (5.19t)–(5.19x) limit the number of switchings.

5.8 Results

The results compare the application of the optimal assignment of off-peak hours to the classic off-peak pattern 21h–6h. First, the parameters of the test cases are detailed. Second, the classical off-peak pattern is compared to the optimal one on a sunny summer day. Finally, the second test shows the advantages of a monthly-optimal off-peak pattern in a summer month. The solutions are obtained using *CPLEX* 12.6 on a computer with two *Intel Core i7*, 3.33 GHz and 24 GB of RAM.

5.8.1 Parameters

This subsection describes the parameters that are used for the two tests. A range of values is indicated for parameters generated using a uniform distribution. The tests are based on a typical distribution network's structure. Behind a HV/MV transformer, the following structure is considered: 12 main buses are present behind the transformer, two buses are connected to each one of the 12 buses. To each of these 24 buses, three buses are connected to a MV/LV transformer. Figure 5.3 gives a visual representation of this network.

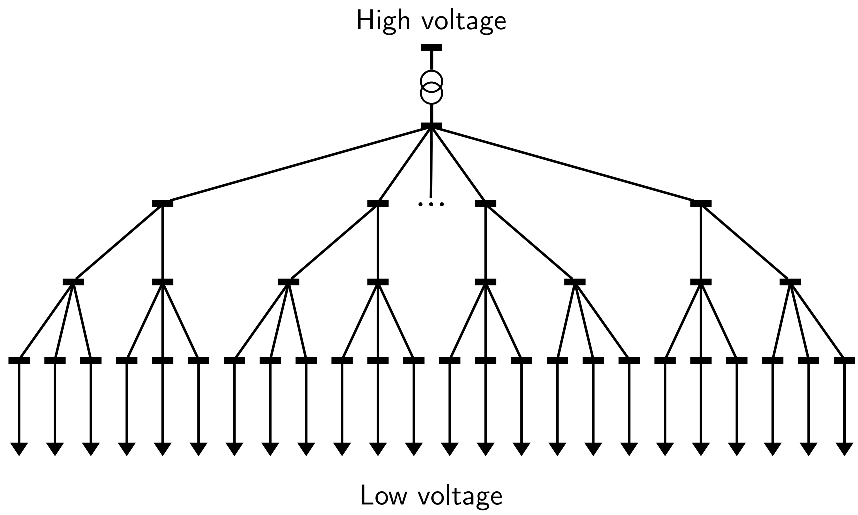


Figure 5.3: Visual representation of the considered medium voltage network.

We consider 70 houses below each MV/LV transformer. Four of these 70 houses are equipped with a night-only meter, each belonging to one of the four considered groups. To these night-only meters are connected an electric boiler of nominal power $\theta \in [2.1, 4.2]$ kW. The users' demands in kWh for the electric boiler during summer are indicated in Table 5.6. These demands should be transposed to their equivalent in periods, $\beta_{j,t}$ in function of the nominal power of the flexible load. Power production coefficients are obtained from [44] by computing ratios between hourly power production and maximal power production. Energy prices π_t^E are taken from [12], and static power consumptions originate from Synthetic Load Profiles values [136]. Remaining parameters are given in Table 5.7.

Table 5.6: Electrical boiler's energy demand in kWh.

t	0	1	2	3	4	5	6	7	8
$\Delta t \theta \beta_{j,t}$	0	0	0	4.05	0	0	0.45	0.45	1.17

Table 5.7: Additional parameters.

Parameter	Value		Unit
	5.8.2	5.8.3	
T	8	248	periods
Δt	3		h
π^{VSC}, π^{p}	$0.003, 1.8$		€/kWh
μ, ξ, τ, γ	$150, 3, 0, 3$		periods
Link capacity, HV/MV capacity	$20000, 40000$		kVA
Yearly static power consumption	$[3010, 5160]$		kWh/year
$\eta_{n,t}$	79		kW
Proportion of PV	30		%
Solar production/house with PV	$[4.2, 6]$		kW
$\beta_{j,t}$	see Table 5.6		periods

5.8.2 Optimal off-peak pattern in a sunny summer day

This subsection describes the results obtained on a sunny summer day: the 24th July 2012. Interesting parts of the solutions are given in Table 5.8. The optimization program places the off-peak hours between 12h and 18h and between 21h and 24h. In particular, placing off-peak hours in the fifth period decreases shedding costs by 44%, from €504.46 to €284.46.

Table 5.8 shows the power balance in each period for bus 7 for which a shedding happens in period 5. These power balances are compared to the maximum injection through the MV/LV transformer, $\eta_{7,t}$. When the optimal pattern is used, period 6 is off-peak, and the power balance is smaller than in the classical pattern, for which this is an on-peak period. Period 8 is an off-peak period for both settings but there is less consumption with the optimal pattern. This is a consequence of the consumption's shifting to periods 5 and 6.

Changing the assignment of off-peak hours modifies the flow going through the the HV/MV transformer. The maximum infeed to the MV network is 15% smaller with the optimal pattern and the maximum infeed to the HV network is 5% smaller. This solution could therefore help to reduce the needs of investment in transformers.

5.8.3 Monthly-optimal off-peak pattern in a summer month

Instead of the classical off-peak hours assignment 21h-6h, one could use a monthly-optimal pattern, i.e. a pattern identical for every day of the time horizon. The off-peak patterns obtained in the monthly-optimal case on July 2012 are given in Table 5.9. They are identical for any group $g \in \mathcal{G}$. The

Table 5.8: Solution for the sunny summer day test.

	Optimal pattern		Classical pattern	
Surplus for energy [€]	1021.38		1001.83	
Curtailment cost [€]	284.46		504.46	
Load wellbeing [€]	−2.46		−6.22	
Welfare value [€]	734.46		491.15	
Off-peak signal [periods]				
$u_{g,1}$	0		1	
$u_{g,2}$	0		1	
$u_{g,3}$	0		0	
$u_{g,4}$	0		0	
$u_{g,5}$	1		0	
$u_{g,6}$	1		0	
$u_{g,7}$	0		0	
$u_{g,8}$	1		1	
Power balance $q_{7,t}$ **[kW]**				
$q_{7,1}$	−14.85	$< \eta_{7,t}$	−14.85	$< \eta_{7,t}$
$q_{7,2}$	−11.79	$< \eta_{7,t}$	−11.79	$< \eta_{7,t}$
$q_{7,3}$	2.78	$< \eta_{7,t}$	2.78	$< \eta_{7,t}$
$q_{7,4}$	58.46	$< \eta_{7,t}$	58.46	$< \eta_{7,t}$
$q_{7,5}$	79	$= \eta_{7,t}$	79	$= \eta_{7,t}$
$q_{7,6}$	66.16	$< \eta_{7,t}$	66.76	$< \eta_{7,t}$
$q_{7,7}$	25.85	$< \eta_{7,t}$	25.85	$< \eta_{7,t}$
$q_{7,8}$	−25.94	$< \eta_{7,t}$	−31.94	$< \eta_{7,t}$
Max. infeed to MV [kW]	2523.53		2955.53	
Max. infeed to HV [kW]	4662.77		4929.35	
Time to solve [s]	0.82		0.21	

optimization program places again the off-peak hours between 12h and 18h and between 21h and 24h.

