1 Section 2.1 – Exercise 8 (Strauss)

The "spherical wave equation" is given by

$$u_{tt} = c^2 \left(u_{rr} + \frac{2}{r} u_r \right). \tag{1}$$

(a) Using the change of variables $v = ru$, show that 1 can be written as $v_{tt} = c^2 v_{rr}$,

(b) Find the general solution of the spherical wave equation,

(c) Using the initial conditions $u(r,0) = \phi(r)$, $u_t(r,0) = \psi(r)$, solve the spherical wave equation.

2 Section 2.1 – Exercise 10 (Strauss)

Solve the equation $u_{xx} + u_{xt} - 20u_{tt} = 0$ using the auxiliary conditions $u(x,0) = \phi(x)$ and $u_t(x,0) = \psi(x)$.

3 Section 2.2 – Exercise 5 (Strauss)

The equation of motion for a damped string is given by (Strauss, Eq. 1.3.3)

$$u_{tt} - c^2 u_{xx} + r u_t = 0, \quad r > 0.$$

Show that, in this case, the energy decreases.

4 Inhomogeneous wave equation (Bonus)

(Exercise 2.4.11 from P. Olver, *Introduction to Partial Differential Equations*, Springer, 2014)

(a) Solve the initial value problem

$$\partial_{tt} u - c^2 \partial_{xx} u = 0, \quad u(x,0) = \sin(x), \quad \partial_t u(x,0) = \cos(x), \quad -\infty < x < \infty, \quad t \geq 0.$$

(b) Is $u(x,t)$ a periodic function of t ?

(c) Solve the forced initial value problem

$$\partial_{tt} u - c^2 \partial_{xx} u = \cos(\omega t), \quad u(x,0) = \sin(x), \quad \partial_t u(x,0) = \cos(x), \quad -\infty < x < \infty, \quad t \geq 0.$$

(d) Does the solution exhibits resonance ?

(e) What would happen if the forcing function is $\sin(\omega t)$ instead of $\cos(\omega t)$?