Cameras

The world is flat.

How is the 3D world mapped onto the 2D image?
The Human Sensor

- Most refraction takes place at the outer surface of the cornea.
- Space-variant resolution.

**Rods**
- wide spectral band
- highly sensitive
- pooled by retinal neurons
- absent in the fovea

**Cones**
- 3 narrow-band types (roughly, RGB)
- less sensitive
- less pooled by retinal neurons
- rare in the periphery
Simplified Model: Pinhole Camera

- The pinhole (at the camera center) is assumed to be of infinitesimal size.
- The optical axis passes through the image plane at the image center.
- The image plane can be assumed behind or in front of the pinhole.
- Perspective Effects:
  - Farther objects appear smaller.
  - Parallel lines intersect in vanishing points.
- Most of computer vision uses this model.
  - Deviations are addressed by correcting the image.

Perspective Projection Equations

By similarity of triangles:

\[ x' = \lambda x \]
\[ y' = \lambda y \]
\[ f' = \lambda z \]

\[ \lambda = \frac{x'}{x} = \frac{y'}{y} = \frac{f'}{z} \]

\[ x' = \frac{f' x}{z} \]
\[ y' = \frac{f' y}{z} \]
Further Simplification: Affine Projection

- **Weak Perspective**: scene depth is small with respect to the distance, and is assumed constant:
  
  \[
  x' = -mx \\
  y' = -my \quad \text{where} \quad m = -\frac{f'}{Z_0}
  \]

- **Orthographic Projection**: \( m = -1 \)

Projection Matrices

- **Affine projection**
- **Perspective** projection?
  
  *Homogeneous coordinates* make our life easier!
Thin Lenses

Lenses:
- gather light from a large area,
- focus it onto a point.

Thin Lenses:
- refract light at their medial axis.
- Rays from infinity focus at the *focal point* located at the *focal length* from the lens.
- The *depth of field* increases with the *f number* \((f/d_{\text{lens}})\).
- The *field of view* depends on the focal length and on the size of the retina.

The Thin Lense Equation

If \(z' = f'\), the thin-lens model corresponds to the pinhole model.

However, for a point to be in focus under the thin-lens model, the following relation must hold (by the 2nd Intercept Theorem):

\[
\frac{f}{z'} = \frac{-z-f}{-z} = \frac{1}{z} - \frac{1}{f} = \frac{1}{f}
\]
Real Lenses

Approximations of the Thin Lens model:
- The incident angles of light rays are small.
- Light is refracted at a single plane.
- Refraction is independent of the wavelength.

Consequences (worse for shorter focal lengths):
- radial distortion
- chromatic aberration
- (longitudinal and transverse) spherical aberration
- vignetting
- ...

Radial distortion

Reason: The magnification varies over the field of view.
Fix: Unwarping:
\[
\begin{align*}
\hat{x} &= x_c + L(r)(x - x_c) \\
r^2 &= \|x - x_c\| \\
\text{Taylor: } L(r) &= 1 + k_1 r + k_2 r^2 + \ldots \\
&\text{(assuming square pixels)}
\end{align*}
\]
**CCD Sensor: Electrons Recorded**

Number of electrons recorded at a site:

\[
l(r, c) = \int_T \int_{\lambda} \int_{p \in S(r, c)} E(p, \lambda) \, R(p) \, q(\lambda) \, dp \, d\lambda \, dt
\]

- \(S\) spatial domain of the cell
- \(E\) *irradiance* (power per unit area and unit wavelength)
- \(R\) spatial response of the site
- \(q\) *quantum efficiency* (number of electrons generated per unit of incident light energy)

**CCD Sensor: Signal Generated**

Amplitude of digital signal:

\[
D(r, c) = \gamma (N_i(r, c) + N_{DC}(r, c) + N_B(r, c) + R(r, c)) + Q(r, c)
\]

- \(\gamma\) amplifier gain
- \(N_i\) number of electrons generated by photoconversion (uncertain due to quantum effects; Poisson with mean \(\beta(r, c)\))
- \(N_{DC}\) number of electrons generated by *dark current* (Poisson)
- \(N_B\) number of electrons generated by CCD electronics (bias, Poisson)
- \(R\) amplifier read-out noise (Gaussian)
- \(Q\) quantization noise (uniform over quantization interval)
Quantal → Analog → Digital

Digital Cameras:
• $D(r, c)$ is quantized immediately after read-out and is sent directly to digital output (IEEE1394, CameraLink, USB2.0, IP).
• The image data are directly available for processing.

Analog Cameras:
• $D(r, c)$ is converted to an analog signal (PAL, NTSC).
• Digitization by a frame grabber is another source of noise/quantization/calibration problems.
Summary

- The human eye
- Pinhole cameras; perspective and affine projection
- Thin lenses
- Real lenses, aberrations, and distortions
- A CCD sensor model