Table 5.9: Off-peak hours assignment for every group $g \in \mathcal{G}$, July 2012.

t	1	2	3	4	5	6	7	8
Classic pattern	1	1	0	0	0	0	0	1
Monthly-optimal pattern	0	0	0	0	1	1	0	1

The simulation over the month provides the solutions given in Table 5.10. Using the monthly-optimal pattern decreases the shedding cost by 47%. Concerning the maximum infeeds, it can be seen that the monthly-optimal pattern always yields lower infeeds than the classical pattern. In particular, there is a decrease of 14% for the maximum infeed to the MV network, and a decrease of 5% for the maximum infeed to the HV network. Notice that it takes only five minutes to solve to optimality the optimization problem on the whole month.

Table 5.10: Solutions obtained on the summer month test for the monthly-optimal and the classical pattern.

	Monthly-optimal	Classical
Surplus for energy	−€ 18909.87	−€ 18947.38
Curtailment cost	€ 1135.82	€ 2143.68
Load wellbeing	−€ 76.33	−€ 192.84
Welfare value	−€ 20122.02	−€ 21283.91
Maximum infeed to the MV network	2549.03kW	2981.03kW
Maximum infeed to the HV network	4662.77kW	4929.35kW
Time to solve	5min	10.07s

5.9 Conclusion

This chapter proposes a mixed-integer linear model to assign optimally the off-peak hours considering automatic shedding of inverters and constraints of the electrical distribution networks so as to minimize a societal cost. Simulation results show that the new disposition of off-peak hours can reduce by 50% the PV energy curtailed in the summer. The solution also helps to reduce the power flow going through the HV/MV transformer. This scheme has the main advantage of being practically implemented with very few to none investments in the current infrastructure.

Future work should focus on field testing the proposed scheme on a real part of the distribution network. Based on DSO data, it should also be worth investigating the exact cost of the strategy that has been proposed in this chapter. The optimization model could be extended to include more than two different tariffs, other load models and a more detailed network model. A stochastic formulation might also be considered to assess the impact of indirect response of consumers to tariff modifications.

Chapter 6

Flexibility from heat pumps

This chapter addresses the problem of an aggregator controlling heat pumps in households to offer a direct control flexibility service for network management purposes, such as solving system balance or congestion issues. The service is defined by a 15 minutes power modulation, upward or downward, followed by a payback effect on a fixed duration that models the corresponding relative decrease or increase of consumption. The service modulation is relative to an optimized baseline that minimizes the energy costs given a forward price signal. The amount of modulable power and payback effect is computed by solving a series of mixed integer linear programs. Within these optimization problems, the building thermal behavior is modeled as an equivalent thermal network made of thermal resistances and lumped thermal capacitances whose parameters are identified from more complex and validated models. With an average heat pump nominal power of 4.3 kW installed in one hundred houses, results show that it is possible to harvest an average upward modulation of 1.2kW per house with a payback of 600Wh leading to 150Wh of overconsumption. An average downward modulation of 500W per house can be achieved with a payback of 420Wh leading to 120Wh of overconsumption. In this case, the payback effect can be contained in a one hour and 15 minutes period.

6.1 Nomenclature

This section defines the main symbols used in this chapter. Others are defined as required in the text.

Sets

$\mathcal{H}$	Periods of the optimization horizon
$\mathcal{K}(\tau, k)$	Payback horizon starting in τ and lasting k periods

Parameters

H	Number of periods in the horizon
k	Number of payback periods
$\mathbf{A}^i$, $\mathbf{B}^i$, $\mathbf{E}^i$	Parameters of state-space model
C	Thermal capacitance
c_i, d_i, f_i	Parameters of the heat pump model
dt	Period duration
COP_t	Heat pump coefficient of performance
ϵ	Penalty for the payback imbalance
Γ_t	Exogenous power consumed
Q_t^g, Q_t^{sol}	internal heat gains, solar gains
π_t^+	Buying price of electricity
π_t^-	Selling price of electricity
R	Thermal resistance
σ	State deviation tolerance
T_t^a	Ambient temperature
T_t^{su}	Water supply temperature
$\mathbf{u}_t$	State-space model parameters
$\mathbf{x}^i$	Initial state

Variables

δ_t	Modulation amplitude
I^+	Maximum positive deviation after a modulation
I^-	Maximum negative deviation after a modulation
P_t	Total consumption
$\hat{P}_t$	Total Baseline consumption
P_t^+	Power bought from the grid
P_t^-	Power sold to the grid
Q_t	Heat pump thermal capacity
T_t	Temperature
W_t	Compressor electrical power consumption
$\hat{W}_t$	Compressor baseline electrical consumption
$\mathbf{x}_t$	State variable

$\hat{\mathbf{x}}_t$	Baseline state variable
y_t	Heating mode
$\hat{y}_t$	Baseline heating mode

Powers are taken positive when consumed and negative when produced. A positive modulation corresponds to an increase of the consumption.

Superscripts

g	gain
sol	solar
n	nominal
a	ambient
w	water
su	supply
s	space heating
max	maximum
min	minimum

6.2 Introduction

The previous chapters take as basis devices able to provide flexibility from their consumption. However, very few details are given on how an actual consumption processes can be turned into loads able to provide flexibility services. This chapter gives an example of such processes with heat pumps. Heat pumps are among the most promising devices to offer flexibility [8]. For the equipped household, they represent the larger part of their consumption and therefore a significant amount of energy to tap into. This energy is converted into heat which is stored by the building and, depending on its thermal inertia, allows consumption to be shifted without noticeable impact on the end-user comfort. A single heat pump does not represent a sufficient amount of consumption to provide a useful and cost efficient service to the electrical system. However, an aggregator can harness a significant amount of heat pumps to provide a coherent flexibility service.

This chapter shows how a specific direct control flexibility service can be provided by an aggregator controlling heat pumps within residential buildings. This specific flexibility service consists in an upward or a downward modulation for one time period followed by a fixed number of periods, called payback time, corresponding to the time for the system to go back to initially predicted state without modulation. The amplitudes of the modulations and of the paybacks are well defined within the service so that the quantity of flexibility activated is well known. This flexibility service takes as reference a baseline for each heat pump optimized to minimize the energy procurement costs considering a model of occupancy of the buildings.

This work has been done in collaboration with Emeline Georges, Ph.D. student in thethermodynamics laboratory of the university of Liège, who is responsible for the thermal model of the buildings and heat pumps. My contributions in this joint work are the definition of the flexibility service, its formalization as an optimization problem for an aggregator and the quantification of the volume of flexibility that can be obtained.

The chapter is organized as follows. The relevant literature is reviewed in Section 6.3. Section 6.4 defines the flexibility services considered in this chapter. The amplitudes of modulation are obtained by solving two optimization problems presented in Section 6.5. These optimization problems are based on a thermal model described in Section 6.6. In Section 6.7, the proposed methodology is applied to an academic case study composed of a hundred buildings representative of freestanding houses built after 1971 in Belgium. Finally, Section 6.8 concludes.

