Discontinuous Galerkin Method

MATH0471 – Spring 2019

v1 (04/02/2019)

This project consists in studying a hyperbolic system of equations in its conservation form. Spatial discretization will be performed using the Discontinuous Galerkin (DG) method and Lagrange nodal basis functions on unstructured meshes. An explicit time-marching method will be chosen. The book of J.S. Hesthaven and T. Warburton entitled *Nodal Discontinuous Galerkin Methods*[1] (Springer 2008) will be the main reference for this project.

Since the DG method requires a more elaborated mesh data structure than the classical finite element method, the numerical scheme will be implemented with the help of the **gmsh SDK**[2] so that meshing, computation of surface integrals and postprocessing of results will be simplified.

The numerical scheme will be first developed, tested and validated for the solution of a scalar transport equation. Then, a more complex (possibly nonlinear) system of equations will be investigated in a second phase. Target applications include shallow water problems, propagation of electromagnetic waves, gas dynamics, ice sheet modeling, advection-diffusion problems (octopus spitting ink into the sea), ...

The project is organized as follows:

1. Students will be divided in 3 groups. Each group will write its own solver.
2. Three intermediate deadlines are given, with a mandatory (but not graded) 8-page progress report that should detail the computer implementation and the mathematical, numerical and physical experiments.
3. The final report (about 60 pages) will present the method and numerical results, the computer implementation and a detailed analysis of physical experiments on non-trivial configurations.
4. An oral presentation of the main project results will be organized during the June exam session (June 4th); individual theoretical and practical questions will be asked to each member of the 3 student groups.

[1] https://link.springer.com/book/10.1007%2F978-0-387-72067-8
[2] http://gmsh.info

The important dates of the project are:

1. **March 6th: Intermediate deadline #1: DG method for a scalar transport equation**: Implementation of the DG method using Lagrange shape functions of arbitrary order for a scalar conservation equation in 2D. The implementation should take advantage of the gmsh library for reading the mesh, computing values of shape functions and jacobians, as well as writing results to files. At this stage, the code is sequential and the time integration is performed using a simple forward-Euler method.

2. **March 19th:** Discussion about the final application for each group. It could be the solution of one of the target applications modeled by a coupled system of 2D equations, or the extension of the code to 3D for handling large-scale problems with MPI.

3. **March 20th: Intermediate deadline #2: Improvement and optimization of the solver**: Extension to a system of n uncoupled transport equations. Time integration is now performed with a 4th order Runge Kutta. The algorithm must be parallelized using OpenMP.

4. **May 1st: Intermediate deadline #3: Extension to a realistic problem**: Modification of the code for solving the target application.

5. **May 17th: Final deadline:** Final report and code.

6. **June 4th: Exam:** Oral presentation of the projects.

The full C/C++ code (in a single ZIP archive with a corresponding reference to the code on Github, directly configurable and compilable on the NIC4 CECI cluster with CMake) should be sent before each deadline to both `cgeuzaine@uliege.be` and `r.boman@uliege.be`.