Introduction to computability
Tutorial 2

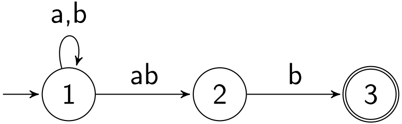
Finite Automata

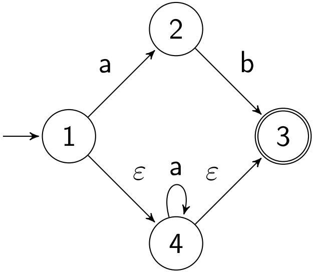
25 September 2018

1. For both of the following automata:

   a) give a regular expression of the language accepted by this automaton (without using the $R(i, j, k)$ - method);

   b) give an equivalent deterministic finite automaton.





2. Give a deterministic finite automaton that accepts

a) the language with regular expression $(ab \cup ba)^+$;

b) the language with regular expression $ab \cup (aab)^*$;

c) the language containing the words defined on $\{a, b\}$ where the number of a's is a multiple of 4.

2. Give a deterministic finite automaton that accepts

a) the language with regular expression $(ab \cup ba)^+$;

b) the language with regular expression $ab \cup (aab)^*$;

c) the language containing the words defined on $\{a, b\}$ where the number of a's is a multiple of 4.

3. Show, using the theory on finite automata that the regular expressions $(a^*b)^*$ and $\varepsilon \cup (a \cup b)^*b$ denote the same language.

4. Let $L \subseteq \Sigma^*$. Show that if L is a regular language, than so is $Pref(L)$, the language containing all the prefixes of the words of L. Formally:

$$Pref(L) = \{w \in \Sigma^* \mid (\exists x \in L)(\exists z \in \Sigma^*)x = wz\}$$

4. Let $L \subseteq \Sigma^*$. Show that if L is a regular language, than so is $\mathit{Pref}(L)$, the language containing all the prefixes of the words of L. Formally:

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5. Let $M = (Q, \Sigma, \Delta, s_0, F)$ be a deterministic finite automaton and L the language accepted by this automaton.
How can M be modified in order to obtain a non-deterministic finite automaton M' that accepts $L^R = \{w \mid w^R \in L\}$, where w^R is the mirror of the word w. (Example: $w = abc$ and $w^R = cba$).

Bonus Exercise 2

Let $L \subseteq \Sigma^*$. Show that if L is a regular language, than so is $\overline{L}$, the the complement of L.
Formally:

$$\overline{L} = \{w \in \Sigma^* \mid w \notin L\}$$