# Regression on fixed-rank positive semidefinite matrices: a geometric approach

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In this talk, we adopt a geometric viewpoint to tackle the problem of learning a regression model whose parameter is a fixed-rank positive semidefinite (PSD) matrix.

An important instance of that problem is the learning of a distance function parameterized by a fixed-rank PSD matrix. This task is a central issue for many machine learning applications where a data-specific distance has to be constructed, or where an existing distance needs to be improved based on additional side information.

Learning low-rank matrices is a typical solution to reduce the computational cost of subsequent algorithms. Indeed, the complexity generally decreases from $O(d^3)$ to $O(d\,r^2)$ where the approximation rank $r$ is generally much smaller than the problem size $d$.

Whereas efficient convex formulations exist in the full-rank case, the problem is no longer convex as soon as the rank constraint is introduced. Nevertheless, the set of rank-$r$ PSD matrices has a rich Riemannian geometry that can be exploited for algorithmic purposes.

We discuss the choice of two particular geometries of fixed-rank PSD matrices and we derive the corresponding gradient descent algorithms. In contrast to previous contributions in the literature, the range space of the matrix is free to evolve during the optimization and the resulting algorithms enjoy important invariance properties.

We apply the two proposed algorithms to the distance learning problem. The good performance of the algorithms is illustrated on several well-known classification and clustering benchmarks.