Systems analysis of oscillator models in the space of phase response curves

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Oscillators-whose steady-state behavior is periodic rather than constant-are observed in every field of science. While they have been studied for a long time as closed systems, they are increasingly regarded as open systems, that is, systems that interact with their environment. Because their functions involve interconnection, the relevance of input-output systems theory to model, analyze, and control oscillators is obvious.

Yet, due to the nonlinear nature of oscillators, methodological tools to study their systems properties remain scarce. In particular, few studies focus on the interface between two fundamental descriptions of oscillators, namely the (internal) state-space representation and the (external) circle representation. Starting with the pioneering work of Arthur Winfree, the phase response curve of an oscillator has emerged as the fundamental input-output characteristic linking both descriptions.

The present dissertation aims at studying the systems properties of oscillators through the properties of their phase response curve. The main contributions of this dissertation are the following.

We distinguish between two fundamental classes of oscillators. These classes differ in the local destabilizing mechanism that transforms the stable equilibrium of a globally dissipative system into a periodic orbit. To address input-output systems questions in the space of response curves, we equip this space with the right metrics and develop a (local) sensitivity analysis of infinitesimal phase response curves. This main contribution of the thesis is completed by the numerical tools required to turn the abstract developments into concrete algorithms. We illustrate how these analysis tools allow to address pertinent systems questions about models of circadian rhythms (robustness analysis and system identification) and of neural oscillators (model classification). These two biological rhythms are exemplative of both main classes of oscillators. We also design elementary control strategies to assign the phase of an oscillator. Motivated by an inherent limitation of infinitesimal methods for relaxation type of oscillators, we develop the novel geometric concept of "singularly perturbed phase response curve" which exploits the time-scale separation to predict the phase response to finite perturbations.

In conclusion, the present dissertation investigates input-output systems analysis of oscillators through their phase response curve at the interface between their external and internal descriptions, developing theoretical and numerical tools to study models arising in the biology of cellular rhythms.

Webpage: `http://www.montefiore.ulg.ac.be/~pierresacre/`