Embedded systems

Exercise session 6

Hybrid Systems

Today

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Hybrid Systems I

Definition

Hybrid systems are a modelling formalism that is able to express:

- Operations on integer and real variables
- Discrete state transitions as well as continuous evolution laws
- Composition of elementary systems into a more complex entity

A hybrid system is composed of :

- a finite number p of **processes** $P_1, ..., P_p$
- a finite number n of **variables** $x_1, ..., x_n \Rightarrow \vec{x} \in \mathbb{R}^n$
- a finite set L of **synchronization labels**

Each process P_i is represented by a graph (V_i, E_i) where :

- V_i is a finite set of **control locations**
- $E_i \subseteq V_i \times V_i$ is a finite set of **transitions**

Hybrid Systems II

Each control location $\nu \in V_i$ is associated with:

- An activity $dif(\nu)$ = linear constraints over $x_1, \ldots, x_n$ and $\dot{x}_1, \ldots, \dot{x}_n$
- An invariant $inv(\nu)$ = linear constraints over $x_1, \ldots, x_n$

Each transition $e \in E_i$ is associated with:

- A guard $guard(e)$ = condition to satisfy to enable this transition
- An action $act(e)$ = constraints specifying how variables are changed by following this transition
- An optional label $sync(e) \in L$ used to synchronize this transition with a transition of another process

Exercise 1

This exercise examines a water tank controlled by a microcontroller. This microcontroller monitors the water level and controls it by switching a water pump on or off. We know that:

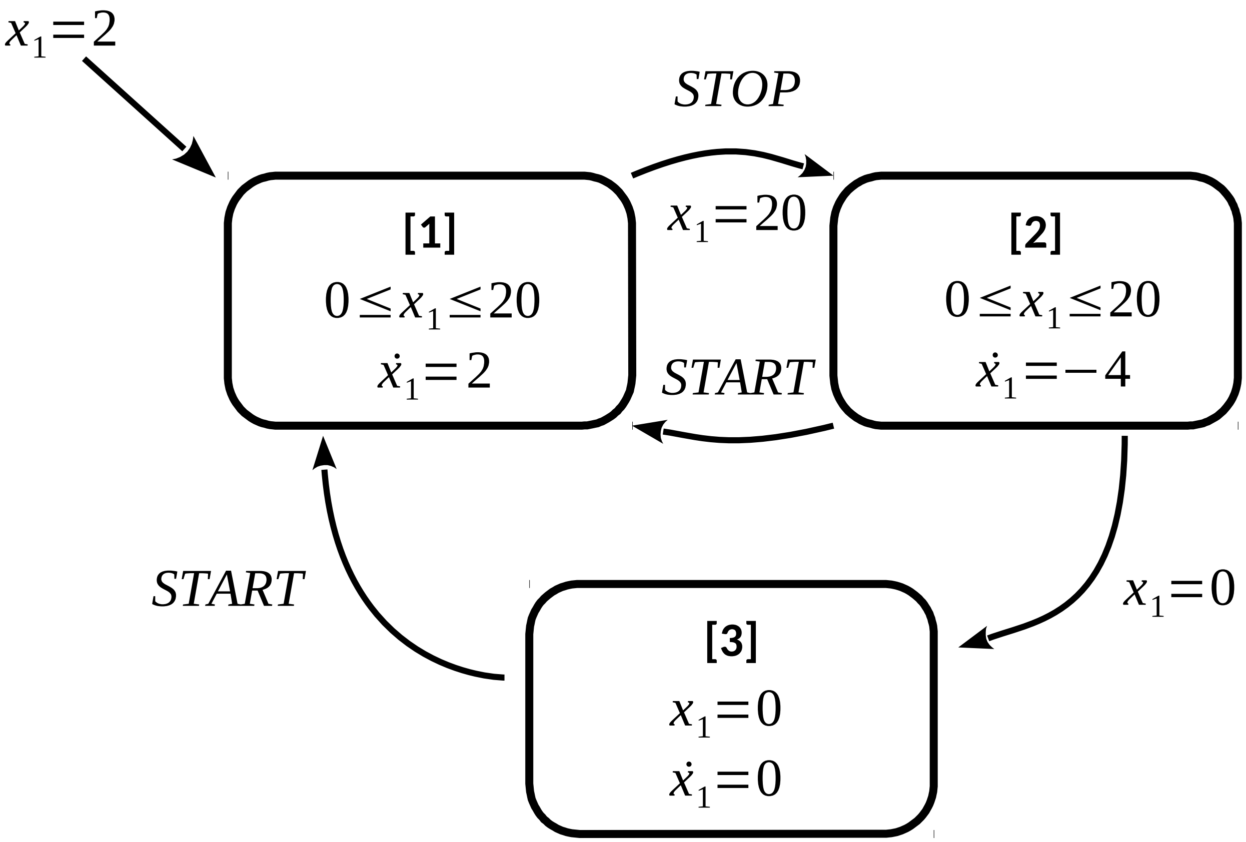
- When the pump is off, the tank loses water at a rate of 4cm/minute;
- When the pump is on, the water level raises of 2cm/minute;
- There is a 2-second delay between the moment at which the pump is ordered to start and when it actually starts working.