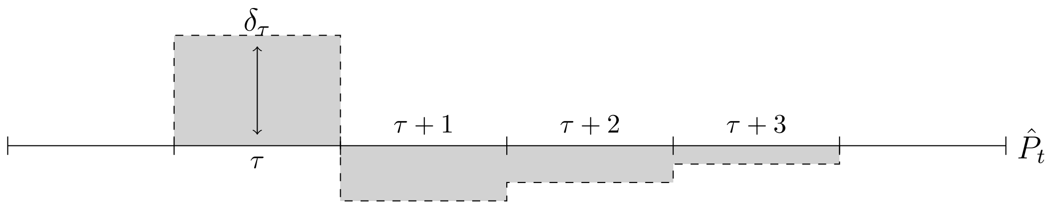
6.3 Literature review

Amongst the available flexible loads, thermostatically controlled loads (TCL) have been shown to present suitable characteristics to perform load following [83]. The following studies focus on detailed demand side models with TCLs. A methodology to build detailed and verified aggregated models to study demand-side management for a cluster of houses equipped with heat pumps is proposed in [121]. Article [110] presents validated physics-based thermal models of residential buildings and equipments with direct energy consumption minimization. Article [123] investigates the potential of using the thermal mass of office buildings to minimize peak demand. A day-ahead multi-objective optimization is implemented to provide the modulation service at minimum cost for the end-user and minimum frequency regulation cost. The optimization also determines the optimal time period to activate the load modulation. The study is extended to a portfolio of office buildings in [122] and the possible additional benefits retrieved from synergies between buildings are outlined. De Coninck and Helsen [35] develop a bottom-up approach to determine the flexibility of buildings and heating, cooling and air-conditioning systems. Three optimal control problems are solved to determine, first, a cost-optimal baseline for the consumer, and second the maximum upwards and downwards modulations available during a given time span of the day. Article [5] proposes a similar optimization scheme to [35] applied to residential demand response [5]. The cost-optimal day-ahead prediction of the baseline is followed by an intraday modulation with the introduction of “bonus” price-incentives. A sensitivity study of the percentage of storage capacity allocated respectively to the day-ahead and to the intra-day optimizations is carried out.

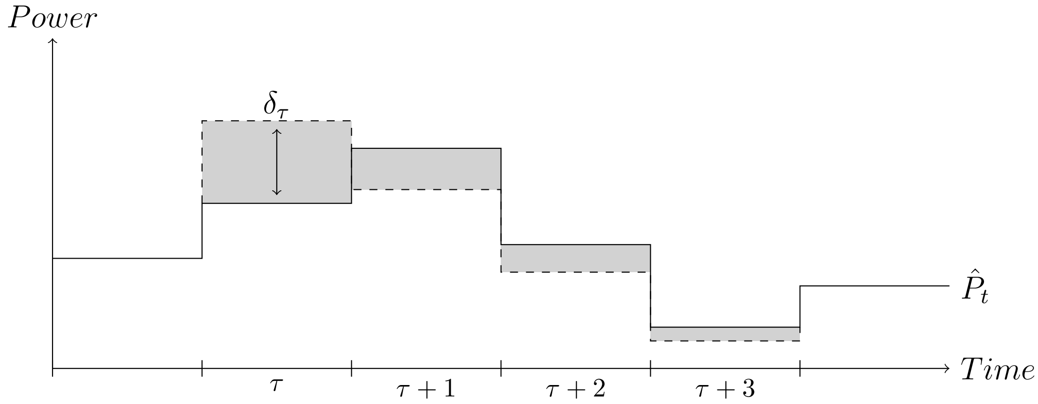
In light of this literature review, the first contribution of this chapter lies in the investigation of a flexibility service with detailed models of the thermostatically controlled loads. The second contribution is the characterization of the payback following the activation of the upward and downward power modulation service and of its influence on the achievable modulation amplitude for different periods of the day. The methodology is therefore complementary to the methods presented in [35] and [5] by constraining the payback time and characterizing the rebound effect in terms of costs and energy volumes, and differs from [123] in which the payback time is a result of the optimization scheme with a unique daily value.

6.4 Flexibility service

The product considered in this chapter is a flexibility service with a modulation in a given period τ and a payback in k following periods. A graphical representation is provided in Figure 6.1. The objective of the aggregator is to obtain the maximum modulation δ_τ, positive for an upward modulation and negative for a downward modulation, with the minimum payback in the following k periods.



(a) Modution signal.



(b) Modulation added to the baseline.

Figure 6.1: Example of upward modulation with three payback periods.

A modulation must be defined with respect to a reference consumption pattern [69]. In this chapter, we take as reference a baseline $\hat{P}_t$ which minimizes the electricity cost for the consumer [96]. This choice has two motivations. First, the use of flexible heat pumps should benefit the end-user. Minimizing the energy cost appears as a good incentive for consumers to enroll in flexibility programs proposed by aggregators. The second motivation lies in the possibility for an aggregator to be a balancing responsible party, which compels it to state its positions to the transmission system operator on the form of baselines. In this work, these baselines are computed by the aggregator and used as references to quantify the power modulations and resulting imbalances.

The flexibility service considered here is the results of the aggregation of a set of houses equipped with heat pumps. The aggregator proposes the service detailed in this section to another actor. The actual volume activated by the other actor, inferior or equal to the total potential, is application dependent and is out of the scope of this paper. The service provides all necessary information: the available potential of flexibility and the cost and deviations entailed by the activation of the service. With these information, the other actor is able to take a decision without having to directly manage each individual heat pumps. A typical case is an electricity retailer using the flexibility of its clients to balance its own portfolio as a balancing responsible party. Another example is an aggregator proposing its services to a distribution system operator willing to relieve a congestion in a line or a transformer, like in Chapter 4, or to a transmission system operator for its secondary reserve, like in Chapter 3.

6.5 Optimization problem

In this section, the thermal states transition model and the state constraints are summarized by

$$\mathbf{x}_{t+1} = f(\mathbf{x}_t, W_t^s, W_t^w, \mathbf{u}_t) \tag{6.1a}$$
$$\mathbf{x}_t^{min} \leq \mathbf{x}_t \leq \mathbf{x}_t^{max} \tag{6.1b}$$

The details of this model are given in Section 6.6. The variables $\mathbf{x}_t, W_t^s$ and W_t^w are the vector of the state variable, the heat pump consumption for space heating and for domestic hot water heating respectively. $\mathbf{u}_t$ represents the set of time dependent input parameters of the building model.

