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# Embedded systems

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Exercise session 5

Task Scheduling

# Today

- Simplified programming environment
- Scheduling
- Exercise 1
- Exercise 2
- Exercise 3

# Simplified programming environment I

## Hypotheses

- Fixed number of tasks
- Each task is characterized by a distinct and constant priority
- Execution requests occur periodically
- Execution times are constant
- Context switches are instantaneous and preemptive

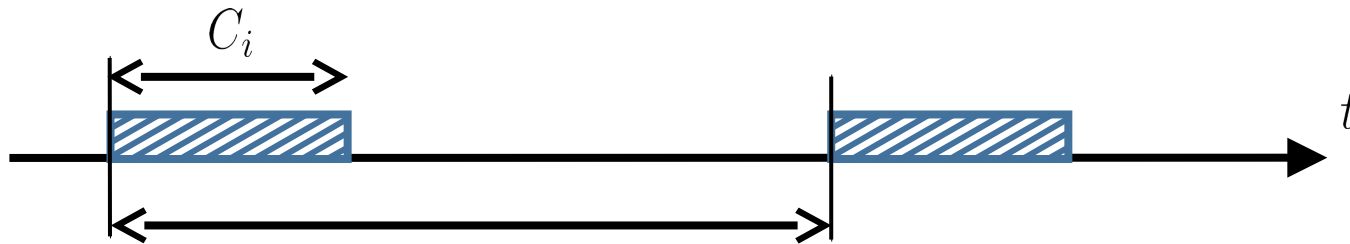
Additionally, the following real-time constraint must be satisfied:

*Each execution of a task must finish before or at the same time as the next execution request for this task*

# Simplified programming environment II

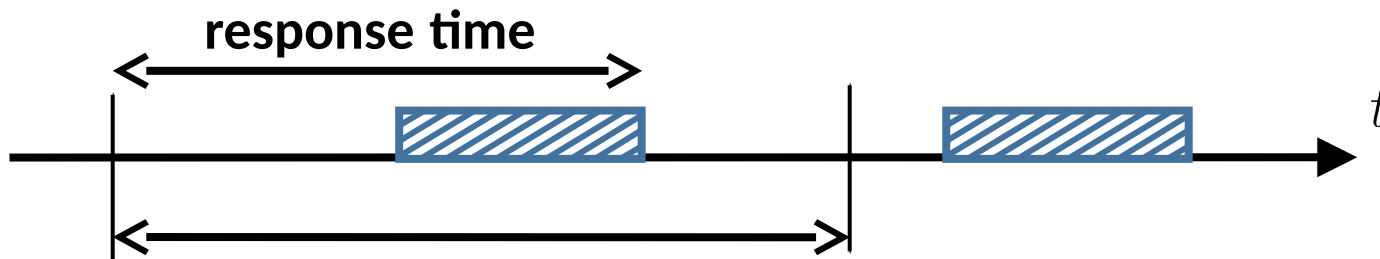
## Task

A task  $\tau_j$  is characterized by a period  $T_j$  and an execution time  $C_j$ .



## Response time

The response time of an execution request for a task  $\tau_j$  is the delay between this request and the end of the execution of this task.



# Simplified programming environment III

## Critical instant

A critical instant for a task  $\tau_i$  is an occurrence of an execution request for that leads to the *largest possible response time* for this task.

Theorem 1: A critical instant for  $\tau_i$  occurs when an execution request for this task happens at the same time that the execution of all higher priority tasks is requested.

## Processor load factor

The processor load factor of a set of tasks with respective periods and execution times and is defined by:

$$U = \sum_{i=1}^n \frac{C_i}{T_i}$$

# Scheduling I

## Schedulable tasks

A set of tasks is schedulable (with respect to a given priority assignment) if the response time of each task is always less than or equal to its period.

## Rate-Monotonic Scheduling

Given a set of tasks  $\tau_1, \tau_2, \dots, \tau_n$  with respective periods  $T_1, T_2, \dots, T_n$ , the Rate-Monotonic Scheduling (RMS) strategy consists in assigning distinct priorities  $P_1, P_2, \dots, P_n$  to the task, such that for all  $i, j$  :

$$T_i < T_j \Rightarrow P_i > P_j$$

Theorem 2: If a set of tasks is schedulable with respect to some priorities assignment, then is it schedulable with the RMS priorities assignment.