To control the water level, the microcontroller shall order the pump to start when the water level is below 10 cm and to stop when the water level reaches 20cm.

1) Model the controller by using a complex timed system
2) Run the model (as far as at least 3 control locations). Initially, there are 2cm of water in the tank and the pump is on.

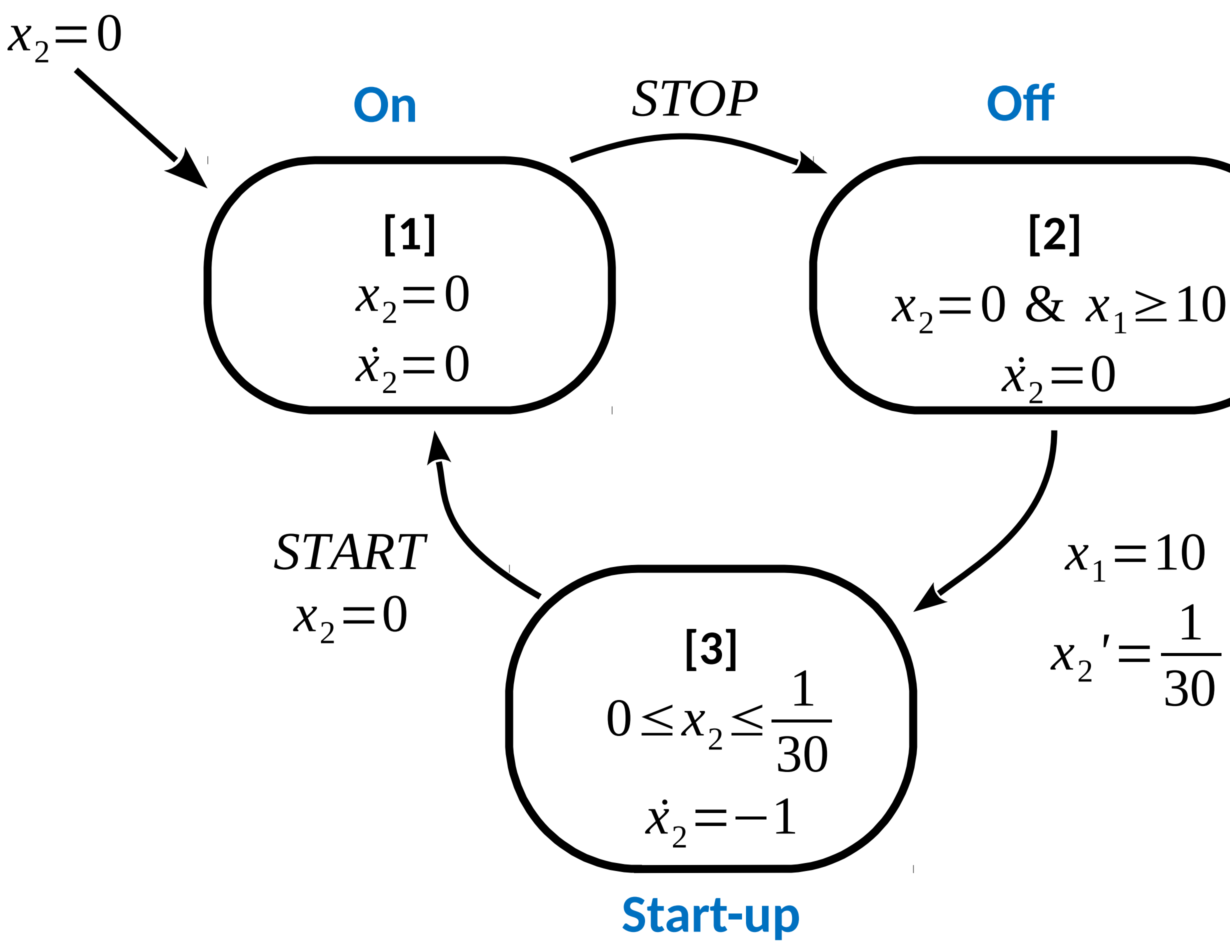
Exercise 1 : Solution I

Tank $x_1 = \text{Water level1}[cm]$



Exercise 1 : Solution II

Pump $x_2 = \text{Delay}\,[minutes]$



Exercise 1 : Solution III

$$
\begin{array}{rll}
 & ([1],[1]): & x_1 = 2,\ x_2 = 0 \\
\Longrightarrow & ([1],[1]): & 2 \le x_1 \le 20,\ x_2 = 0 \\
\xrightarrow{\text{STOP}} & ([2],[2]): & x_1 = 20,\ x_2 = 0 \\
\Longrightarrow & ([2],[2]): & 10 \le x_1 \le 20,\ x_2 = 0 \\
\xrightarrow{x_1=10} & ([2],[3]): & x_1 = 10,\ x_2 = \frac{1}{30} \\
\Longrightarrow & ([2],[3]): & 0 \le x_1 \le 10,\ 0 \le x_2 \le \frac{1}{30} \\
\xrightarrow{\text{START}} & ([1],[1]): & x_1 = 10 - \frac{4}{30},\ x_2 = 0 \\
\Longrightarrow & ([1],[1]): & 10 - \frac{4}{30} \le x_1 \le 20,\ x_2 = 0
\end{array}
$$

Exercise 2

In a chemical factory, a robotised system is used to mix 2 products, $P1$ and $P2$. Each product is contained in a pipette of 80ml and flows out at a constant rate. The outflow rate of $P1$ is 1ml/s and the one of $P2$ is 2ml/s.

