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INFO-0064 Embedded systems

Exercise session 9

Hybrid Systems

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Planning for today

- Hybrid systems
- Exercises

Hybrid systems

Hybrid systems are a modeling formalism that is able to express:

- Operations on integer and real variables
- Discrete state transitions as well as continuous evolution laws
- Composition of elementary systems into a more complex entity

Hybrid systems

A hybrid system is composed of

- a finite number p of **processes** $P_1, ..., P_p$
- a finite number n of **variables** $x_1, ..., x_n \Rightarrow \vec{x} \in \mathbb{R}^n$
- a finite set L of **synchronization labels**

Each process P_i is represented by a graph (V_i, E_i) where

- V_i is a finite set of **control locations**
- $E_i \subseteq V_i \times V_i$ is a finite set of **transitions**

Hybrid systems

Each control location $\nu \in V_i$ is associated with:

- An activity $dif(\nu)$ = linear constraints over $x_1, ..., x_n$ and $\dot{x_1}, ..., \dot{x_n}$
- An invariant $inv(\nu)$ = linear constraints over $x_1, ..., x_n$

Each transition $e \in E_i$ is associated with:

- A guard $guard(e)$ = condition to satisfy to enable this transition
- An action $act(e)$ = constraints specifying how variables are changed by following this transition
- An optional label $sync(e) \in L$ used to synchronize this transition with a transition of another process

Exercise 1

This exercise examines a water tank controlled by a microcontroller (μc). This μc monitors the water level and controls it by switching on or off a water pump.

We know that:

- when the pump is switched off, the water level falls of 4cm/minute;
- when the pump is switched on, the water level raises of 2cm/minute;
- there is a 2 seconds delay between the moment where the pump is ordered to start and when it actually starts working.

To control the water level, the μc shall order the pump to start when the level is below 10 cm, and to stop when the level is above 20cm.

1. Model the controller using a complex timed system;
2. Run the model (as far as at least 3 control locations); initially, the water level is 2cm and the pump is working..

Exercise 2

In a chemical factory, a robotized system is used to mix 2 products, P_1 and P_2. Each product is contained in a pipette of 80ml and flows out with a constant manner.

The flow out rate of P_1 is of 1 ml/s and that of P_2 is of 2 ml/s. The system uses a mobile arm which is able to move above each pipette and to fill it with new product. When the level of P_1 or P_2 is lower than 20 ml, the arm moves in order to fill the appropriate pipette. This move takes 2 seconds at the most and can only be done if the arm is free. Then, the arm pours product with a 10 ml/s rate, until the level reaches 60 ml. At that moment, the arm becomes free for another movement.

1. Model the controller using a complex timed system.
2. Run the model (as far as at least 3 control locations); initially, both pipettes are full.

Exercise 3

We now study the functioning of an electrical oven.

In this oven, When the heating resistors are turned on:

