

Logic

Tutorial 2

04 October 2018

Exercises

1. Five people (a, b, c, d, e) have put their money into the same safe. They however have no confidence in each other and decided therefore that the safe can only be opened in the presence of a and b , or b and c , or b, d and e . How many locks does the safe have? How many keys are needed? And who has them?

Hint: Consider the formula

$$\phi(p_a, p_b, p_c, p_d, p_e) \triangleq \text{“the safe can be opened”},$$

where p_x is true if x is present.

Logical Consequences

2. Show that $(X \wedge Y) \Rightarrow Z$ et $X \Rightarrow (Y \Rightarrow Z)$ are logically equivalent.
3. If Jordan or Algeria joins the union then, if Syria or Kuwait boycott the union then, even though Iraq does not boycott the union, Yemen does. If Iraq or Morocco do not boycott the union, Egypt will join the union. Therefore, if Jordan joins the union then, if Syria boycotts the union, Egypt will join the union.
4. Logic is intriguing me. Everything that is comprehensible never intrigues me. Therefore logic is incomprehensible.
5. Whoever says that I am a man says true. Whoever says that I am stupid says that I am a man. Therefore whoever says that I am stupid says true.

6. Let A , B , X and Y be formulas. If $A \models B$, what can you say, in general, about
- $C \triangleq X \Rightarrow (A \Rightarrow Y)$
 - $D \triangleq X \Rightarrow (B \Rightarrow Y)$
- ($C \models D$? $D \models C$? $C \leftrightarrow D$? No logical consequence?)
7. Let A , B , X and Y be formulas. If $A \models B$, what can you say, in general, about
- $C \triangleq \neg(X \Rightarrow A) \vee Y$
 - $D \triangleq \neg(X \Rightarrow B) \vee Y$
8. Let A , B , X and Y be formulas. If $A \models B$, what can you say, in general, about
- $C \triangleq X \Rightarrow (A \equiv Y)$
 - $D \triangleq X \Rightarrow (B \equiv Y)$
9. Let A , B , X and Y be formulas. If $A \models B$, what can you say, in general, about
- $C \triangleq X \Rightarrow (\neg A \wedge Y)$
 - $D \triangleq X \Rightarrow (\neg B \wedge Y)$
10. Consider a set of five propositional variables $P \triangleq \{a, b, c, d, e\}$.
- (a) How many formulas, up to logical equivalence, exist that are satisfied by exactly seventeen interpretations?
 - (b) How many formulas, up to logical equivalence, exist that are logical consequence of the formula $a \wedge b$?
11. He said that he would come if it does not rain. It rains. Therefore he does not come.
- Is the conclusion really a logical consequence of the facts?

12. If he does not tell her, she will never discover it. If she does not ask him, he will not tell her about it. She discovered it. Therefore she asked about it.

Is the conclusion really a logical consequence of the facts?

13. If one considers that the people that study extrasensory perceptions are honest, then one must admit the existence of such perceptions. Further, if one puts to test extrasensory perceptions, one needs to seriously consider clairvoyance. To admit the existence of extrasensory perceptions will push us to put them to test and to explain them.

Clairvoyance needs to be seriously considered if one is willing to consider seriously occult phenomena. And if one is willing to consider seriously those phenomena, one need to respect psychic media. Further, if we respect these people, we also need to take seriously their ability to talk to the deceased. Finally, if we take seriously their ability to talk to the deceased, we must believe in ghosts.

Therefore, considering that the people who study extrasensory perceptions are honest forces us to believe in ghosts.

Semantic Tableaux

14. Using the semantic tableaux method, determine whether the following formula is valid, consistent or inconsistent.

$$(p \Rightarrow q) \Rightarrow [(\neg p \Rightarrow q) \Rightarrow q]$$

15. Using the semantic tableaux method, determine whether the following formula is valid, consistent or inconsistent.

$$[(p \vee q) \wedge (p \Rightarrow r) \wedge (q \Rightarrow s)] \Rightarrow (r \Rightarrow s)$$

Give a model of the formula if possible.

16. Using the semantic tableaux method, determine whether the following formula is valid, consistent or inconsistent.

$$[p \Rightarrow (q \Rightarrow r)] \Rightarrow [(p \Rightarrow q) \Rightarrow (p \Rightarrow r)]$$

Exercises

17. Determine whether the following formulas are valid, consistent or inconsistent using three different methods.

- (a) $(\neg p \Rightarrow q) \vee (p \Rightarrow \neg q)$
- (b) $(p \wedge q) \vee (q \wedge r) \vee (r \wedge p)$
- (c) $[(p \wedge q) \vee (\neg p \wedge \neg q)] \vee [(\neg p \wedge q) \vee (p \wedge \neg q)]$
- (d) $[(p \wedge q) \Rightarrow (r \wedge s)] \Rightarrow [(p \wedge q) \Rightarrow (r \wedge s)]$
- (e) $(a \equiv (b \Rightarrow c)) \equiv [(a \wedge c) \vee (\neg(a \equiv b) \wedge \neg c)]$

18. (Interro 2006 Q3)

During the construction of a semantic tableau, the label of a node should be $\{A, X\}$ but due to an error it becomes $\{A, Y\}$. The construction of the tableau finishes without another error. What can you conclude from your tableau if

- (a) $X \leftrightarrow Y$;
- (b) $X \models Y$ but $Y \not\models X$;
- (c) $Y \models X$ but $X \not\models Y$;
- (d) $A \wedge X \leftrightarrow A \wedge Y$ but $X \not\models Y$ et $Y \models X$;
- (e) $A \vee X \leftrightarrow A \vee Y$ but $X \not\models Y$ et $Y \models X$.

19. Brown, Jones and Smith are three irish salesmen in New York. They are on trial for the fabrication and sale of alcohol (during prohibition). They swore on the Bible and declared:

Brown: Jones is guilty, and Smith est innocent.

Jones: If Brown is guilty then Smith is guilty as well.

Smith: I am innocent, but at least one of the two others is guilty.

Let b, j, s be the tree propositions “Brown is innocent”, “Jones is innocent”, “Smith is innocent”. Give a logical formula for each of the statements.

- (a) Are the statements consistent?
- (b) One statement is a logical consequence of another one. Which one?

- (c) If everyone is innocent, who made a false declaration?
- (d) As the irish are very religious people, one could assume that they tell the truth. In this case, who is guilty?
- (e) If the innocent tell the truth and the guilty lie, who is guilty?