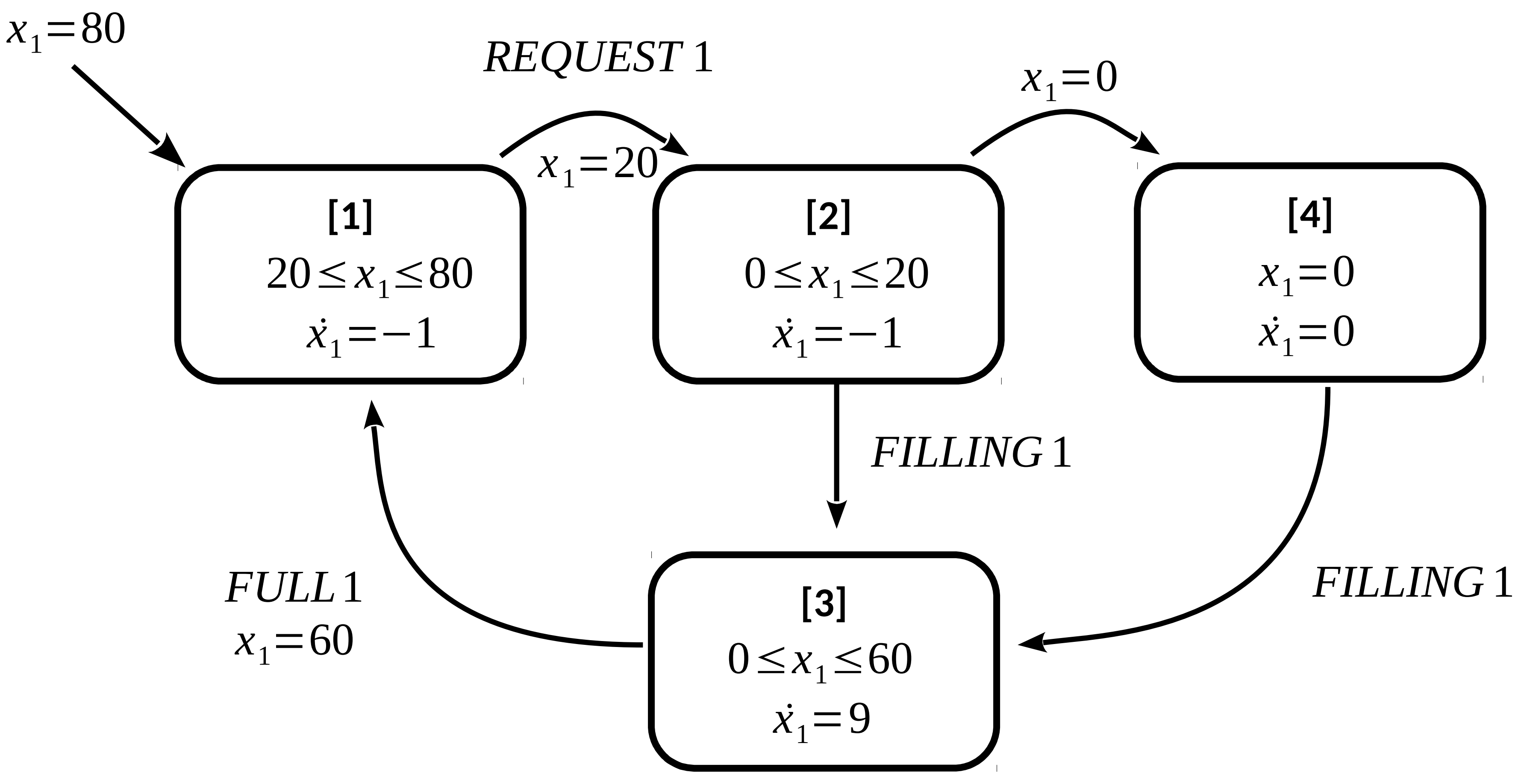
The system uses a mobile arm which is able to move above each pipette in order to pour product to refill the appropriate pipette. This move takes 2 seconds at the most and can only be done if the arm is free (i.e. not busy). Each pipette needs to be refilled when its product level is lower than 20 ml. The arm pours the products at a rate of 10ml/s until the level reaches 60ml. At that moment, the arm becomes free for another duty.

1)Model the controller by using a complex timed system

2)Run the model (as far as at least 3 control locations). Initially, both pipettes are full.

