INFO0013 Computer Vision

Extending Stereo Geometry to Multiple Views

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The Final Phase

We know how to derive Euclidean cameras automatically from a stereo pair of images (given the internal calibrations).
Now we would like to extend this to $N$ views.
Three Views

![Diagram of three views]

[Figure from Forsyth/Ponce lecture notes]

N-View Geometry

A third view adds an important new constraint: Given a two-way point correspondence and the three fundamental matrices, the position of the corresponding point in the third view is fixed (at the intersection of the two epipolar lines).

These three-way constraints are conveniently expressed by the trifocal tensor. (We will not talk about it here.)

More views do not add any more independent constraints.
Point Correspondences

Objective: Establish point correspondences over as many images as possible.

Note
Points need not be visible in all images.
Compute an Initial Stereo Reconstruction
How to choose a suitable pair of *key frames*?

Extend the Reconstruction by More Key Frames
How to place all cameras into the same world frame?
Bundle Adjustment

**Given:** In views $i = 1, \ldots, m$, images $x_j^i$ of world points $X_j$, $j = 1, \ldots, n$, and initial estimates of the $P^i$ and the $X_j$.

**Sought:** Our best global estimates $\hat{P}^i$ and $\hat{X}_j$ such that $x_j^i = P^i \hat{X}_j$ and the estimated $\hat{x}_j^i$ are as close as possible to the measured $x_j^i$, that is,

$$\arg\min_{\hat{P}^i, \hat{X}_j} \sum_{i=1}^{m} \sum_{j=1}^{n} d(\hat{P}^i \hat{X}_j, x_j^i)$$

where $d$ is the geometric image distance. Solve this by nonlinear optimization.

**Note**
Points need not be visible everywhere.
**Parametrization**

Conventional parametrization of $\hat{P}^j$ yields a *projective* reconstruction and independent cameras. However, we know more:

- Camera does not change (constant aspect ratio and skew, perhaps constant focal length and principal point offset)
- Camera is calibrated (we (approximately) know these parameters), allowing a *Euclidean* reconstruction

Thus, parametrize the $\hat{P}^j$ as $\hat{P}^j = K'[R^i|t^i]$, where the $K^i$ share some or all parameters, some or all of which are fixed.

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**Soft Constraints**

Specify additional, approximate constraints by adding penalty terms to the *objective function*:

- Internal camera parameters not fixed by parametrization
**Bundle Adjustment in Practice**

Our rather large optimization problem over $11m + 3n$ parameters (or $6m + 3n$ in the Euclidean case) may choke Levenberg-Marquardt.

**Interleaved optimization:** Iterate over a sequence of $m + n$ smaller optimization problems:

- For each $i$, compute $\arg\min_{\hat{p}^i} \sum_{j=1}^{n} d^2(p^j, \hat{x}^j)$.
- For each $j$, compute $\arg\min_{\hat{x}^j} \sum_{i=1}^{m} d^2(p^i, \hat{x}^j)$.

**Sparse methods:** See Hartley and Zisserman 2003 Appendix 6.3, and existing code.

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**Completion**
Intermediate Cameras

How to fill in the cameras between the key frames?

Radial Distortion

During all nonlinear estimates (bundle adjustments in particular), pass the $x^i_j$ through a function that corrects for radial distortion whose parameters are added to the list of unknowns.

*Note*

If the focal length is fixed, the radial distortion should be fixed as well.
Merging Temporally Distant Views

**Situation:** The camera returns to past poses, but no points are tracked for the entire time between visits. Thus, we are seeing the same world points, but we do not know this! (And reconstruction errors accumulate.)

1. Detect such situations by comparing $R$ and $t$ to other cameras.
2. For each such pair of views:
   a. Find correspondences by computing $F$ (or even $H$ if $|t|$ is very small) using RANSAC.
   b. Take the corresponding pairs of world points to be identical.

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**Summary**
Our Algorithm

Implementation Plan

Start simple, and then add more elements, one at a time, for example:

1. Stereo reconstruction from a pair of manually-chosen key frames
2. Insert your graphical object
3. More key frames; choose all of them automatically
4. Bundle adjustment
5. Intermediate cameras
6. Radial distortion
7. Distant views

All of these are not required for a good grade.
We now know how to insert an artificial character into a movie! (How?)

References