# Scheduling II

## How to know if a set of task is schedulable

The following algorithm is used to check efficiently whether a set of  $n$  periodic tasks with a processor load factor  $U$  is schedulable or not:

- 1) If  $U > 100\%$  → Not schedulable
- 2) If  $U < 69\%$  → Schedulable
- 3) If  $U < n(2^{1/n}-1)\%$  → Schedulable
- 4) Otherwise, perform an exact scheduling simulation based on a RMS priority assignment to find out.

# Exercise 1

- Every task is represented by  $(T, C)$ .
- Are the following set of tasks schedulable ?
  - 1)  $\{ (5,1) ; (6,2) ; (7,4) \}$
  - 2)  $\{ (5,2) ; (16,3) \}$
  - 3)  $\{ (7,1) ; (8,2) ; (9,3) \}$
  - 4)  $\{ (2,1) ; (4,1) ; (8,1) ; (16,2) \}$
  - 5)  $\{ (8,1) ; (6,1) ; (6,1) ; (15,1) ; (5,1) ; (10,2) \}$
  - 6)  $\{ (5,4) ; (5,1) \}$



# Exercise 1

1.  $\{ (5,1) ; (6,2) ; (7,4) \}$

$$U = \frac{1}{5} + \frac{2}{6} + \frac{4}{7} \approx 110\% \quad \Rightarrow \quad U > 100\% \quad \rightarrow \text{Not Schedulable}$$

2.  $\{ (5,2) ; (16,3) \}$

$$U = \frac{2}{5} + \frac{3}{16} = 58.75\% \quad U < 69\% \quad \rightarrow \text{Schedulable}$$

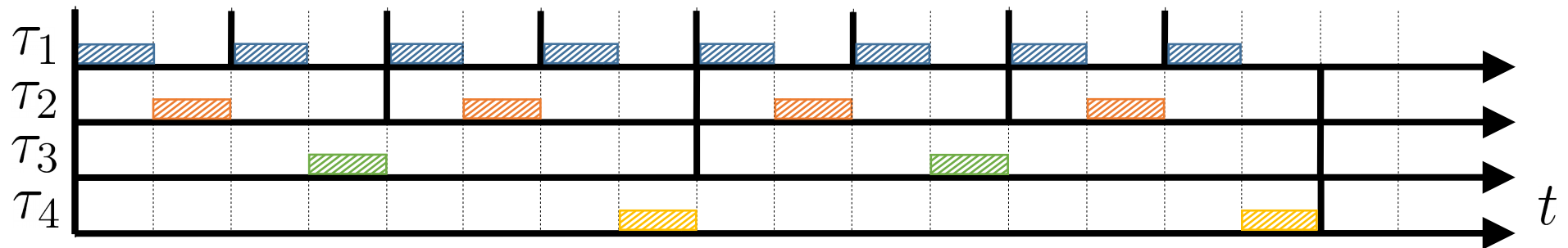
3.  $\{ (7,1) ; (8,2) ; (9,3) \}$

$$U = \frac{1}{7} + \frac{2}{8} + \frac{3}{9} = 72\% \quad U > 69\% \quad \rightarrow \text{Schedulable}$$
$$U < 3(2^{\frac{1}{3}} - 1) \approx 78\%$$

# Exercise 1

4.  $\{ (2,1) ; (4,1) ; (8,1) ; (16,2) \}$

$$U = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{2}{16} = 100\% \Rightarrow \text{RMS priorities assignment}$$



