Advanced computer programming

Exercise session 7: Problem solving II

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Exercice 1

Let $G = (V, E, W)$ be a weighted graph, where V is the set of vertices, $E \subseteq V \times V$ be the set of edges and $W : E \to \mathbb{R}_+$ be the weighting function of the graph. How can we build a minimum spanning tree. i.e., a tree (connected acyclic graph) $T = (V, E', W)$ with $E' \subseteq E$ such that the sum of the selected edges is the smallest possible.

Exercise 2

Let B be a boolean expression composed of the symbols `true`, `false`, `and` and `or`. Give a algorithm which determines the number of ways to parenthesize B such that it is evaluated to `true`.

Exercise 3 (adapted from CLRS, 16.2-4)

Julien is going to take part in a race which consists in going from town A to town B. Considering the capacity of his can, he knows that he make as far as m meters without running out of water. Consequently, he would like to plan his stops beforehand. Quite logically, he want to minimize them. To help him in his decision, he disposes of a list a refilling location. How Julien should proceed ? What is the complexity of your solutions ?

Exercise 4

Task scheduling comes up often in the life of a computer scientist : from a set $S = \{s_1, s_2, ..., s_n\}$ of n tasks, we need to produce an optimal sequence of tasks such as to minimize the average response time. Each task s_i is associate to an execution time t_i. The response time is the time ranging from the start till the end of the given task.

For instance, let us consider the case of $S = s_1, s_2$ with execution times $t_1 = 3$ and $t_2 = 5$. If there are ordered $< s_1, s_2 >$, the response time C_i of each task would be:

- $C_1 = t_1 = 3$
- $C_2 = C_1 + t_2 = 3 + 5 = 8$

and consequently, the mean response time would be $\bar{C} = \frac{1}{2}C_1 + C_2 = 5.5$.

(a) What would be the complexity of a brute-force algorithm for this task ?

(b) Can we do better ? (*i.e.* does the problem benefit from a substructure property ?)

(c) In that case, propose an efficient algorithm for this problem.

Exercise 5

Draw the recursion tree of MergeSort for the following problem:

$$A =< 5, 2, 4, 7, 1, 3, 2, 6 >$$

Why memoization is not helpful for this problem ?

Exercise 6 (adapted from CLRS, 15-2)

Give a efficient pseudo-code for finding the longest palindromic subsequence within a word.

Exercise 7 (adapted from CLRS, 15.5)

Formulate the recurrence for building the optimal binary search tree by dynamic programming once the search frequency is known for each key.