Dimension Reduction and Measure Transformation in Stochastic Multiphysics Modeling

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In numerous critical areas from across science and engineering, models and simulations share a common base of mathematical formulations and algorithms that are multi-physics, multi-scale and/or multi-domain in nature. The crucial demand for predictive computational results in these areas motivates the development of uncertainty quantification approaches for coupled models that feature multiple physics, scales and/or domains.

We investigate in this work a computational framework for the efficient propagation of uncertainties through coupled models based on a general stochastic expansion methodology. We address the following two challenges that arise in extending stochastic expansion methods to coupled problems. The first challenge is in that the formulation and implementation of stochastic coupled models requires care in adequately facilitating the communication of information across physics interfaces or scale boundaries. This information can include that of physical properties, energetic quantities, or solution patches, among other quantities. In a probabilistic context, any exchanged information is statistical in nature, and thus requires an adequate probabilistic representation. The second challenge is in the curse of dimensionality, in that the computational cost of stochastic expansion methods grows quickly with the stochastic dimension of the problem, which can rapidly become large when uncertainties affect multiple model components.

While many sources of uncertainty may affect a coupled model, our contention is that statistical information passed between model components often exhibits a much lower effective stochastic dimension. In this work, we thus propose an adaptation of the Karhunen-Loeve decomposition that can be used in solution algorithms to extract a low-dimensional representation of information as it is passed from component to component in a stochastic coupled model, thus mitigating the growth in stochastic dimension as uncertainties are exchanged. After a transformation of measure, the reduced-dimensional interface thus created between the model components enables a more efficient solution of the model components in a reduced-dimensional space.

The presentation will demonstrate the effectiveness of our reduced-dimensional interfacing approach on a multiphysics problem relevant to nuclear reactors. This problem consists of a coupled system of two second-order elliptic equations, which describes the stationary transport of neutrons in a reactor with a temperature feedback. We accommodate uncertainties in the parameters of both the neutron-transport and heat-transfer equations, and solve for the solution of the fully coupled problem by an iteration procedure that only involves the successive solution of the model components in their reduced dimensional space.