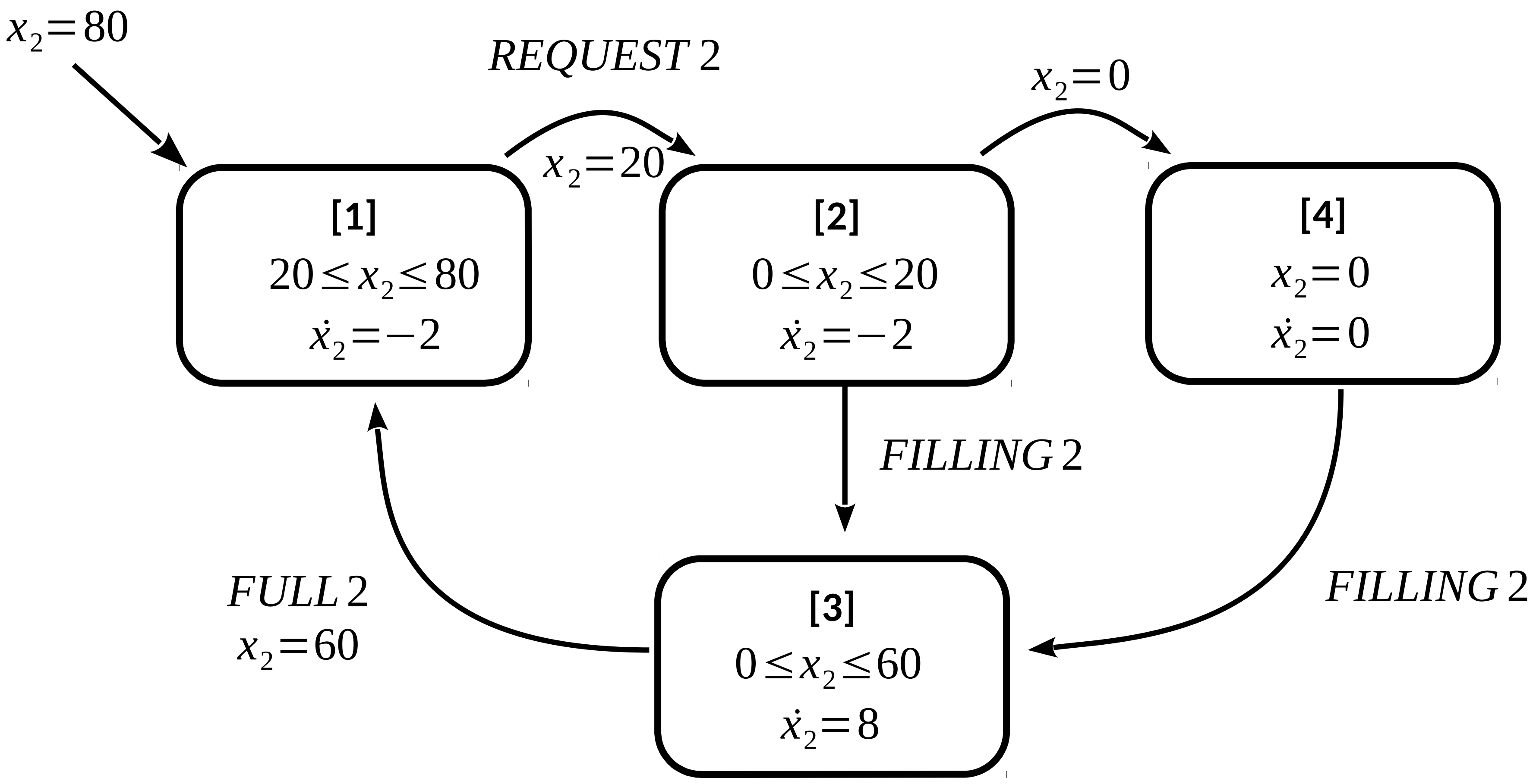
Exercise 2 : Solution I

Pipette 1 x_1 = Liquid volume in pipette 1 $[ml]$



Exercise 2 : Solution II

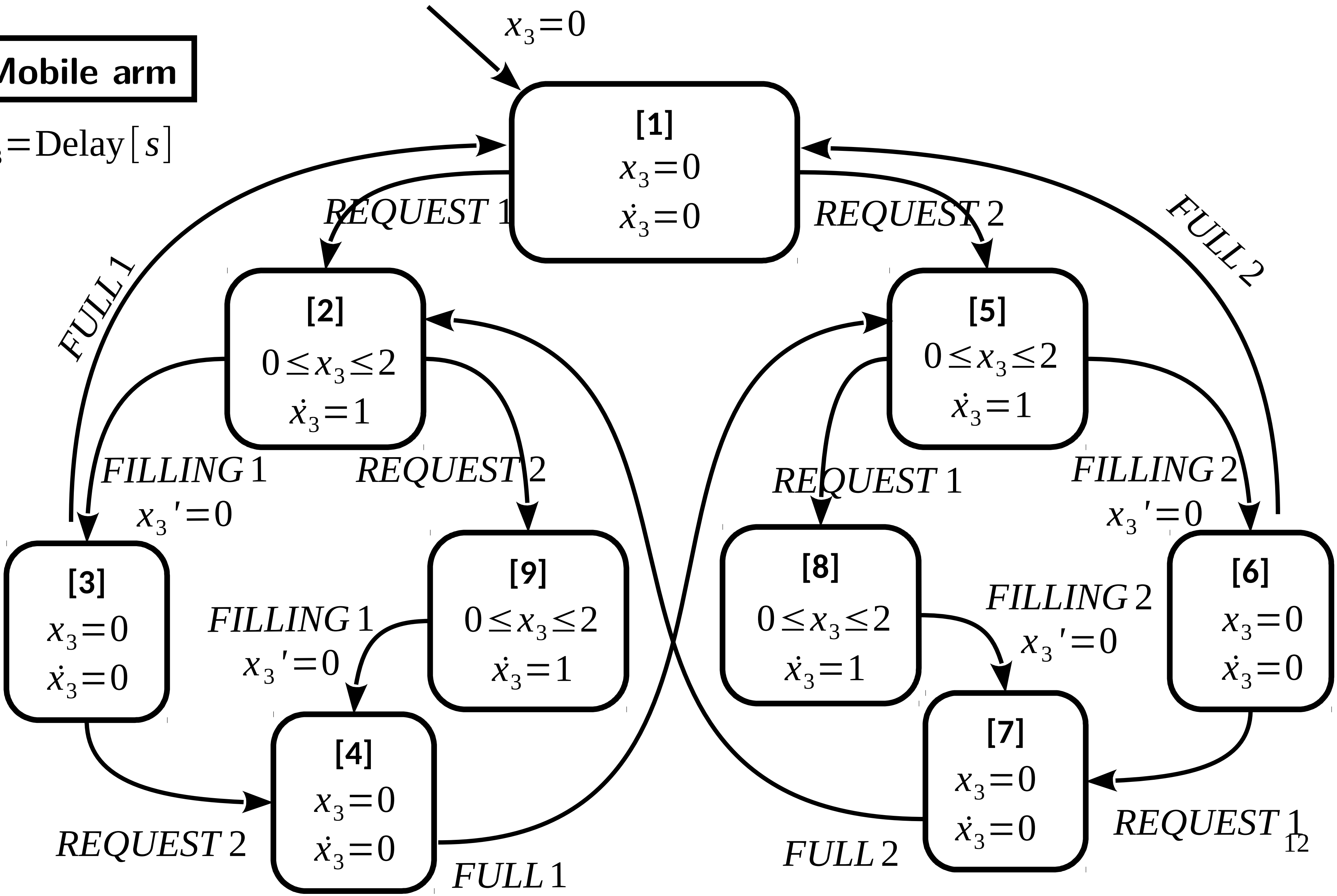
Pipette 2 $x_2 = \text{Liquid volume in pipette } 2\,[ml]$



Exercise 2 : Solution III

Mobile arm

$x_3 = \text{Delay}[s]$



Exercise 2 : Solution IV

$$
\begin{array}{rll}
& ([1],[1],[1]): & x_1 = 80,\ x_2 = 80,\ x_3 = 0 \\