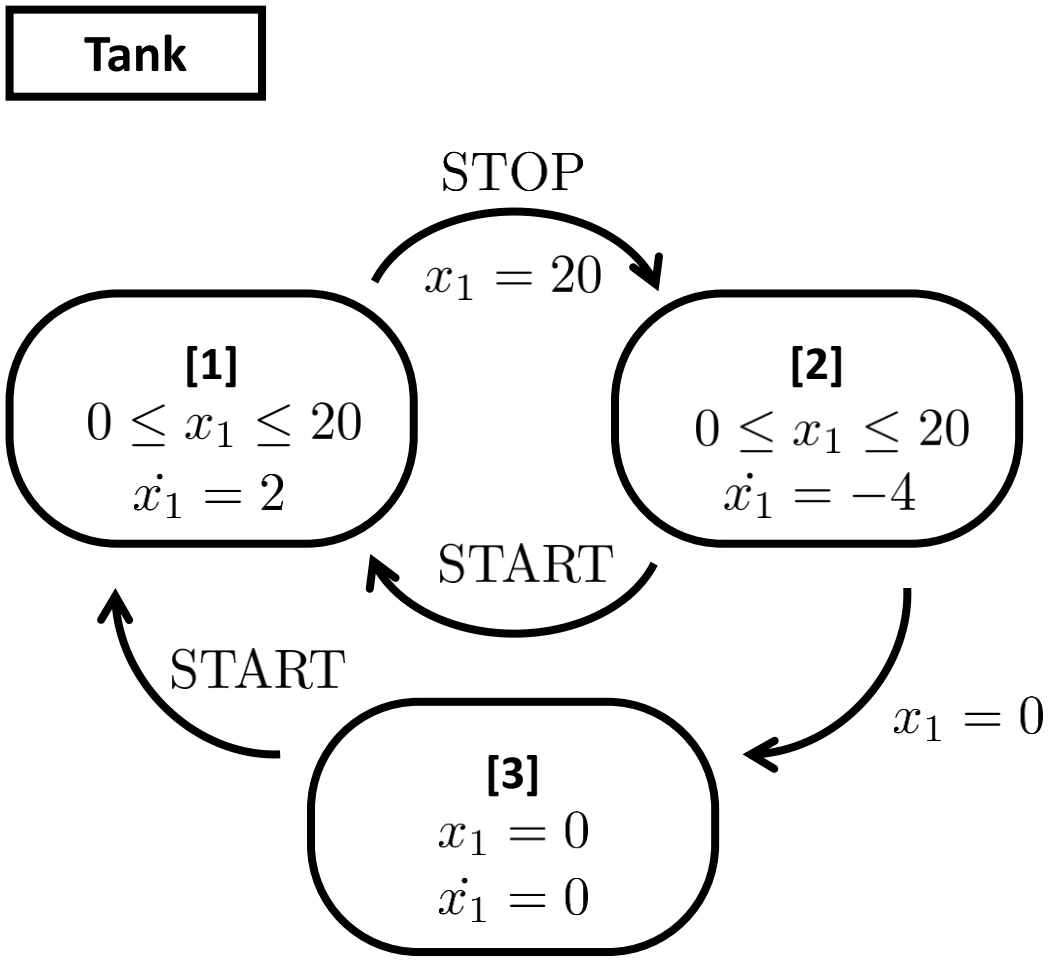
- if the internal temperature is below 150 degrees, it raises of 5 degrees per minute;
- if the internal temperature is between 150 and 170 degrees, it raises of 2 degrees per minute;
- if the internal temperature is above 170 degrees, it raises of 1 degrees per minute.

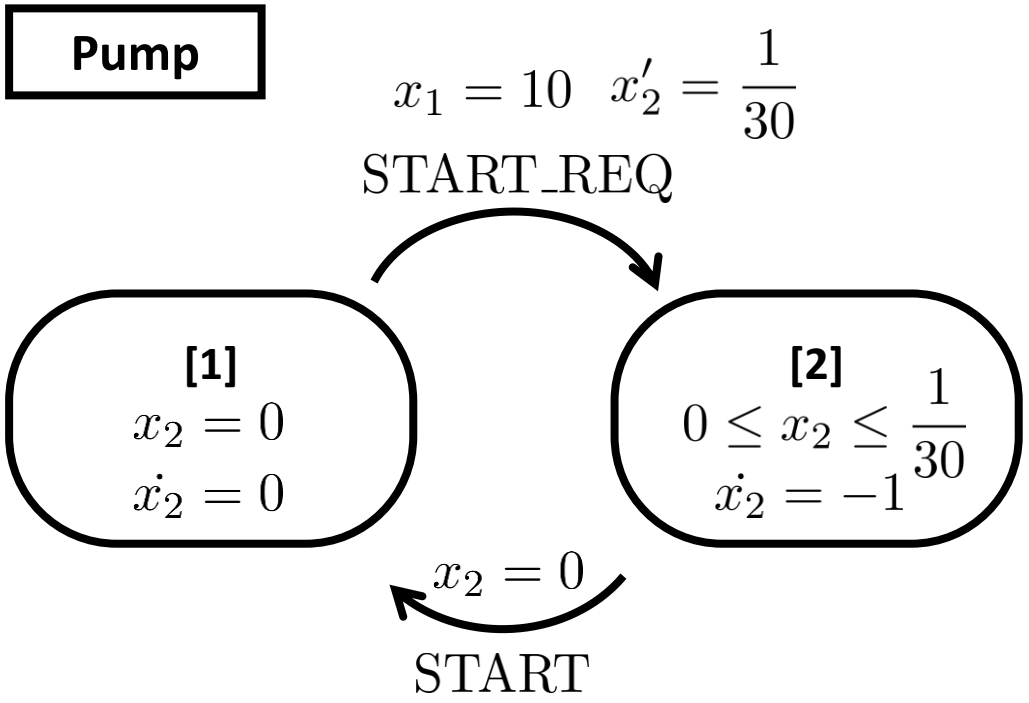
When the resistors are turned off, temperature falls of 0.3 degree per minute, until it reaches the room temperature which is 20 degrees.

