Introduction to computability

28 October 2013

Open-book. Duration : 1h30

Please answer each question on a separate sheet where you indicate your name and section. Justify and detail all your answers.

1. a) Is the set $\{2^n \mid n \in \mathbb{N}\}$ denumerable ?

   b) Give a set $S \subseteq \mathbb{R}$ such that S is not denumerable and $\mathbb{R} \setminus S$ is denumerable.

2. Let L be the language generated by the grammar

$$
\begin{aligned}
S &\rightarrow AbaBS \mid \varepsilon \\
A &\rightarrow aA \mid a \\
B &\rightarrow bB \mid \varepsilon.
\end{aligned}
$$

   where S is the start symbol. Give a regular grammar that generates L and one that generates its complement.

3. Show that the language $L = \{a^i b^j c^k \mid k \neq i \cdot j\}$ is not regular.
   (Hint : Use the pumping lemma)

4. Is the language $L = \{a^i b^j c^k d^l \mid i + k = j + l\}$ context-free ?
   Explain your intuition and then prove your statement.