A computational survey of best-case gap closure for various relaxations

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Introduction

The process of generating valid inequalities for a mixed integer problem

$$P = \min\{c^T x : Ax = b,\ \ x \geq 0,\ \ x_j \in \mathbb{Z}\ \ \forall j \in J\}$$

generally involves first considering a relaxation of the problem by dropping some of the constraints (integrality, non-negativity, or general linear constraints). Then, cuts can be computed for the relaxed problem. Therefore, by examining how tight the relaxed problem is, we get what is attainable in the best case with cuts based on that relaxation.

In practice, we measured the gap closed by five relaxations in the following way:

$$\%\text{gc} = 100\,\frac{z^*_{\text{relaxation}} - z^*_{\text{LP}}}{z^*_{\text{P}} - z^*_{\text{LP}}}$$

on most problems of the in the miplib3 [5] and miplib2003 [1].

The extent of this information has three important limitations:

- we do not know the gap actually closed by the cuts: cutting-plane algorithms may or may not converge to the underlying relaxation.
- all the relaxations we consider are based on the status of the variables at the LP optimum. Therefore, our measurements only regard independent inequalities added at the root node of the MIP.
- due to the use of the objective function as an indicator, when the relaxation deals only with part of the rows of the problem, we still need to consider the others. This in order to keep the basis matrix full-rank in the original problem, and have defined values for all our variables. An alternative would be to compute all the facets of the relaxation that are binding at its optimal value, and add them to the MIP. But that would not be conceivable for large-scale problems.

However, this provides us with upper-bounds on the gap closure obtained using cuts derived from each relaxation. We thus have a partial indication of the usefulness of each type of cut for each problem we study.

In this approach, we mainly extend the work of Fischetti and Monaci [11] on the *group* and *corner* relaxations, and part of our experiments overlaps with theirs (specifically, on the *group* relaxation), with similar outcomes.

Relaxations

The *group* relaxation [15][13][14][16], consists in dropping non-negativity constraints on all basic variables, i.e. given B and N the index set of respectively basic and nonbasic variables in the optimal solution of P_{LP},

$$P_{\text{group}} = \min\{c^T x : Ax = b,\ \ x_j \geq 0\ \ \forall j \in N,\ \ x_j \in \mathbb{Z}\ \ \forall j \in J\}$$

The mixed integer set P_I, presented in [6] and [8], is related to the one suggested in [3] for deriving inequalities from two rows of the simplex tableau

$$P_I = \min\{c^T x : Ax = b,\ \ x_j \geq 0\ \ \forall j \in N \cup \overline{J},\ \ x_j \in \mathbb{Z}\ \ \forall j \in J \cap B\}$$

The set "P_I+lifting" adds back to P_I the integrality constraints on the non-basic variables [9][7].

The relaxation "x-bounded P_I" adds lower and upper bounds (when available in the original problem) on the basic variables of P_I [10][4][12], while "s-bounded P_I" considers bounds on the non-basic variables [2].

Results

In summary, we have:

	basic		non basic		additional
	integer	continuous	integer	continuous	constraints
LP	continuous		continuous		
group	unbounded	unbounded			
P_I	unbounded		continuous		
P_I + lifting	unbounded				
x-bounded P_I			continuous		$0 \leq x_B \leq u$
s-bounded P_I	unbounded		continuous		$0 \leq x_N \leq u$