\Longrightarrow & ([1],[1],[1]): & 20 \le x_1 \le 80,\ 20 \le x_2 \le 80,\ x_3 = 0 \\
\xrightarrow{\text{REQUEST 2}} & ([1],[2],[5]): & x_1 = 50,\ x_2 = 20,\ x_3 = 0 \\
\Longrightarrow & ([1],[2],[5]): & 20 \le x_1 \le 50,\ 0 \le x_2 \le 20,\ x_3 = \mu \\
& & 0 \le \mu \le 2 \\
\xrightarrow{\text{FILLING 2}} & ([1],[3],[6]): & x_1 = 50 - \mu,\ x_2 = 20 - 2\mu,\ x_3 = 0 \\
& & 0 \le \mu \le 2 \\
\Longrightarrow & ([1],[3],[6]): & 20 \le x_1 \le 50 - \mu,\ 20 - 2\mu + 8\lambda \le x_2 \le 60,\ x_3 = 0 \\
& & 0 \le \mu \le 2,\ 0 \le \lambda \le 5 + \frac{1}{4}\mu \\
\xrightarrow{\text{FULL 2}} & ([1],[1],[1]): & x_1 = 50 - \mu - \lambda,\ x_2 = 60,\ x_3 = 0 \\
& & 0 \le \mu \le 2,\ \lambda = 5 + 0.25\mu \\
\Longrightarrow & ([1],[1],[1]): & 20 \le x_1 \le 45 - \frac{5}{4}\mu,\ 20 \le x_2 \le 60,\ x_3 = 0 \\
& & 0 \le \mu \le 2
\end{array}
$$

