Introduction to computability
Tutorial 4

Finite Automata and Grammars

09 October 2014
Pattern search

We want to look in a text on the alphabet $\Sigma$ for the word $u$. We thus want an automaton that accepts $L(\Sigma^* u)$. 

\[
\begin{array}{c}
\Sigma^* \\
\downarrow \\
u_1 \\
\downarrow \\
u_2 \\
\downarrow \\
u_3 \\
\downarrow \\
\ldots \\
\downarrow \\
u_{n-1} \\
\downarrow \\
u_n \\
\end{array}
\]
We want to look in a text on the alphabet \( \Sigma \) for the word \( u \). We thus want an automaton that accepts \( L(\Sigma^* u) \).

1. Give a deterministic finite automaton that recognizes the pattern "nano".
Grammars

2. For each of the following languages, give a grammar that generates it:
   a) \( L((a \cup b)^* (bab \cup b^*)(aab)^*) \);
Grammars

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   a) \( L((a \cup b)^*(bab \cup b^*)(aab)^*) \);
   b) \( \{a^m b^n c^p \mid m + n = p\} \);
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   b) \( \{a^mb^n c^p \mid m + n = p\}; \)
   c) the language of the palindromes on \( \Sigma = \{a, b\} \), i.e. the language containing the words \( w = w_0w_1 \ldots w_n \) such that for all \( i, 0 \leq i \leq n \) we have that \( w_i = w_{n-i}; \)
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   c) the language of the palindromes on \( \Sigma = \{a, b\} \), i.e. the language containing the words \( w = w_0 w_1 \ldots w_n \) such that for all \( i, 0 \leq i \leq n \) we have that \( w_i = w_{n-i} \);
   d) the language accepted by the following automaton:
3. Describe the languages generated by the following grammars:
   
a) \[ S \rightarrow aSa \]
   \[ S \rightarrow bSb \]
   \[ G \rightarrow \varepsilon \]
   
b) \[ S \rightarrow aS \]
   \[ S \rightarrow bS \]
   \[ S \rightarrow \varepsilon \]
   
c) \[ S \rightarrow LaR \]
   \[ L \rightarrow LD \]
   \[ Da \rightarrow aaD \]
   \[ DR \rightarrow R \]
   \[ L \rightarrow \varepsilon \]
   \[ R \rightarrow \varepsilon \]
4. Using the second version of the pumping lemma, prove that the language \( \{a^m b^n \mid m > n\} \) is not regular.
Additional exercises

- Give a deterministic finite automaton representing in base 3 the set of even numbers.
- Give a deterministic finite automaton representing in base 3 the set \( \{ x \geq 4 \mid x \in \mathbb{N} \} \).
- Give a deterministic finite automaton representing in base 3 the set \( \{ 2x \mid x \in \mathbb{N} \} \cup \{ x < 4 \mid x \in \mathbb{N} \} \).
Bonus Exercise 4

Show that the language $L = \{ www \mid w \in \{a, b\}^* \}$ is not regular.