miplib 3

problem	group %gc	P_I %gc	P_I + lifting %gc	x-b. P_I %gc	s-b. P_I %gc
10teams	42.86	0	14.29	0	0
air03	90.28	100	100	100	100
air04	1.09	10.9	10.9	100	10.9
air05	3.7	19.82	19.82	100	19.82
arki001	99.45	24.78	100	38.72	46.49
bell3a	83.79	100	100	100	100
bell5	4.4	96.11	100	96.26	96.11
blend2	37.98	80.9	80.9	80.9	80.9
cap6000	76.92	56.01	100	56.01	56.01
dano3mip					
danoint	1.74	99.89		99.89	99.89
dcmulti		100	100	100	99.99
dsbmip					
egout	100	100	100	100	100
enigma					
fast0507	46.08		46.08		
fiber	72.25	7.26	93.81	11.43	7.35
fixnet6	78.87	100	100	100	100
flugpl	97.38	100	100	100	100
gen		100	100	100	100
gesa2	32.3	69.21	100	69.22	69.22
gesa2_o	32.33	69.36	100	69.73	69.36
gesa3	49.31	98.76	100	99.12	99.12
gesa3_o	49.2	97.17	100	98.76	97.17
gt2	46.79	57.12	57.12	100	57.12
harp2		32.28	77.18	37.3	32.28
khb05250	62.07	100	100	100	100
l152lav	0.98	16.1	22.3	70.29	18.6
lseu		10.42	83.88	19.03	10.42
markshare1		0		0	0
markshare2		0		0	0
mas74		17.31		20.69	17.31
mas76		14.14	72.64	14.4	14.14
misc03	7.24	19.43	44.83	43.71	23.16
misc06	2.19	99.84	100	100	100
misc07	0.72	0.72	27.6	6.93	0.72
mitre		78.29		80.04	78.29
mkc		21.52		81.83	35.68
mod008	50.22	4.65	93.78	5.18	4.65
mod010	49.75	37.19	37.19	100	37.19
mod011	3.5	100	100	100	100
modglob	31.28	48.56	100	48.57	48.57
noswot	0	0	100	0	0
nw04	51.03	3.99	41.6	4.9	4.11
p0033		6.61		24.2	6.61
p0201		56.76		56.76	56.76
p0282	8.9	46.02	64.85	90.85	46.02
p0548	0.02	85.74	93.54	88	85.74
p2756	0.29	2.34	40.95	2.34	2.34
pk1	0	0	81.82	7.27	0
pp08a		89.38	100	89.38	89.38
pp08aCUTS		100	100	100	100
qiu	0	100	100	100	100
qnet1	6.4	72.17	95.63	79.62	72.17
qnet1_o		54.48		77.1	59.22
rentacar	0	100	100	100	100
rgn	0	0	100	0	0
rout		17.27		34.23	18.76
set1ch		100		100	100
seymour	6.02				
stein27	0	100	100	100	100
stein45	0	100	100	100	100
swath	33.01	3.5		3.69	3.5
vpm1		100		100	100
vpm2		100		100	100