You have also seen during the course that the sets of tasks such that

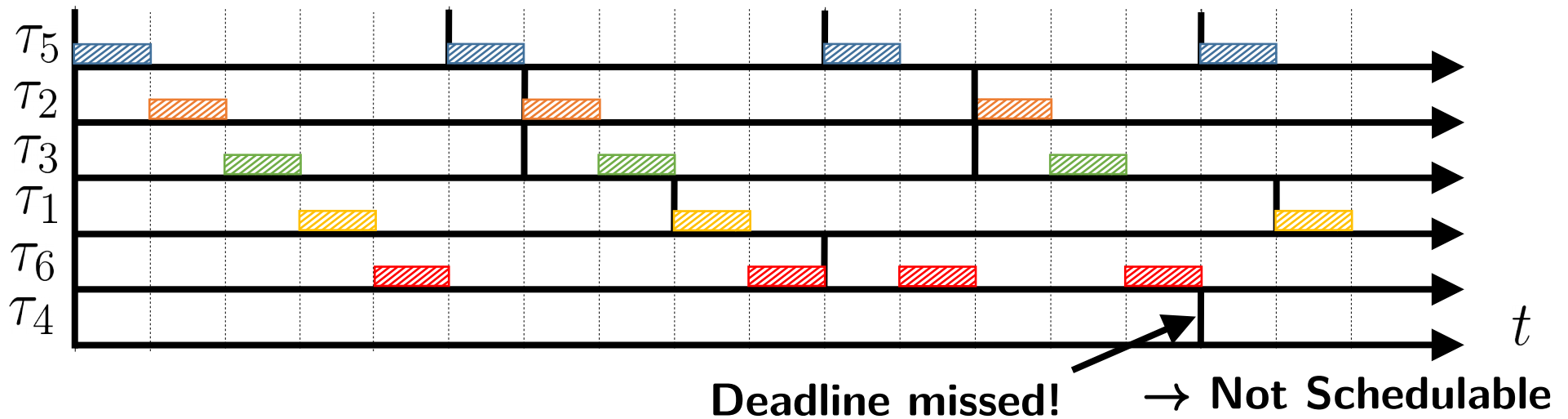
$\forall i, j : i < j \Rightarrow T_j$  is an integer multiple of  $T_i$   
are schedulable if  $U \leq 100\%$ .

→ **Schedulable**

# Exercise 1

5.  $\{ (8,1) ; (6,1) ; (6,1) ; (15,1) ; (5,1) ; (10,2) \}$

$$U = \frac{1}{8} + 2\frac{1}{6} + \frac{1}{15} + \frac{1}{5} + \frac{2}{10} = 92.5\% \Rightarrow \text{RMS priorities assignment}$$



6.  $\{ (5,4) ; (5,1) \}$

$$U = \frac{4}{5} + \frac{1}{5} = 100\% \quad \text{cf. Ex 4.} \quad \rightarrow \text{Schedulable}$$

## Exercise 2

Two periodic tasks  $\tau_1$  and  $\tau_2$  are characterized by their respective periods  $T_1=1\text{ms}$  and  $T_2=3.5\text{ms}$ . The execution time  $C_2$  of  $\tau_2$  is equal to  $1.2\text{ms}$ . For which value(s) of  $C_1$  does this pair of tasks fully use the processor? (Justify all steps of your reasoning)

(Short answer:  $C_1=0.6\text{ms}$ )

# Exercise 3

1. Determine graphically the maximum execution time  $C$ , for which the following set of tasks  $\{(T_i, C_i)\}$  is schedulable.

$$\{ (5,1) ; (10,3) ; (15,4) ; (16,C) \}$$

2. Determine graphically the maximum execution time  $C$ , for which the following set of tasks  $\{(T_i, C_i)\}$  is schedulable.

$$\{ (5,1) ; (10,3) ; (15,4) ; (11,C) \}$$

3. Determine graphically the minimum period  $T$ , for which the following set of tasks  $\{(T_i, C_i)\}$  is schedulable.

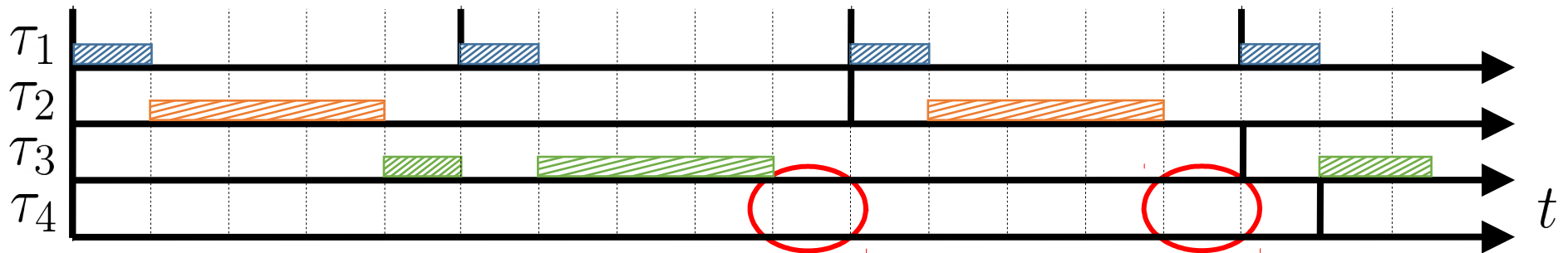
$$\{ (5,1) ; (8,2) ; (10,2) ; (T,2) \}$$