Exercise 3

We now study the functioning of an electrical oven. In this oven, when the heating resistors are turned on :

- If the internal temperature is below 150 degrees, it raises of 5 degrees per minute
- If the internal temperature is between 150 and 170 degrees, it raises of 2 degrees per minute
- If the internal temperature is above 170 degrees, it raises of 1 degree per minute

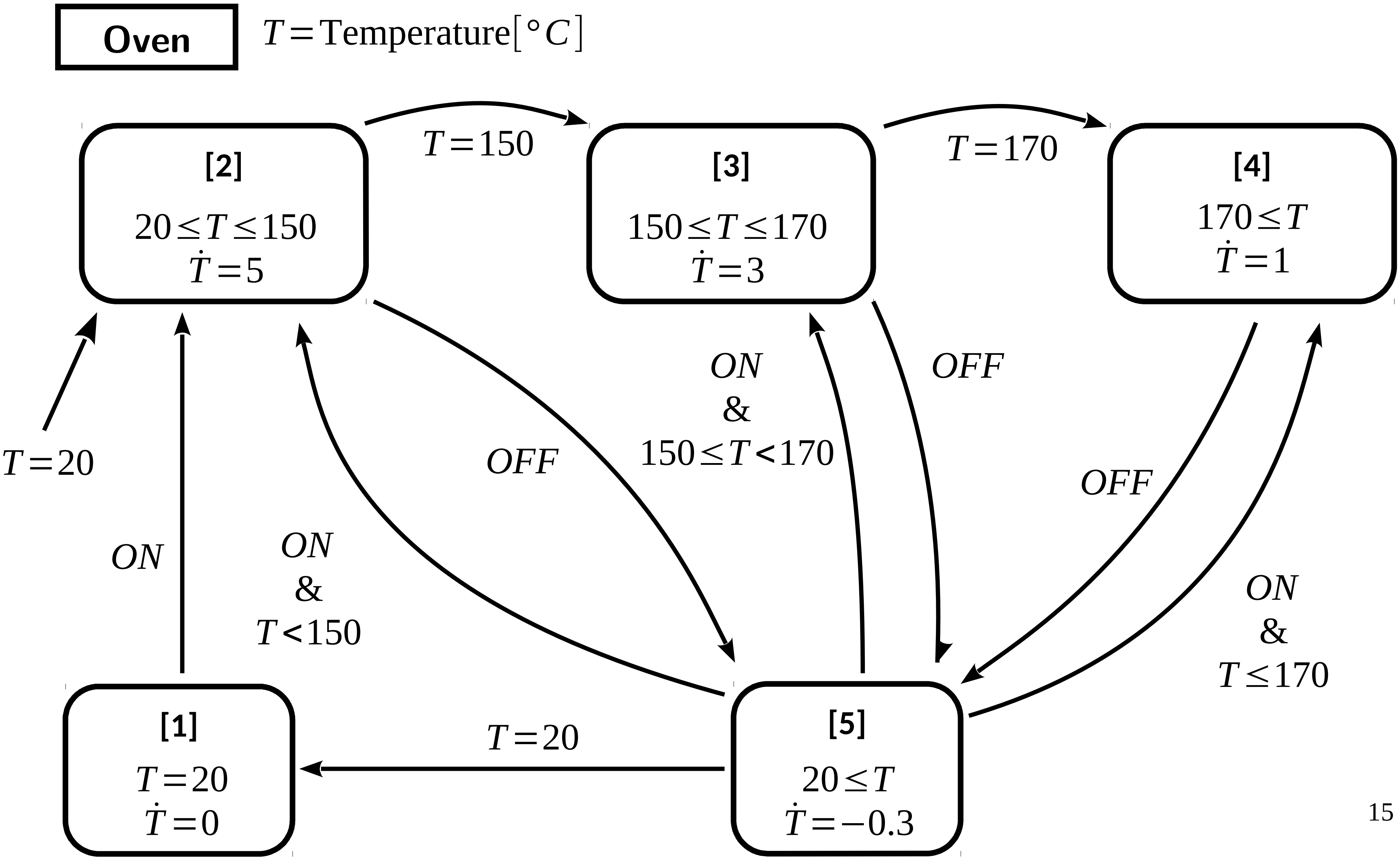
When the resistors are turned off, the temperature falls of 0.3 degrees per minute until it reaches the room temperature which is 20 degrees.

