Introduction to computability
Tutorial 5

Finite Automata, Grammars and Pushdown Automata

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1. Take the languages generated by a grammar $G$ whose production rules are of the form

$$A \rightarrow Bw, \quad A \rightarrow w$$

where $A$, $B$ are non-terminal symbols and $w \in \Sigma^*$.  

- Show that the class of these languages coincides exactly with the class of regular languages.  
- What happens if we also allow production rules of the form $A \rightarrow wB$?
2. Show that the language \( L = \{ zzz \mid z \in \{a, b\}^* \} \) is not regular.
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3. Using the second version of the pumping lemma for regular languages, prove that the language $\{ a^n! \mid n \in \mathbb{N} \}$ (where $n!$ is the factorial of $n$) is not regular.
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4. Show that the language \( L = \{ a^i b^j c^k \mid k \neq i \cdot j \} \) is not regular.
How to draw pushdown automata

A transition \(((q, u, \beta), (q', \gamma))\) is represented by

\[(u, \beta/\gamma)\]
3. Describe the language accepted by the pushdown automata $M = (Q, \Sigma, \Gamma, \Delta, Z, s, F)$ where:

- $Q = \{s, p, q, f\}$;
- $\Sigma = \{a, b, c\}$;
- $\Gamma = \{A, Z\}$;
- $F = \{f\}$;
- $\Delta$ contains the following transitions:
  - $(s, a, \varepsilon) \rightarrow (s, A)$,
  - $(s, b, \varepsilon) \rightarrow (q, \varepsilon)$,
  - $(s, c, A) \rightarrow (p, \varepsilon)$,
  - $(q, b, \varepsilon) \rightarrow (q, \varepsilon)$,
  - $(q, c, A) \rightarrow (p, \varepsilon)$,
  - $(p, c, A) \rightarrow (p, \varepsilon)$,
  - $(p, \varepsilon, Z) \rightarrow (f, \varepsilon)$. 
4. Give a pushdown automata accepting each of the following languages:

- The language generated by the grammar
  \[ S \rightarrow aSa \]
  \[ S \rightarrow bSb \]
  \[ S \rightarrow \varepsilon; \]

- \[ L = \{a^n b^{2m+n} c^m | n, m \geq 0\}; \]

- The language of the words on the alphabet \{a, b\} that contain as many a’s as b’s;

- \[ L = \{a^n b^m | 0 < n \leq m \leq 2n\}. \]
Bonus Exercise 5

Let $L$ be the language of the words on the alphabet $\{a, b\}$ that contain exactly twice as many times the letter $a$ than the letter $b$ (in an arbitrary order). Formally,

$$L = \{ w \in \{a, b\}^* \mid N_a(w) = 2 \cdot N_b(w) \}$$

where $N_\sigma(w)$ is the number of letters $\sigma$ contained in the word $w$.

- Show that $L$ is not regular.
- Give a pushdown automaton that accepts $L$. 