The first unknown to obtain is a base profile which minimizes the energy costs of the heat pump owner. This base profile is denoted $\hat{P}_t$ and the corresponding states are denoted $\hat{\mathbf{x}}_t$. They are obtained by solving the following optimization problem for each house.

$$\min \sum_{t \in \mathcal{H}} \left(\pi_t^+ P_t^+ - \pi_t^- P_t^- \right) dt \tag{6.2a}$$

subject to,

$$\hat{P}_t = P_t^+ - P_t^- \qquad \forall t \in \mathcal{H} \tag{6.2b}$$
$$\hat{P}_t = \hat{W}_t^s + \hat{W}_t^w + \Gamma_t \qquad \forall t \in \mathcal{H} \tag{6.2c}$$
$$\hat{\mathbf{x}}_{t+1} = f(\hat{\mathbf{x}}_t, \hat{W}_t^s, \hat{W}_t^w, \mathbf{u_t}) \qquad \forall t \in \mathcal{H} \tag{6.2d}$$
$$\mathbf{x}_t^{min} \leq \hat{\mathbf{x}}_t \leq \mathbf{x}_t^{max} \qquad \forall t \in \mathcal{H} \tag{6.2e}$$
$$0 \leq \hat{W}_t^s \leq \hat{y}_t W_t^{s,max} \qquad \forall t \in \mathcal{H} \tag{6.2f}$$
$$0 \leq \hat{W}_t^w \leq (1 - \hat{y}_t) W_t^{w,max} \qquad \forall t \in \mathcal{H} \tag{6.2g}$$
$$P_t^-, P_t^+ \geq 0 \qquad \forall t \in \mathcal{H} \tag{6.2h}$$
$$\hat{y}_t \in \{0, 1\} \qquad \forall t \in \mathcal{H} \tag{6.2i}$$

The duration of a period is given by dt which, for one quarter, equals 0.25h. The power bought from or sold to the grid in period t, P_t^+ and P_t^-, respectively at the prices π_t^+ and π_t^- in €/kWh, is defined from the heat pump consumption for space heating, $\hat{W}_t^s$, or domestic hot water heating, $\hat{W}_t^w$, and the power consumed or produced by other electric appliances Γ_t in (6.2b). We assume

$\pi_t^+ > \pi_t^-$. The case of an equality can be handled by removing constraint (6.2b) and using $\hat{P}_t$ in the objective function. The fact that heat pumps cannot be used simultaneously for space heating and domestic hot water heating is modeled by a binary variable $\hat{y}_t$ equal to one if the heat pump is used for space heating and to zero for domestic hot water production.

In the following, the optimization problem to solve in order to obtain the potential maximum upward modulation in a period τ with a payback effect in the k following periods is detailed. The maximum modulation available in one house at a given period is denoted δ_τ, and, in the case of an upward modulation, is obtained by solving

$$\max \delta_\tau - \epsilon I^+ - \epsilon I^- \tag{6.3a}$$

subject to,

$$
\begin{aligned}
& P_t = W_t^s + W_t^w + \Gamma_t && \forall t \in \mathcal{K}(\tau, k) && \text{(6.3b)}\\
& P_t = \hat{P}_t + \delta_t && \forall t \in \mathcal{K}(\tau, k) && \text{(6.3c)}\\
& 0 \leq W_t^s \leq y_t W_t^{s,max} && \forall t \in \mathcal{K}(\tau, k) && \text{(6.3d)}\\
& 0 \leq W_t^w \leq (1 - y_t) W_t^{w,max} && \forall t \in \mathcal{K}(\tau, k) && \text{(6.3e)}\\
& -I^- \leq \delta_t \leq I^+ && \forall t \in \mathcal{K}(\tau, k) \setminus \{\tau\} && \text{(6.3f)}\\
& I^-, I^+ \geq 0 && && \text{(6.3g)}\\
& \mathbf{x}_\tau = \hat{\mathbf{x}}_\tau && && \text{(6.3h)}\\
& \mathbf{x}_{t+1} = f(\mathbf{x}_t, W_t^s, W_t^w, \mathbf{u_t}) && \forall t \in \mathcal{K}(\tau, k) && \text{(6.3i)}\\
& \mathbf{x}_t^{min} \leq \mathbf{x}_t \leq \mathbf{x}_t^{max} && \forall t \in \mathcal{K}(\tau, k) && \text{(6.3j)}\\
& -\sigma \leq \hat{\mathbf{x}}_{\tau+k+1} - \mathbf{x}_{\tau+k+1} \leq \sigma && && \text{(6.3k)}
\end{aligned}
$$

where I^+ and I^- are the maximum positive and negative deviations with respect to the baseline on the payback horizon. These deviations are penalized by a parameter ϵ arbitrarily set in our tests to 10^{-2}.

Equation (6.3c) defines the modulation that can be achieved in each house with respect to its baseline. The initial condition on the state is given by (6.3h). Equality (6.3k) ensures that the state at the end of the modulation horizon is close enough to the one given by the baseline. As the state transition only depends on the previous state and the power consumed by the heat pump, this condition ensures that there is no major deviations from the baseline after the payback horizon. The case of maximum downward modulation is obtained by replacing (6.3a) by

$$\min \delta_\tau + \epsilon I^+ + \epsilon I^- \tag{6.4}$$

The total potential of modulation of the portfolio of an aggregator is obtained by summing the individual potential of each house.

6.6 Buildings and heat pumps

Heat demand of buildings can be determined using models containing different levels of details. Grey-box models are simplified models which provide an accurate representation of the thermal response of a building at significantly reduced computational requirements [41]. The building thermal behavior is modeled by an equivalent thermal network consisting of thermal resistances, R in K/W, and lumped thermal capacitances, C in J/K. The RC parameters of the network can be identified from validated models with higher level of details. For the purpose of this study, a single zone 5R3C structure, illustrated in Figure 6.2 and presented in [96], is used.

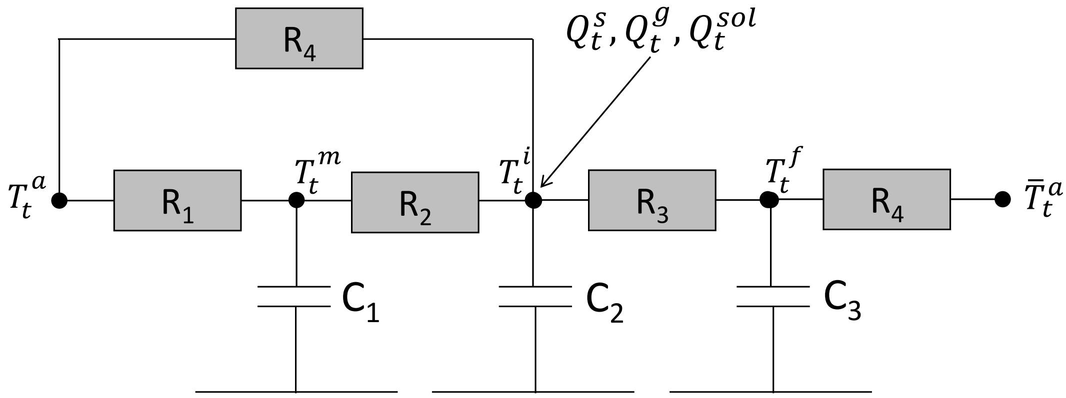


Figure 6.2: 5R3C Grey-box model structure

Such models allow a straightforward linear state-space formulation of the governing differential equations as follows

$$\mathbf{x}_{t+1}^s = \mathbf{A}^s \mathbf{x}_t^s + \mathbf{B}^s \mathbf{u}_t^s + \mathbf{E}^s Q_t^s \tag{6.5}$$

