Introduction to computability
Tutorial 1

Regular Expressions and Denumerable Sets

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Regular expressions

1. Let $R$ and $S$ be the following regular expressions:

$$R = a^* \cup b^*$$

$$S = ab^* \cup ba^* \cup b^*a \cup (a^*b^*)^*$$

(a) Find a word that belongs to $L(S)$ but not to $L(R)$.
(b) Find a word that belongs to both $L(R)$ and $L(S)$.
(c) Find a word that belongs to $L(R)$ but not to $L(S)$.
(d) Find a word that belongs neither to $L(R)$ nor to $L(S)$. 
2. Determine whether the following statements are correct or not on the alphabet $\Sigma = \{a, b\}$:
(a) $aab \in L(b^*a^*b^*a^*)$;
(b) $L(\emptyset^*) = L(\epsilon)$;
(c) $(L_1 \cup L_2)^* = L_1^* \cup L_2^*$;
(d) $(L_1.L_2)^* = L_1^*.L_2^*$.

3. Give a regular expression of the following languages (defined on $\Sigma = \{a, b\}$):
(a) the language whose words contain an odd number of $a$’s;
(b) the language whose words contain exactly once the factor $aaa$;
(c) the language whose words do not end with $b$;
(d) the language whose words contain an even number of symbols.
Denumerable sets

4. Are the following sets denumerable?
(a) The set \( \mathbb{Z} \) containing all integers.
(b) The set \( \mathbb{N} \times \mathbb{N} \) of all pairs.
(c) The set \( 2^{\mathbb{N}} \) of all subsets of \( \mathbb{N} \).
(d) The set of all polynomials with integer coefficients.

5. Show that
(a) every infinite subset of a denumerable set is itself denumerable;
(b) the difference between a non-denumerable and a denumerable set is not denumerable.
Bonus Exercise 1

Are the following sets denumerable?
(a) The set $\mathbb{Q}$ containing all rational numbers.
(b) The set $\mathbb{I} = \mathbb{R} \setminus \mathbb{Q}$ containing all irrational numbers.