The user can select a temperature between $[20, 200]$ degrees by using a rotary knob (T_{ins}). The microcontroller compares this instruction with the temperature inside the oven (T_{meas}). The resistors are switched on when T_{meas} is lower than ($T_{ins} - 5$ degrees). They are turned off when T_{meas} is above ($T_{ins} + 5$ degrees).

1. Model the controller using a complex timed system.
2. Run the model (as far as at least 3 control locations); initially, the oven is at room temperature.

Exercise 1: solution

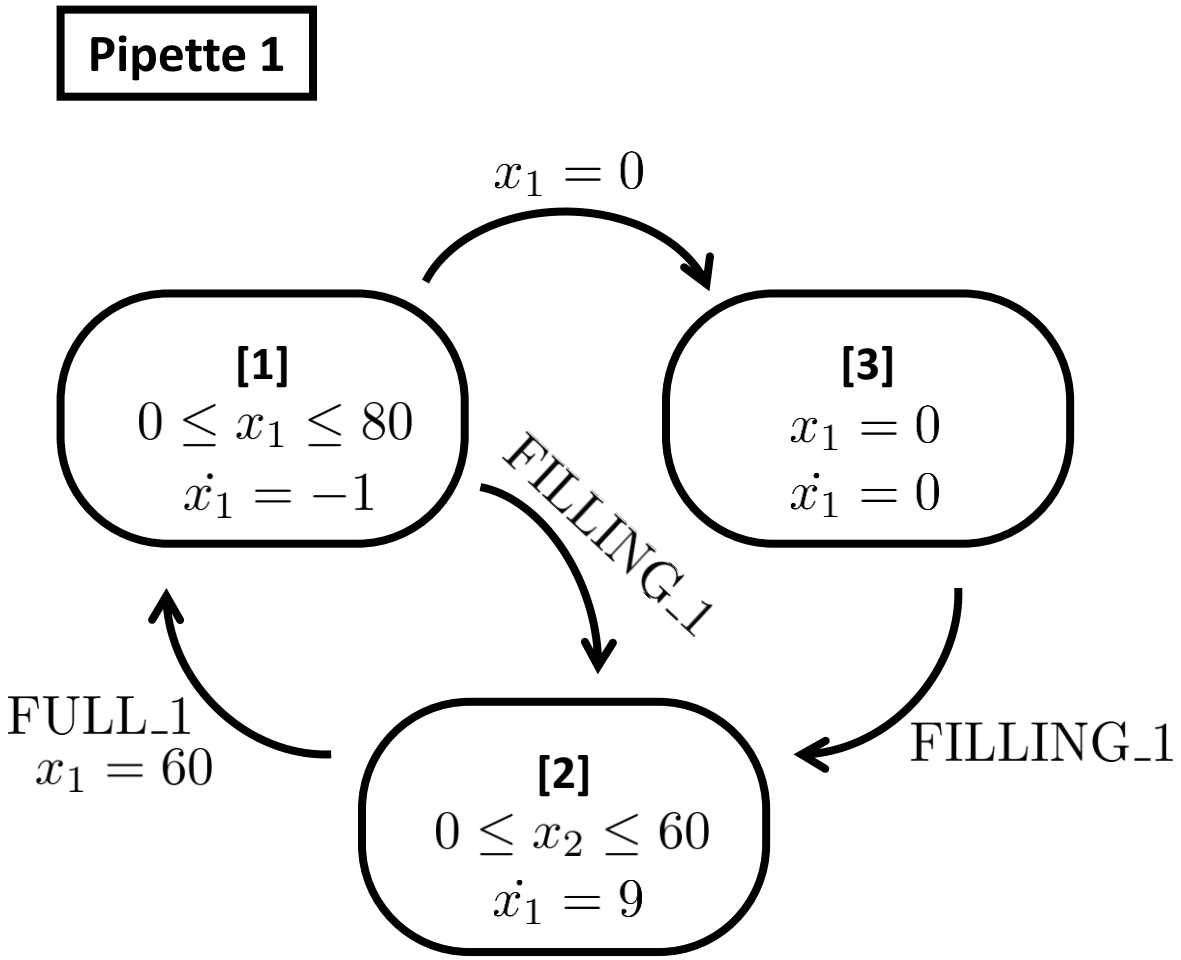


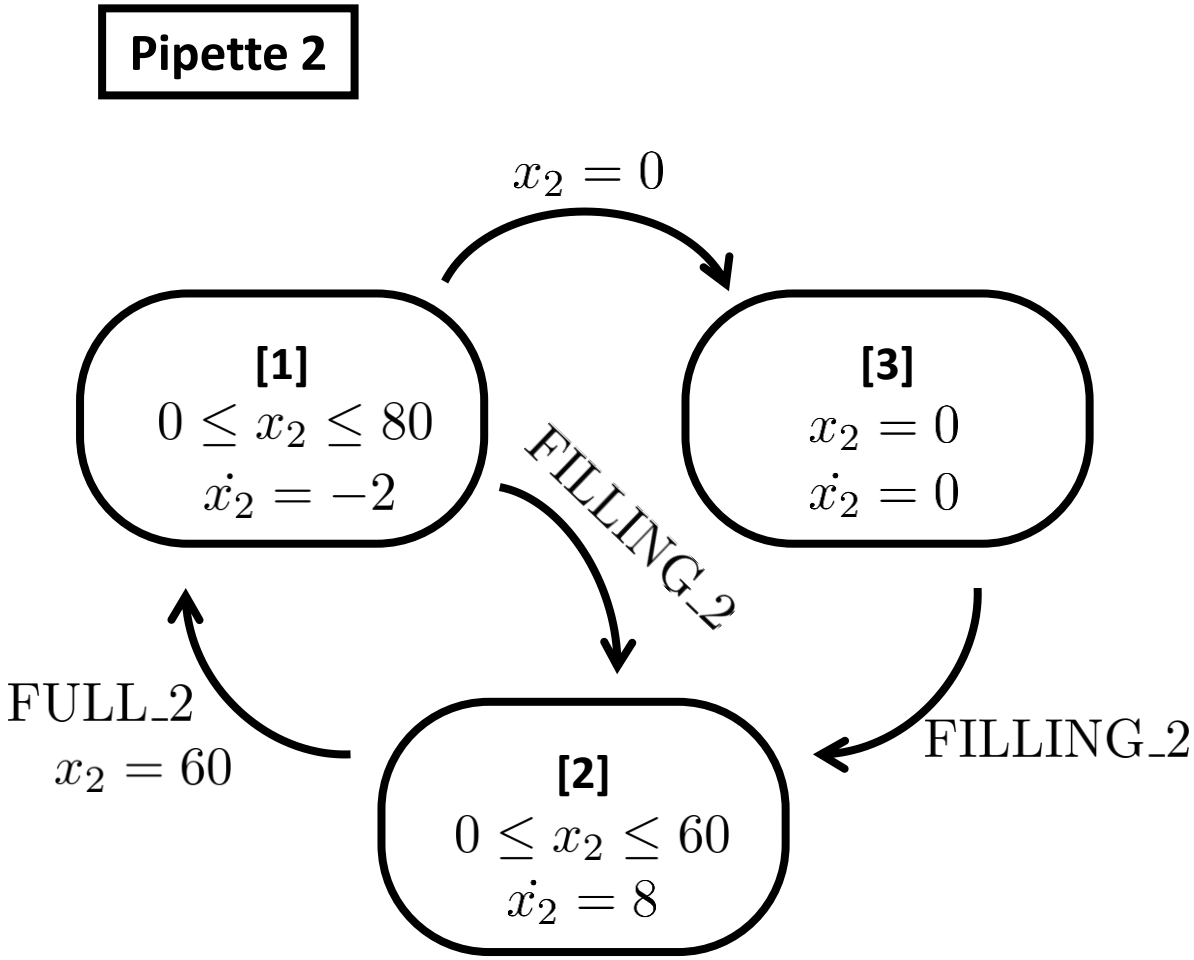


Exercise 1

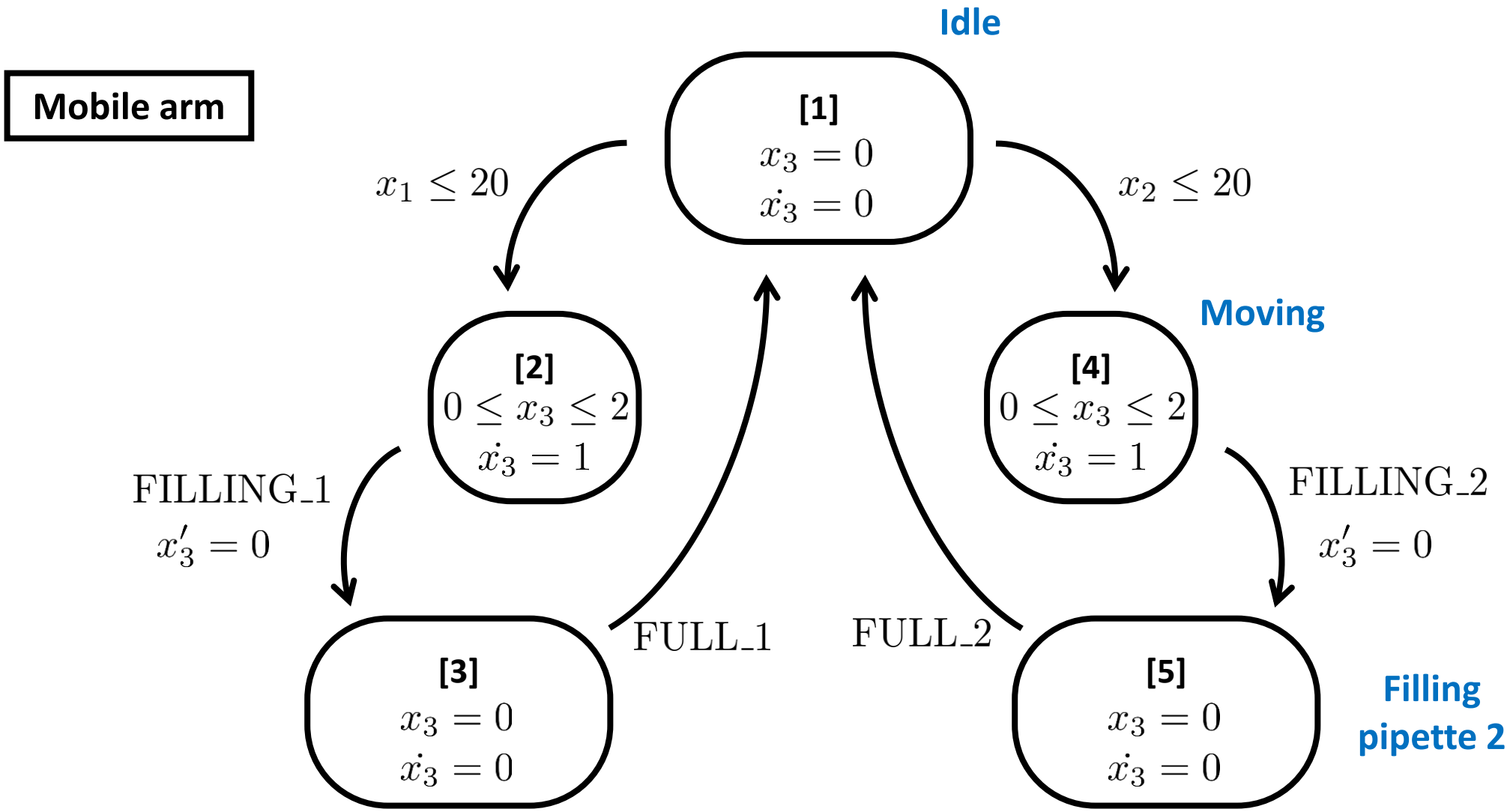
$$
\begin{array}{rlcl}
 & ([1],[1]) & : & x_1 \;=\; 2,\; x_2 \;=\; 0 \\
\Longrightarrow & ([1],[1]) & : & 0 \le x_1 \le 20, \\
 & & & x_2 \;=\; 0 \\
\xrightarrow{STOP} & ([2],[1]) & : & x_1 \;=\; 20, \\
 & & & x_2 \;=\; 0 \\
\Longrightarrow & ([2],[1]) & : & 0 \le x_1 \le 20, \\
 & & & x_2 \;=\; 0 \\
\xrightarrow{START\_REQ} & ([2],[2]) & : & x_1 \;=\; 10, \\
 & & & x_2 \;=\; \frac{1}{30} \\
\Longrightarrow & ([2],[2]) & : & 0 \le x_1 \le 20, \\
 & & & 0 \le x_2 \le \frac{1}{30}, \\
\xrightarrow{START} & ([1],[1]) & : & x_1 \;=\; 10 - \frac{4}{30} \\
 & & & x_2 \;=\; 0
\end{array}
$$