where Q_t^s is the space heating thermal power in period t. The state vector $\mathbf{x}_{t+1}^s$ is a three-elements vector composed of the indoor air temperature, T_t^i, the wall mass temperature, T_t^m, and the floor temperature, T_t^f. $\mathbf{u}_t^s$ is a four-elements vector composed of the uncontrolled model inputs, i.e. the outdoor air temperature, T_t^a, the yearly average outdoor air temperature, $\overline{T}_t^a$, the solar gains, Q_t^{sol} and the internal gains due to occupants and electrical appliances, Q_t^g. The matrices $\mathbf{A}^s, \mathbf{B}^s$ and $\mathbf{E}^s$ are equivalent RC parameters of the state space model dependent of the house modeled. Indoor thermal comfort for the occupants should be satisfied at any time as imposed by the constraint

$$T_t^{min} \leq T_t^i \leq T_t^{max}. \tag{6.6}$$

The domestic hot water tank is modeled using a one-node capacitance model with homogeneous water temperature $\mathbf{x}_t^w$. Heat losses to the ambiance are considered. The energy conservation law can be expressed by the following state-space formulation

$$\mathbf{x}^w_{t+1} = \mathbf{A}^w T^w_t + \mathbf{B}^w \mathbf{u}^w_t + \mathbf{E}^w Q^w_t \tag{6.7}$$

where Q^w_t is the domestic hot water heating thermal power in period t and T^w_t is the water temperature in the tank constrained by

$$T^{min} \leq T^w_t \leq T^{max} \tag{6.8}$$

The input vector $\mathbf{u}^w_t$ is composed of the outdoor air temperature and the mains water temperature. The matrices $\mathbf{A}^w, \mathbf{B}^w$ and $\mathbf{E}^w$ are parameters of the state space model dependent on the house modeled.

Variable-speed air-to-water heat pumps are used to cover domestic hot water and heating needs of the houses. They are modeled using a linear empirical model based on ConsoClim method [13]. The same model is used for space heating and domestic hot water, and only differs by the temperature of the water supplied to the house and of the water tank, T^{su} in K. The model determines the parameter $W^{\max}_t$ linked to the coefficient of performance (COP) of the heat pump which are used later to obtain the relation between the electrical power W_t and the heat demand Q_t.

$$Q^{max}_t = (d_0 + d_1(T^a_t - T^{a,n}) + d_2(T^{su}_t - T^{su,n}))Q^n \tag{6.9a}$$

$$\Delta T_t = \frac{T^a_t}{T^{su}_t} - \frac{T^{a,n}}{T^{su,n}} \tag{6.9b}$$

$$COP^{\max}_t = \frac{COP^n}{c_0 + c_1 \Delta T_t + c_2 \Delta T^2_t} \tag{6.9c}$$

$$W^{max}_t = \frac{Q^{max}_t}{COP^{max}_t} \tag{6.9d}$$

Equation (6.9a) determines the maximum thermal power that can be supplied by the heat pump for given ambient and water supply temperatures. The coefficient of performance is determined by Equation (6.9c) and the corresponding maximal electrical consumption of the compressor is given by Equation (6.9d). The part-load electrical consumption of the variable-speed compressor, W_t is expressed as a function of the heat demand, Q_t, using a piecewise linear approximation

$$W_t = f_1 \frac{Q_t}{Q^{max}_t} W^{max}_t \qquad for \ \frac{Q_t}{Q^{max}_t} \leq 0.3 \tag{6.10a}$$

$$W_t = f_2 \frac{Q_t}{Q^{max}_t} W^{max}_t \qquad for \ \frac{Q_t}{Q^{max}_t} > 0.3 \tag{6.10b}$$

In terms of technical constraints, the heat pump cannot work simultaneously to supply heat to the domestic hot water tank and to the space heating system. Furthermore, to prevent damage of mechanical components in the long term, decisions to start or stop the heat pumps should not occur more than eight times an hour. This precaution is ensured by a decision time step of 15 minutes.

The thermal states transition, given by equations (6.5), (6.7) and (6.10), and the state constraints, given by equations (6.6) and (6.8), are summarized by

$$\mathbf{x}_{t+1} = f(\mathbf{x}_t, W_t^s, W_t^w, \mathbf{u}_t) \tag{6.11a}$$

$$\mathbf{x}_t^{min} \leq \mathbf{x}_t \leq \mathbf{x}_t^{max} \tag{6.11b}$$

where $\mathbf{x}_t = [\mathbf{x}_t^s \; T_t^w]$ and $\mathbf{u}_t = [\mathbf{u}_t^s \; \mathbf{u}_t^w]$.

6.7 Results

6.7.1 Generation of the test cases

The methodology presented in the previous sections is applied to an academic case study composed of a hundred buildings representative of freestanding houses built after 1971 in Belgium. The characterization of the residential builds in terms of buildings geometry and envelope structure comes from the study [65]. The average heat pump nominal power is 4.3 kW. The nominal conditions are defined as in [19] for a 7°C outdoor temperature and a water temperature adapted to the house insulation level. Additional resistances of 3 to 5kW depending on the house insulation level are used as back up to cover the heat demand for space heating during the coldest days of the year. The control horizon is set to 24 hours divided into 96 periods.

The number of inhabitants in each house is drawn from a normal distribution of average three and a standard deviation of two with a maximum of five occupants. The exogenous consumption profiles associated to lighting and appliances are obtained from article [67], as well as the domestic hot water draw-off events. Indoor temperature set points schedules are intermittent temperature profiles generated based on normal distribution laws for morning, mid-day and evening start-up times. All profiles have a weekly average indoor set point above 18°C. Occupancy profiles are derived from the latest. Indoor thermal comfort for the occupants should be satisfied at any time. During the heating season, the indoor air temperature is constrained to deviated of maximum 1°C from the imposed set point during occupied periods of the day time and from the extreme limits of the daily set point during the night. In the summer, the lower limit of the indoor air temperature is set to 1°C below the imposed set point and the upper limit is set to 25°C, since no cooling system is considered.

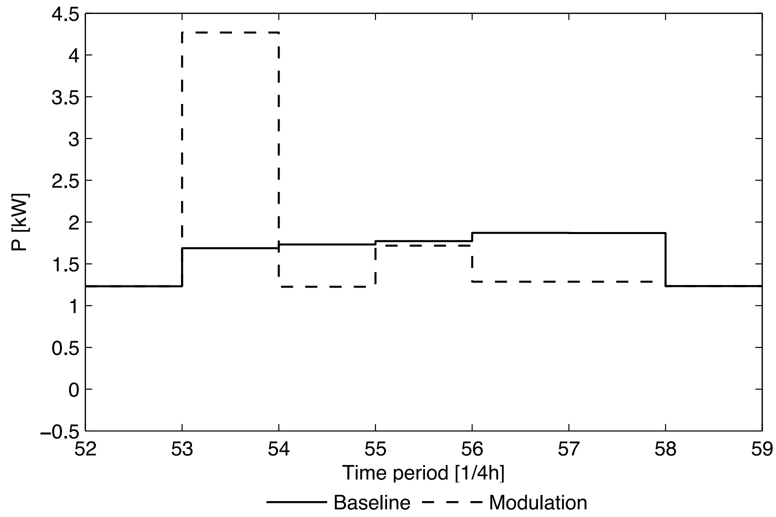
Buildings are equipped with conventional hydronic radiators. The temperature of the water supplied to the radiator is adjusted according to the insulation level of each building. The radiators are assumed to be sized so that they are able to supply the thermal power required by the building at any time and the dependency of the emitted heat on the water supply temperature is not modeled. The domestic hot water tank lower limit in (6.8) is imposed by sanitary constraints to 50°C, whereas the upper limit of 65°C is imposed by the heat pumps design. The tank volume is adapted for each house based on a water consumption of 50 liters per person per day and with an additional safety volume of 50 liters. It is therefore comprised between 50 and 300 liters. The supply temperature is set to 65°C which underestimates the performance of the heat pump. The parameters of the heat pump model detailed in (6.9) and (6.10) are calibrated based on manufacturer data.

