Introduction to the Theory of Computation

Final exam

16 August 2017

Closed-book. Duration: 3h30.
Please answer each question on a separate sheet with your name and section. ***Motivate all your answers and give sufficient details.***

1. a) Is the set of all functions from $\{0, 1\}$ to $\mathbb{N}$ denumerable?

   b) Is the set of all functions from $\mathbb{N}$ to $\{0, 1\}$ denumerable?

2. Let L be the language of the words w over the alphabet $\{0, 1\}$ that respect at least one of the following conditions:

   - w contains the substring 010
   - w does not contain the substring 11.

   a) Give a NFA that accepts L.

   b) Give a DFA that accepts L.

   c) Give a regular grammar that generates L.

3. a) Let L be the language of all words w over the alphabet $\{0, 1\}$ such that the length of w is odd and the middle symbol of w is a 0. Is L regular? Is it context-free?

   b) Is it correct that every regular language is also a context-free language? And vice-versa, is every context-free language also a regular language?

4. a) Show that $L = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geqslant 0\}$ is a context-free language by giving a pushdown automaton that accepts L as well as a context-free grammar that generates L.

   b) State the pumping lemma for context-free languages.

5. a) For Turing machines, define the notions of *configuration, derivation, execution, accepted language* and *decided language.*

   b) State the Church-Turing Thesis. What type of justification can be given for this thesis?

6. a) Show that NbDivs(n) that computes the number of divisors of n is primitive recursive. Is NbDivs μ-recursive?
   Hint: Use an auxilary function NbDivsAux(n, m) that computes the number of divisors of n that are less than or equal to m.

   b) Do there exist computable functions that are not primitive recursive? Justify your answer.

7. a) Let M_1 and M_2 be two Turing Machines that accept the languages L_1 and L_2 respectively. Show that determining whether there exists $w \in L_1$ such that M_2 stops on w is undecidable.

   b) Why are the languages accepted by a Turing machine also called "recursively enumerable"? Prove your statement.

8. a) Define the complexity class NP and the complexity measures used in this definition.

   b) State Cook's theorem. In the proof of Cook's theorem, which problem is encoded by a boolean formula?