Introduction to computability
Tutorial 4

Finite Automata and Grammars

09 October 2018

Grammars

- A grammar is a 4-tuple $G = (V, \Sigma, R, S)$, where
  - V is an alphabet,
  - $\Sigma \subseteq V$ is the set of *terminal symbols* ($V \setminus \Sigma$ is the set of *nonterminal symbols*),
  - $R \subseteq (V^+ \times V^*)$ is a finite set of *production rules*,
  - $S \in V \setminus \Sigma$ is the *start symbol*.
- The language generated by a grammar G is the set

$$L(G) = \{v \in \Sigma^* \mid S \Rightarrow_G^* v\}.$$

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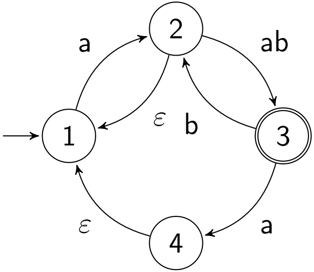
d) the language accepted by the following automaton:



2. Describe the languages generated by the following grammars:

a)
$$\begin{aligned} S &\to aSa \\ S &\to bSb \\ G &\to \varepsilon \end{aligned}$$

b)
$$\begin{aligned} S &\to aS \\ S &\to bS \\ S &\to \varepsilon \end{aligned}$$

c)
$$\begin{aligned} S &\to LaR \\ L &\to LD \\ Da &\to aaD \\ DR &\to R \\ L &\to \varepsilon \\ R &\to \varepsilon \end{aligned}$$

The "pumping" lemma

Let L be a regular language and let $w \in L$ be such that $|w| \geq |Q|$ where Q is the set of states of a deterministic automaton accepting L. Then $\exists x$, u, y, with $u \neq \varepsilon$ and $|xu| \leq |Q|$ such that $xuy = w$ and, $\forall n$, $xu^n y \in L$.

3. Using the second version of the pumping lemma, prove that the language $\{a^m b^n \mid m > n\}$ is not regular.

4. Let L be a regular language. Consider the following algorithm to test if L is finite:

- Construct the DFA A such that $L(A) = L$
- Simulate A on all the words of length between n and $2n - 1$ where n is the number of states of A.
- Output yes if and only if no word is accepted and otherwise output no.

Is this algorithm correct? Motivate your answer!
Can you give a more efficient algorithm?

Bonus Exercise 4

Let L be the language generated by the grammar

$$\begin{array}{lcl} S & \rightarrow & A \mid B \\ A & \rightarrow & YZYA \mid \varepsilon \\ B & \rightarrow & Xc \\ X & \rightarrow & aX \mid bX \mid \varepsilon \\ Y & \rightarrow & cY \mid \varepsilon \\ Z & \rightarrow & a \mid b \end{array}$$

where S is the start symbol.

a) For each non-terminal symbol, give a regular expression for the language generated from that symbol.

b) Give a NFA that accepts L.