6.7.2 Illustration on a single house

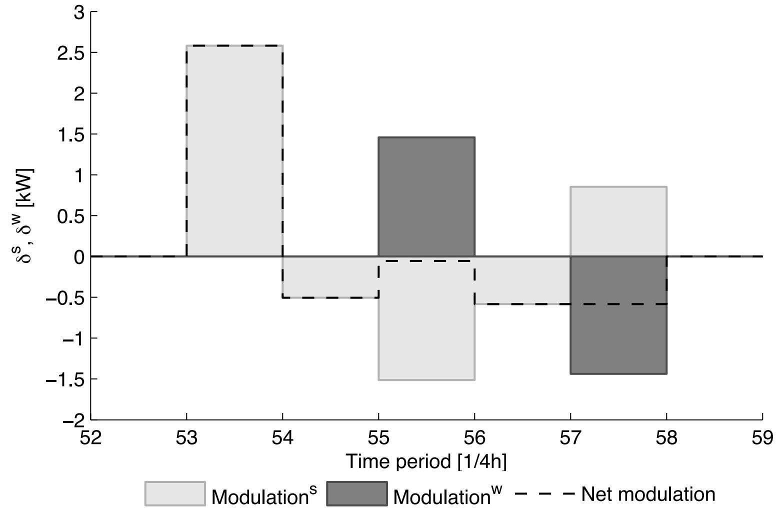
Figure 6.3 shows results for an upward modulation activated at time period 53 for a payback of four periods. A total electrical power consumption increase of 2.5kW can be observed in Figure 6.3a, which corresponds in this example to the upwards activation of the heat pump power for space heating as illustrated in Figure 6.3b. The 2.5kW consumption increase at period 53 is directly followed by a decrease in electricity use for space heating during four periods. In order to minimize the amplitude of the power payback, and since the heat pump cannot work in both space heating and domestic hot water modes simultaneously, the diminution of power demand for space heating is counterbalanced by a shift of consumption for domestic hot water production from period 57 to 55.

6.7.3 Results on the aggregated portfolio

The maximum upwards and downwards modulations for the aggregated portfolio of houses are illustrated in Figure 6.4 for a typical winter weekday and three payback horizon lengths.
The largest upward and downward modulations are obtained in periods 0 to 28, with maximum amplitudes reaching 400kW and 210kW respectively. During that time frame, most of the flexibility is provided by space heating consumption. Most of the profiles present a night set-back where the set point temperature is reduced and the allowed temperature range around the set point is wider. Since during that period outdoor temperature variations and internal heat gains are limited, the upwards and downwards modulation amplitude are fairly constant. For the upward modulation, there is a maximum in periods 16 to 28. This phenomenon is due to the higher room temperature set point for the day time, which allows a faster return to the baseline electricity demand of the house. In the case of downward modulation, the limitation of the heat pump capacity reduces the achievable downward modulation as one gets closer to the set point transition. The upward peaks observed in period 30 and 88



(a) Total power modulation.



(b) Space heating and domestic hot water power modulation.

Figure 6.3: Power modulation of a freestanding house built after 2007 for an activation in period 48 and with a payback on four periods.

correspond to the start-up of heat pumps to produce domestic hot water after usual morning and evening water draw-off events. During the day, most of the upward and downward modulations are provided by space heating. The flexibility from domestic hot water tank is mostly restricted by the high inertia of the water tanks caused by its insulation. In addition to this inertia, major hot water draw-off events mainly happen in the morning and evening which limits the consumption needs. The heat pump is more often used for space heating as the dead-band of the room temperature is set to only 2°C during the day. The heat pump being limited to work in one mode at a time, consumption for domestic hot water is mostly concentrated in single periods to give more freedom for space heating. The potential of downward modulation

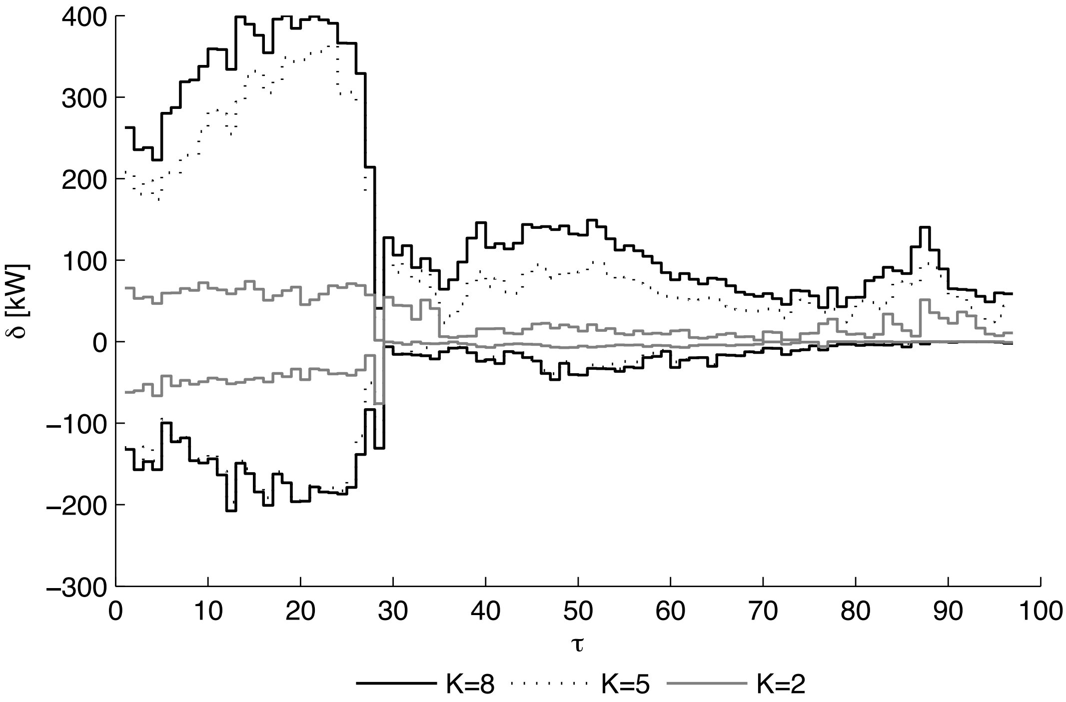


Figure 6.4: Influence of the payback length on the modulation amplitude for each potential activation period on January 24th for 100 houses.

gets close to zero for time periods between 88 and 95. This is due to fact that the first optimization of the baseline drives the system towards minimizing the consumption and therefore the temperatures hit their lower bound.

