# Introduction to computability
# Tutorial 5

Finite Automata and Grammars

23 October 2018

1. Take the languages generated by a grammar G whose production rules are of the form

$$A \to Bw, \qquad A \to w$$

where $A$, $B$ are non-terminal symbols and $w \in \Sigma^*$.

- Show that the class of these languages coincides exactly with the class of regular languages.
- What happens if we also allow production rules of the form $A \to wB$?

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4. Using the second version of the pumping lemma for regular languages, prove that the language $\{a^{n!} \mid n \in \mathbb{N}\}$ (where $n!$ is the factorial of $n$) is not regular.

# Bonus Exercise 5

Let $L$ be the language of the words on the alphabet $\{a, b\}$ that contain exactly twice as many times the letter b than the letter a (in an arbitrary order). Formally,

$$L = \{w \in \{a, b\}^* \mid N_b(w) = 2 \cdot N_a(w)\}$$

where $N_\sigma(w)$ is the number of letters $\sigma$ contained in the word $w$. Show that $L$ is not regular.