# Exercise 3

1. max  $C$  such that the following set of tasks is schedulable.

$$\{ (5,1) ; (10,3) ; (15,4) ; (16,C) \}$$

$$U = \frac{1}{5} + \frac{3}{10} + \frac{4}{15} + \frac{C}{16} < 100\% \Rightarrow C < 3.73$$



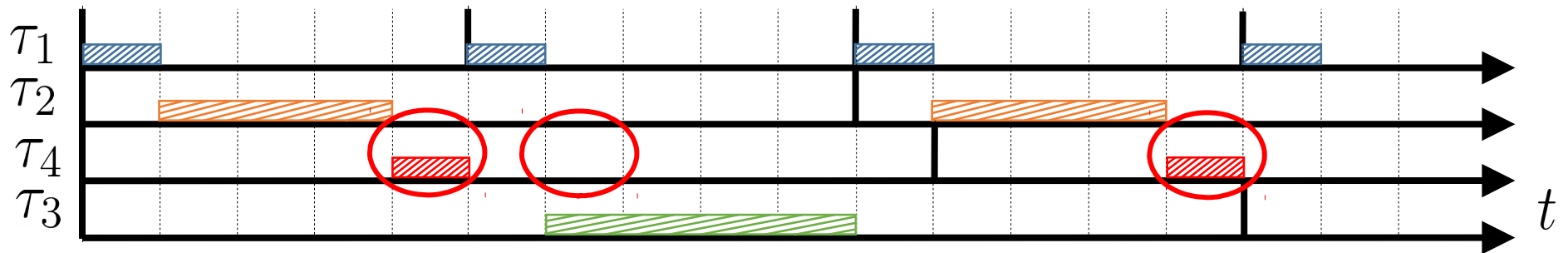
$$\Rightarrow C = 2$$

## Exercise 3

2. max  $C$  such that the following set of tasks is schedulable.

$$\{ (5,1) ; (10,3) ; (15,4) ; (11,C) \}$$

$$U = \frac{1}{5} + \frac{3}{10} + \frac{4}{15} + \frac{C}{11} < 100\% \Rightarrow C < 2.57$$



$$\Rightarrow C = 1$$

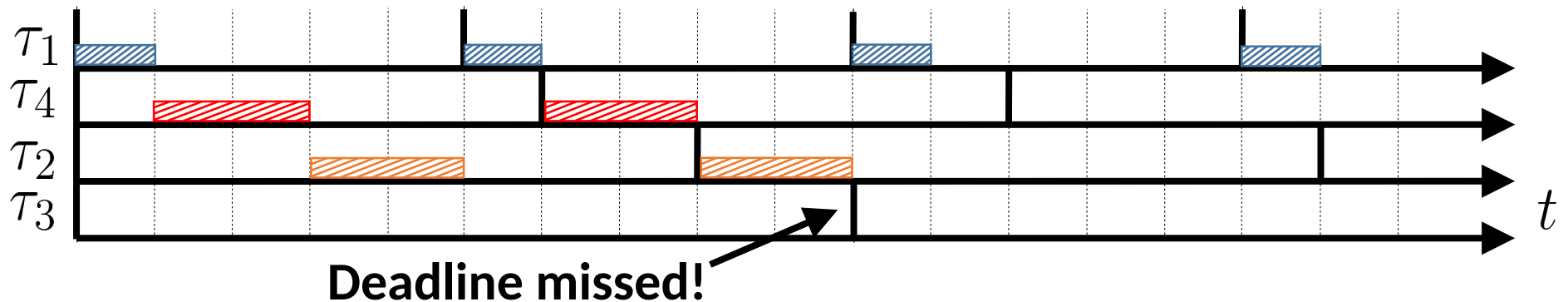
## Exercise 3

3. min  $T$  such that the following set of tasks is schedulable.

$$\{ (5,1) ; (8,2) ; (10,2) ; (T,2) \}$$

$$U = \frac{1}{5} + \frac{2}{8} + \frac{2}{10} + \frac{2}{T} < 100\% \Rightarrow T > 5.71$$