Figure 6.5 illustrates the seasonal modification of the flexibility potential on the 24th of January, April, June and November. The difference in modulation profiles observed between November and January lies in the higher outdoor temperature, which, combined to the night time set-back reduces the flexibility potential for space heating for time periods below 20. The relative share of electricity consumption devoted to domestic hot water production increases in warmer seasons.

Tables 6.5a and 6.5b provide a detailed quantification of the cost, the overconsumption and the deviation following the modulation, respectively for the upward and downward activations on January 24th. The overconsumption is the net difference between the baseline consumption and the consumption with the modulation. The deviation is the sum of the absolute differences, during the pay-back periods, between the baseline consumption and the consumption after modulation. Allowing a payback time of one hour and 15 minutes leads to an average upward modulation of 1.2kW per house with a deviation of 600Wh, whereas the average downward modulation reaches 500W per house with a deviation of 420Wh. Several differences can be observed between upward and downward activations. First, the average downward modulation amplitude is

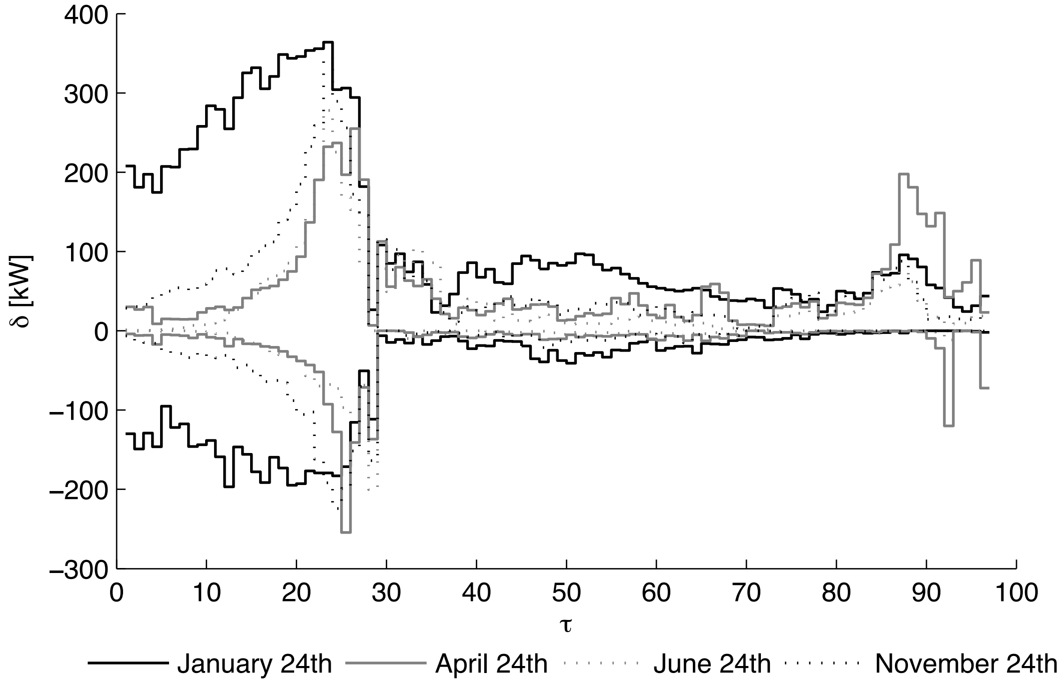


Figure 6.5: Seasonal influence on the modulation amplitude for each potential activation period on January 24th for 100 houses.

smaller than the corresponding upward modulation. The reason is to be found in the choice of a cost-optimal baseline which drives the zone and water tank temperature trajectories closer to the lowest set points, in particular during peak price hours. Second, the deviation from the baseline consumption and the overconsumption are proportionally larger for downward activations. In the case of an upward activation, the resulting higher temperature level entails an increase in heat losses to the ambiance, hence the overconsumption. For a downward activation, and especially if the payback time crosses a transition from a lower temperature set point to a higher set point, the heat pump has to work closer to its maximum capacity and sometimes resort to the back up electrical resistance, which reduces the heating performance and increases the payback consumption. A downward modulation of the electricity demand is therefore more expensive.

Table 6.6 presents the influence of the season on the mean values for a payback on five periods. Results show that it is possible to harvest on average an upward modulation of 400W to 1.2kW per house with a payback of 150Wh to 600Wh, or an downward modulation of 100W to 500W per house with a payback of 60Wh to 420Wh. The overconsumption varies between 60Wh and 150Wh per house for an upward activation and between 40Wh and 100Wh for a downward activation.

Table 6.5: Influence of the payback length on modulations.
(a) Upward activations

	Modulation [**kW**]			**Deviation** [**kWh**]	**Cost** [**€**]	**Overconsumption** [**kWh**]
k	*min*	*mean*	*max*	*mean*	*mean*	*mean*
1	0.0	4.9	48.0	1.1	0.2	0.1
2	1.7	29.2	74.0	11.5	1.3	2.8
3	7.0	70.1	206.1	35.0	3.8	8.7
4	14.8	97.6	295.8	49.1	5.4	12.1
5	23.1	121.3	364.0	59.8	6.7	14.7
6	31.6	136.4	382.2	67.1	7.5	16.2
7	37.5	149.4	396.4	74.2	8.3	17.7
8	40.9	160.1	399.8	80.8	9.0	19.1

(b) Downward activations

	Modulation [**kW**]			**Deviation** [**kWh**]	**Cost** [**€**]	**Overconsumption** [**kWh**]
k	*min*	*mean*	*max*	*mean*	*mean*	*mean*
1	0.0	0.9	61.6	0.2	0.0	0.0
2	0.0	15.4	75.9	11.0	1.0	2.7
3	0.0	38.4	138.7	35.8	3.3	8.4
4	0.0	48.3	178.1	39.5	3.8	9.5
5	0.0	53.5	196.9	42.4	4.1	10.3
6	0.0	55.1	197.9	43.0	4.3	10.5
7	0.0	56.3	203.7	44.1	4.4	10.7
8	0.0	56.9	207.5	44.0	4.4	10.7

Table 6.6: Seasonal variation of modulations amplitude.

	Modulation [**kW**]		**Deviation** [**kWh**]		**Overconsumption** [**kWh**]	
	upward	*downward*	*upward*	*downward*	*upward*	*downward*
Jan.	121.3	53.5	59.8	42.4	14.7	10.3
Apr.	54.9	17.7	25.7	9.7	4.2	1.8
Jun.	37.9	10.9	15.4	6.2	2.4	1.1
Nov.	58.7	24.7	29.4	16.0	5.9	3.6

6.8 Conclusion

This chapter presents a flexibility service provided by a load aggregator controlling domestic heat pumps. The heat pumps are used to supply both domestic hot water production and space heating needs. The flexibility service consists in the upward or downward activation of the heat pumps at a certain time-period with a pay-back effect over a fixed number of periods. A sequential optimization scheme is proposed to determine the maximum modulation amplitude from an optimized baseline for different pay-back durations. The methodology is applied to a case study composed of a hundred freestanding houses representative of the Belgian residential building stock and built after 1971. Simulation results indicates that an average modulation amplitude of 400W to 1.2kW per house, depending on the seasonality, can be obtained in the case of an upward activation. In the case of a downward activation, the average value per house lies between 100W and 500W. About 80% of the flexibility potential comes from the modulation of the heat pump power in space heating mode in the winter, whereas the potentials relative to space heating and domestic hot water production tend to even out in the mid-season. The overconsumption varies between 60Wh and 150Wh per house for an upward activation and between 40Wh and 100Wh per house for a downward activation.

