Synchronization and clustering in models of pulse-coupled firing oscillators: discrete and continuous approaches

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In this seminar, networks of pulse-coupled firing oscillators are considered, which are simple models to describe numerous biological systems (neural networks, pacemaker cells of the heart,...). Each oscillator is characterized by a single state which evolves between two threshold values. As the state reaches the upper threshold, it is reset to the lower threshold (the oscillator is said to fire). The coupling between the oscillators is impulsive: whenever an oscillator fires, it emits a pulse which instantaneously increments by a constant value the state of every other oscillator.

The first part of the seminar focuses on a discrete application, the so-called firing map, which completely characterizes the behavior of the network. In most of the cases, the stability analysis of the firing map reveals an interesting dichotomy of the behavior: either the oscillators asymptotically synchronize or they asymptotically converge to a phase-locked clustering configuration. This dichotomy is investigated in the case of different state dynamics.

In the second part, a continuous model is obtained by a coarse-grained approximation of the network, which leads to the study of a nonlinear partial differential equation (PDE). Through a generalization of the stability results of the previous discrete model, the stability analysis of the PDE establishes a strong parallel between both discrete and continuous models. In particular, the dichotomy is observed under the same conditions for both discrete and continuous models.