Let's try with  $T = 6$





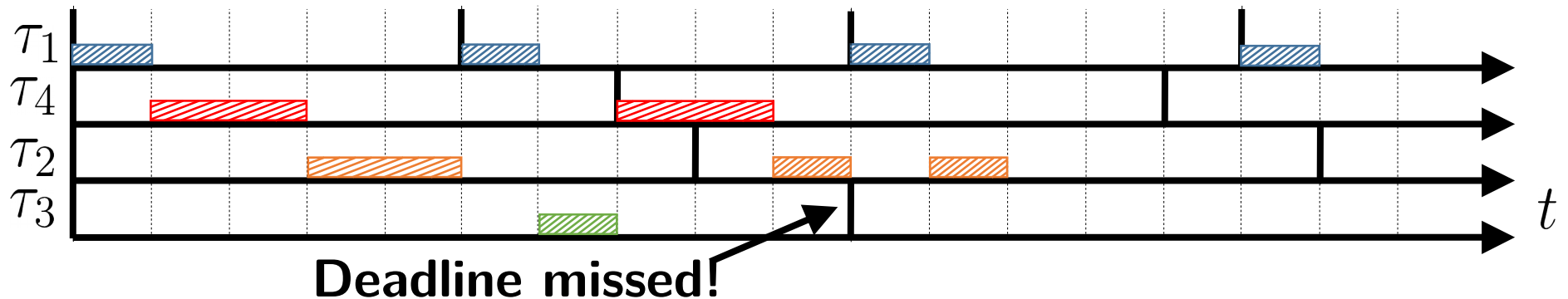
## Exercise 3

3. min  $T$  such that the following set of tasks is schedulable.

$$\{ (5,1) ; (8,2) ; (10,2) ; (T,2) \}$$

$$U = \frac{1}{5} + \frac{2}{8} + \frac{2}{10} + \frac{2}{T} < 100\% \Rightarrow T > 5.71$$

Let's try with  $T = 7$



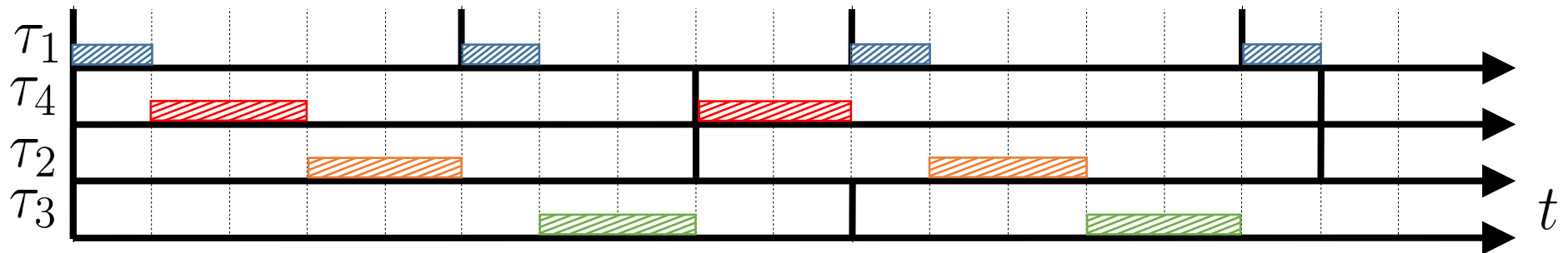
## Exercise 3

3. min  $T$  such that the following set of tasks is schedulable.

$$\{ (5,1) ; (8,2) ; (10,2) ; (T,2) \}$$

$$U = \frac{1}{5} + \frac{2}{8} + \frac{2}{10} + \frac{2}{T} < 100\% \Rightarrow T > 5.71$$

Let's try with  $T = 8$



$$\Rightarrow T = 8$$

## Exercise 4

Two Consider the following set of tasks  $\tau_i = \{C_i, T_i\}$ :

$$\{ \tau_1 = (3, 13), \tau_2 = (1, 3), \tau_3 = (\alpha, 5) \},$$

where  $\alpha$  is a parameter.

- (a) Compute the maximum value of  $\alpha$  for this set of task to be schedulable (justify all steps of your reasoning).
- (b) Verify your answer with a graphical simulation.

(Short answer:  $C1 = 5/3 = 1.666\dots$ )