Exercise 2: solution





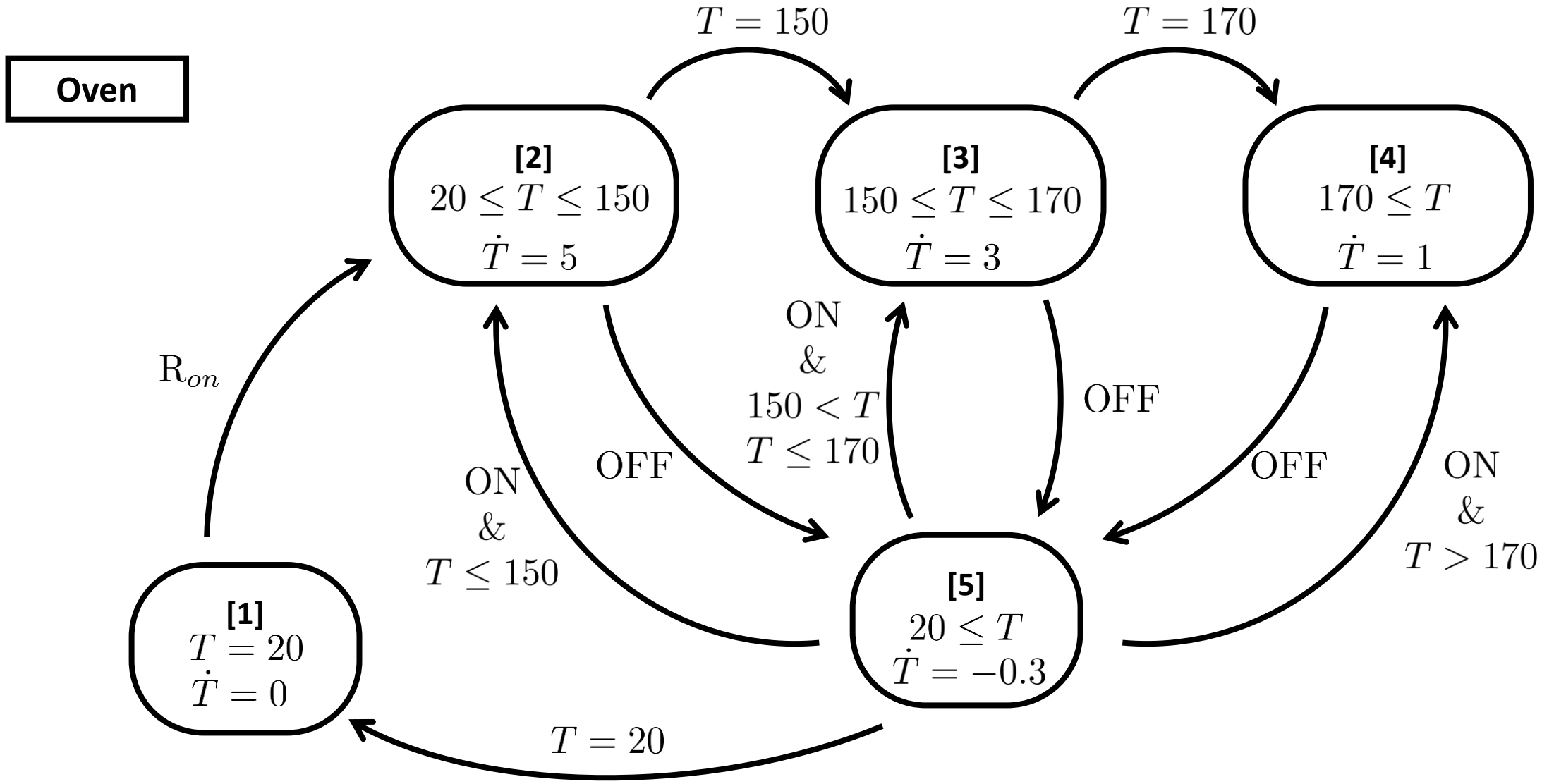
Exercise 2: solution



Exercise 2: solution

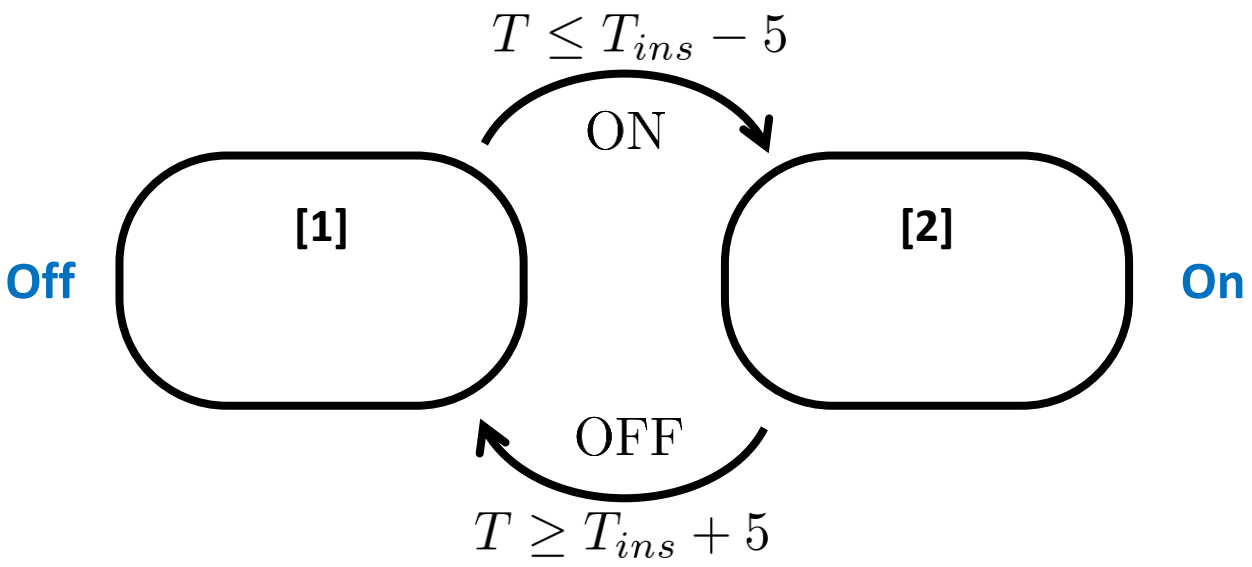
$$
\begin{array}{lll}
 & ([1],[1],[1]) & : x_1 \;=\; 80,\; x_2 \;=\; 80,\; x_3 \;=\; 0 \\
\Longrightarrow & ([1],[1],[1]) & : \; 20 \leq x_1,\; 20 \leq x_2,\; x_3 \;=\; 0 \\
\xrightarrow{x_2=20} & ([1],[1],[4]) & : \; x_1 \;=\; 50,\; x_2 \;=\; 20,\; x_3 \;=\; 0 \\
\Longrightarrow & ([1],[1],[4]) & : \; 50-\mu \leq x_1,\; 20-2\mu \leq x_2,\; x_3 = \mu \\
 & & \quad 0 \leq \mu \leq 2 \\
\xrightarrow{FILLING\_1} & ([1],[2],[5]) & : \; x_1 \;=\; 50-\mu,\; x_2 \;=\; 20-2\mu,\; x_3 \;=\; 0 \\
 & & \quad 0 \leq \mu \leq 2 \\
\Longrightarrow & ([1],[2],[5]) & : \; 0 \leq x_1 = 50-\mu,\; x_2 = 20-2\mu \leq 60,\; x_3 = 0 \\
 & & \quad 0 \leq \mu \leq 2
\end{array}
$$

Exercise 3: solution



Exercise 3: solution

Resistor



Exercise 3: solution

$$T_{ins} = 163$$

$$
\begin{array}{rlcl}
 & ([1],[1]) & : & T = 20 \\
\xrightarrow{ON} & ([2],[2]) & : & T = 20 \\
\Longrightarrow & ([2],[2]) & : & T \leq 150 \\
\xrightarrow{T=150} & ([3],[2]) & : & T = 150 \\
\Longrightarrow & ([3],[2]) & : & T \leq T_{ins} + 5 \\
\xrightarrow{OFF} & ([5],[1]) & : & T = 168 \\
\Longrightarrow & ([5],[1]) & : & T \geq T_{ins} - 5 \\
\xrightarrow{ON} & ([3],[2]) & : & T = 158 \\
 & & \vdots &
\end{array}
$$