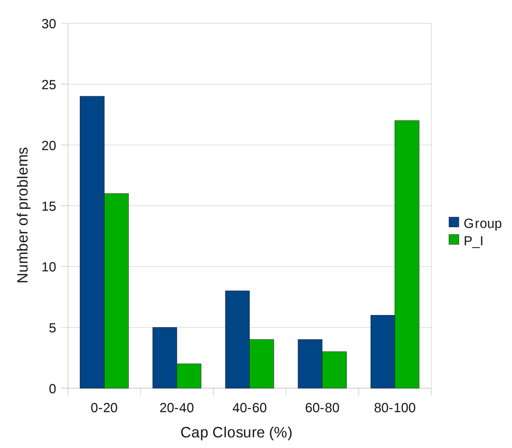
miplib 2003

problem	group %gc	P_I %gc	P_I + lifting %gc	x-b. P_I %gc	s-b. P_I %gc
10teams	42.86	0	14.29	0	0
a1c1s1	24.42				
aflow30a	14.2	92.4	100	92.4	92.4
aflow40b	14.99	95.54	100		
air04	1.09	10.9	10.9	100	10.9
air05	3.7	19.82	19.82	100	19.82
arki001	99.45	24.78	100	38.72	46.49
atlanta-ip					
cap6000	76.92	56.01	100	56.01	56.01
dano3mip					
danoint	1.74	99.89		99.89	99.89
disctom					
ds				8.73	
fast0507	46.08		46.08		
fiber	72.25	7.26	93.81	11.43	7.35
fixnet6	78.87	100	100	100	100
gesa2	32.29	69.2	100	69.21	69.21
gesa2-o	32.32	69.35	99.98	69.72	69.35
glass4	0	0		0	0
harp2		32.28	77.18	37.3	31.51
liu					
manna81	100	100	100	100	100
markshare1		0		0	0
markshare2		0		0	0
mas74		17.31		20.69	17.31
mas76		14.14	72.64	14.4	14.14
misc07	0.72	0.72	27.6	6.93	0.72
mkc		21.52		81.83	35.68
mod011	3.5	100	100	100	100
modglob	31.28	48.56	100	48.57	48.57
momentum1					
momentum2					
momentum3					
msc98-ip					
mzzv11	3.85			100	
mzzv42z	3.05			100	
net12					
noswot	0	0	100	0	0
nsrand-ipx		0		0	0
nw04	51.03	3.99	41.6	4.9	4.11
opt1217	50.27	0.53	50.27	0.53	0.53
p2756	0.29	2.34	40.95	2.34	2.34
pk1	0	0	81.82	7.27	0
pp08a		89.38	100	89.38	89.38
pp08aCUTS		100	100	100	100
protfold	8.74				
qiu	0	100	100	100	100
rd-rplusc-21	0	0		0	0
roll3000					
rout		17.27	21.66	34.23	18.76
set1ch		100	100	100	100
seymour	6.02				
sp97ar					
stp3d	0.45				
swath	33.01	3.5		3.69	3.5
t1717					
timtab1		90.19		90.19	90.19
timtab2					
tr12-30		100	100	100	100
vpm2		100	100	100	100

On the 47 problems for which we could solve to optimality both the *group* and P_I relaxations, we have the following statistics:

	group	P_I
Average (%gc)	31.69	56.58
Standard deviation (%gc)	34.29	42.35

And they are distributed (in function of the gap closure) as illustrated on the right.



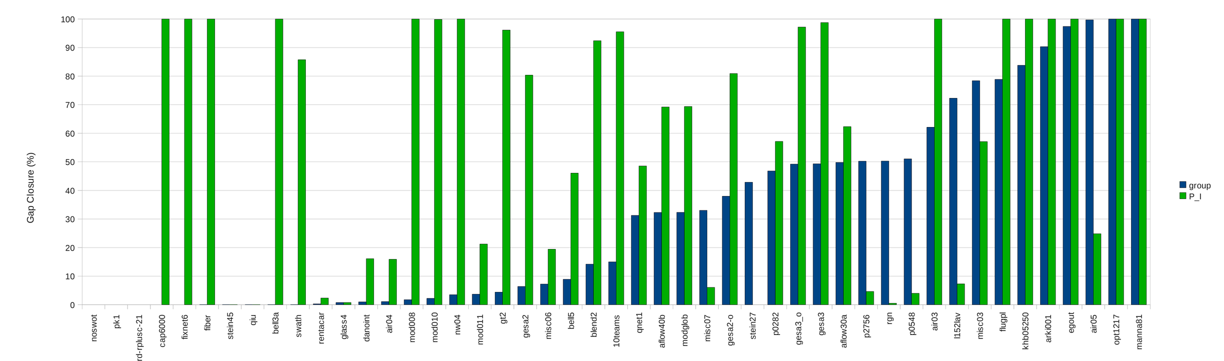
Conclusions

As explained before, the average gap closures presented here must be taken with caution: they are not necessarily directly linked to efficiency in the corresponding cuts. However, an interesting fact might be the apparent non-correlation, given one problem, in the gap closed by the *group* and P_I relaxations (see graph below). This can lead us to think that the per-problem usefulness of a given type of cut might be heavily determined by the tightness of the underlying relaxation.

Further developments

Some refinements can be made in the handling of models.

- The additional bounds on variables could be evaluated by optimizing over the LP when they are not present in the initial formulation.
- The slack variables could be made integer in pure-integer constraints. However, this leads to dramatically higher computing times.
- More complete tables will be available with extended memory and CPU time limits (currently 3Gb and 2h).



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