The user can select a temperature between [20,200] degrees by using a rotary knob (T_{ins}). The microcontroller compares this instruction with the temperature inside the oven (T_{meas}). The resistors are switched an when T_{maes} is lower than (T_{ins}-5 degrees). They are turned off when T_{meas} is above ($T_{ins} + 5$ degrees).

1)Model the controller by using a complex timed system

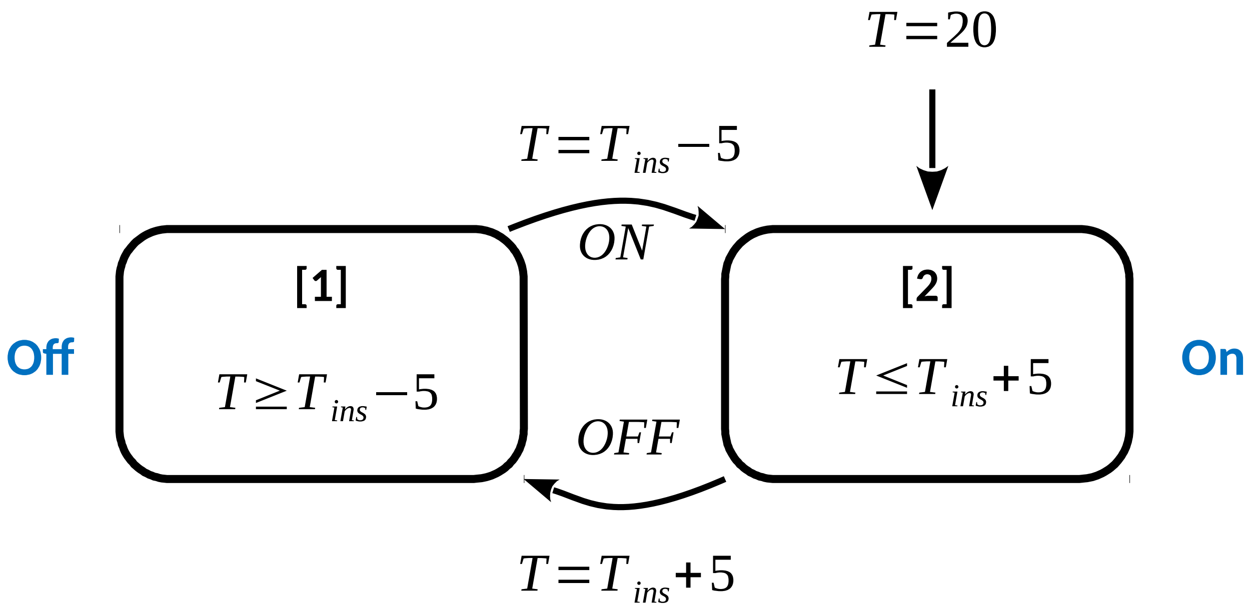
2)Run the model (as far as at least 3 control locations). Initially, the oven is at room temperature and $T_{ins} = 163$ degrees

Exercise 3 : Solution I



Exercise 3 : Solution II

Resistor



Exercise 3 : Solution III

$T_{ins} = 163$

$$
\begin{aligned}
& ([2],[2]): && T = 20 \\
\Longrightarrow\ & ([2],[2]): && 20 \leq T \leq 150 \\
\xrightarrow{T=150}\ & ([3],[2]): && T = 150 \\
\Longrightarrow\ & ([3],[2]): && 150 \leq T \leq T_{ins} + 5 \\
\xrightarrow{\text{OFF}}\ & ([5],[1]): && T = T_{ins} + 5 \\
\Longrightarrow\ & ([5],[1]): && T_{ins} - 5 \leq T \leq T_{ins} + 5 \\
\xrightarrow{\text{ON}}\ & ([3],[2]): && T = T_{ins} - 5 \\
\Longrightarrow\ & ([3],[2]): && T_{ins} - 5 \leq T \leq T_{ins} + 5
\end{aligned}
$$