As for future work, other modulation services may be proposed to tackle scenarios prompted by different grid management constraints. For instance, one could consider extending the modulation on more than one period. This would make the computations more technical as the problem could no longer be decomposed per heat pump. The consequences of optimizing simultaneously the baselines and the flexibility services should be investigated. Finally, the level of details of buildings and system models could be increased to include non-linear behaviors.

Chapter 7

Conclusion

This last chapter provides a brief summary of the work presented in this thesis and gives a general conclusion. More specific conclusions can be found at the end of each of the previous chapters.

7.1 Summary

The work presented in this thesis considers the electrical flexibility from the electric load to its usage as a commodity. The conception of the European electrical system has led to a large amount of actors that are impacted by flexibility exchanges. This thesis assesses the impact of exchanging flexibility in the electrical system and analyzes the complex interactions that may result from these exchanges. The impacts on different parts of the electrical system are presented: the day-ahead energy market, the secondary reserve and the distribution system. The day-ahead energy market is studied on a system where flexibility of the consumption managed by electricity retailers changes the market prices of each hour of the day. This system is mapped to a game theory problem which allows us to obtain analytical results. One of these results is a simple method to compute the price of flexibility to which flexibility of the electrical consumption should be remunerated in electrical power systems. To recover the cost of turning loads into smart appliances, an aggregator may need more than the benefits it could obtain from the day-ahead energy market. One business opportunity for the aggregators is to participate to the secondary reserve market. Evaluating the benefits of introducing load flexibility in the secondary reserve market is the topic of one of the chapters. Using flexibility of the consumption is shown to decrease the cost of the secondary reserve even though the volume of reserve should be increased. As more interactions are involved, the system to model is too complex to obtain an analytical solution of the corresponding market equilibrium and an agent-based model is devised to harness this complexity. One step down into the electrical system, we study how distribution system operators may use the flexibility of the grid users connected to the medium voltage distribution network to perform active network management. First, we propose six interaction models, defining how the flexibility is exchanged. Second, two methods are used to perform the quantitative analysis of the interaction models organizing the exchange of flexibility within the distribution network: a macroscopic analysis and an agent-based model. The results show that the choice of the interaction model has a major impact on the electrical system and on the costs of each actor within the system. In our results, one interaction model safely increases by 55% the amount of distributed generation in the network with respect to a restrictive model representing the currently applied interaction model.

The first part of the thesis is based on devices able to provide flexibility from their consumption. However, very few details are given on how actual consumption processes can be turned into loads able to provide flexibility services. There are mainly two methods to obtain flexibility: direct control of the loads and dynamic pricing. One chapter provides an example of how flexibility can be obtained by the direct control of a portfolio of heat pumps. Results on a case study composed of a hundred freestanding houses representative of the Belgian residential building stock and built after 1971 show that their control can lead to a substantial amount of flexibility. On the other hand, another chapter studies the control of electric heaters and boilers via the use of a simple price signal. The case considered is the modification by the DSO of the tariff of the distribution network to the grid user. One major advantage is that, in most European countries, a system of two electric meters associated with off-peak and on-peak tariffs already exists. Therefore, the latter solution requires no investment in the infrastructure. The main drawback of this method is that there is no guarantee on the actual volume of flexibility that results from a price change. This motivates the choice made in this thesis to favor direct control flexibility services where the volumes are well defined, which ease the trading of flexibility as a commodity.

7.2 Discussion

The thesis provides a complete picture of how the flexibility may be integrated in the European electrical system. The top-down approach adopted in this document shows that the flexibility may be used for different purposes. One of the main contribution is to consider flexibility in an unbundled market-based system. Most works of the scientific literature on flexibility focus on fully integrated methods to obtain flexibility. For instance, there are some works considering that the DSO, the only actor with the knowledge of the state of the distribution network, may directly control heat pumps to solve congestion in its network. In the European unbundled electrical system, the legal framework is under discussion and goes in a direction where the DSO may not control directly the flexible assets but contracts the flexibility throughout dedicated services. This thesis strengthens the state of the art of the scientific literature by formalizing the interactions and information exchanges to use flexibility in the European electrical system.

The work carried out in this thesis could be continued along two major lines: going through further academic researches and jumping from theory to practice. From an academic perspective, the systems modeled in this thesis could be improved with more details considering for instance better network models, more accurate models of production units or uncertainty in the decision of the actors. However, increasing the complexity of the models are very likely to lead to intractable problems from a computational point of view. Core component of this thesis are optimization, in particular mixed-integer linear programming, agent-based modeling and game theory. Other techniques could be investigated such as machine learning, non-linear optimization, Monte Carlo sampling, meta-heuristics, etc. The results obtained in this thesis could be more related to investments. Chapter 2 presents a method to obtain an activation cost of the load flexibility. These costs are obtained from the point of view of the system and should be compared with numbers obtained from the point of view of an actual aggregator. Obtaining the investment cost per megawatt per hour of flexibility is not straightforward and dependent on the type of load which provides the flexibility. Investments in flexibility should also be compared with investments in the network. The thesis mainly focuses on direct control services even though flexibility services based on price signals are investigated. One could be interested in comparing quantitatively the two approaches using the agent-based technique described in this manuscript. The open-source framework DSIMA could be extended to perform such comparison [37]. The agent-based technique could also be used to evaluate the benefits of creating a capacity market which seems to be more and more a necessity to keep gas production units running for days without sun nor wind. Even if the decisions taken by the European Commission are defined for the whole Europe, each country has its specific implementation of the directives. A detailed comparison of the special features of each implementation would worth to be investigated. Finally, a quantitative comparison of the European electrical system and the one of the United States may be of interest. In the United States, the system operator procures its reserve during the clearing of the day-ahead market [47]. The comparison of the European and the United States systems using an agent based modeling approach would be worth investigating. Already one study highlights the imperfections of the United States day-ahead energy market and of the European one [141].

From a practical point of view, there is an ongoing trend for load flexibility. There are currently many projects which are in their final testing phases or even in operation. More and more companies are equipping their factories with sensors to monitor their processes and are therefore only one step away from modulating them upon request. Consumers are more and more equipped with electric cars, home automation, smart washing machines, etc. The scientific literature contains plenty of algorithms able to coordinate these flexible loads in coherent flexibility services. Some retailers are already using flexibility to adjust their portfolio as balancing responsible parties. To jump from theory to practice, my opinion is that one important remaining step to perform is to complete the regulation with definitions of commercial products derived from flexibility that consider the constraints inherent to the process of underlying loads. Fortunately, many regulators of various countries of the European electrical system are currently writing their legislation to allow the usage of flexibility within their electrical system.

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