ELEC0431 Electromagnetic Energy Conversion
Corrective of Exercises

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Exercise 9: Single-phase autotransformer

1) $m = 5$
2) $R_l = 50\ \Omega$; $X_m = 6.3\ \Omega$
3) $R_s = 0.24\ \Omega$; $X_s = 0.32\ \Omega$
4) $\Delta U_2 = 4.61$ V; $U_2 = 95.2$ V
5) $P_2 = 916$ W
6) $I_1 = 62.3$ A
7) $\eta = 95.6$ %
8) $U_1' = 80$ V
9) $I_{1o}' = 0.8$ A
10) $R_s' = 15\ m\Omega$; $X_s' = 20\ m\Omega$
11) RC = 2.39 ms
12) $R_m = 5.41\ \Omega$; $\eta_m = 99.2$ %

4 AC synchronous machines

Exercise 10: Constant air gap alternator

1. Physical explanations and advantages

   The three synchronous alternators are fixed to the same shaft.

   The permanent magnet alternator is the smallest of the three machines (regarding power) and produces some three-phase current alternating current as it rotates.
   The current produced is then rectified (it becomes DC current) and used for the excitation of the second machine.

   The second machine (here the inversed alternator) is used to increase the power between the small permanent magnet alternator and the main alternator.

   It is called an inversed alternator because the fixed part contains the excitation winding (DC current) (normally in the rotor for a conventionnal machine) and the moving part contains the three-phase windings (normally in the stator for a conventionnal machine).

   The three-phase currents are then rectified and used for the excitation of the main alternator. Remark that the rectifier is also rotating along the inversed alternator.

   The main alternator rotor is then excited with the excitation current I_e and the mechanical power applied on the shaft is transfered into three-phase electrical power in the conductors (a, b, c, n).

   Advantages :

   The permanent magnet machine allows an autonomous start (no excitation current is required for this machine). Also, this machine is brushless, meaning that no spark are produced when it is rotating. This is an important point for the design of safe aircraft. A brushed DC generator could not be used instead of the permanent magnet alternator because it would create sparks.

   The electrical connections of the inversed alternator are simpler and do not require any connecting ring either for the excitation winding or the three-phase windings

2. Express f in terms of $\dot{\theta}_e$, k_m and p

   $$\dot{\theta} = k_m\, \dot{\theta}_e \tag{1}$$

   where $\dot{\theta}$ is the machine synchronous speed ; $k_m = 2{,}67$ is the gearbox ratio ; $\dot{\theta}_e$ is the aircraft reactor speed.

   Then,

   $$f = p\, \dot{\theta} = p\, k_m\, \dot{\theta}_e \qquad (\dot{\theta} \text{ and } \dot{\theta}_e \text{ expressed in turn/s}) \tag{2}$$

   $$f = p\, \frac{\dot{\theta}}{60} = \frac{p\, k_m\, \dot{\theta}_e}{60} \qquad (\dot{\theta} \text{ and } \dot{\theta}_e \text{ expressed in rpm}) \tag{3}$$

   $$f = \frac{p\, \dot{\theta}}{2\,\pi} = \frac{p\, k_m\, \dot{\theta}_e}{2\,\pi} \qquad (\dot{\theta} \text{ and } \dot{\theta}_e \text{ expressed in rad/s}) \tag{4}$$

3. Deduce p, f_{min} and f_{max}

The alternator frequency is 370 Hz when its rotation speed is 11 000 rpm. 11 000 rpm = 183,3 turn/s. Then, $p = \frac{f}{\dot{\theta}} = \frac{370}{183{,}3} = 2,018$ and the number of pair of poles $p = 2$.

$\dot{\theta}_{e,min} = 4160$ rpm and $\dot{\theta}_{e,max} = 9000$ rpm. Then,

$$f_{min} = \frac{p\, k_m\, \dot{\theta}_{e,min}}{60} = 370,24Hz \tag{5}$$

$$f_{max} = \frac{p\, k_m\, \dot{\theta}_{e,max}}{60} = 801Hz \tag{6}$$

4. Justify the relevance of working at variable frequency

- The high frequency (higher the industrial 50 Hz) enables the use of smaller components (L and C).
- The mechanic is easier, there is only one gearbox with one ratio.
- The rotation speed of the reactor can vary even if there are only two pairs of poles.

5. Nominal current I_{sn}

$$I_{sn} = \frac{S_n}{3\,V} = \frac{150\,000}{3 \cdot 115} = 434,783 \text{ A} \tag{7}$$

6. Express $e_s(t)$, the emf and deduce E_s

Considering $\dot{\theta}$ in rad/s,

$$\phi = \Phi_M \;\cos\big(p(\dot{\theta}\,t - \theta_0)\big) \tag{8}$$

$$e_s(t) = -\frac{d\phi}{dt} \tag{9}$$

$$= p\,\dot{\theta}\;\Phi_M\;\sin\big(p(\dot{\theta}\,t - \theta_0)\big) \tag{10}$$

$$= 2\pi f\;\Phi_M\;\sin\big(p(\dot{\theta}\,t - \theta_0)\big) \tag{11}$$

$$= E_m\;\sin\big(p(\dot{\theta}\,t - \theta_0)\big) \tag{12}$$

$$\tag{13}$$

and

$$E_s = \frac{E_m}{\sqrt{2}} = \sqrt{2}\pi\, f\, \Phi_M \tag{14}$$

7. Coil factor k_b, induced emf E and E wrt I_e

The coil factor k_b is a global coefficient that takes some non idealities into account, such as : the leakage flux between the rotor and stator, the angular section of the turns of a phase.

$$E = k_b\; N_s\; E_s \tag{15}$$

with

$$E_s = \sqrt{2}\;\pi\; f\;\Phi_M \tag{16}$$

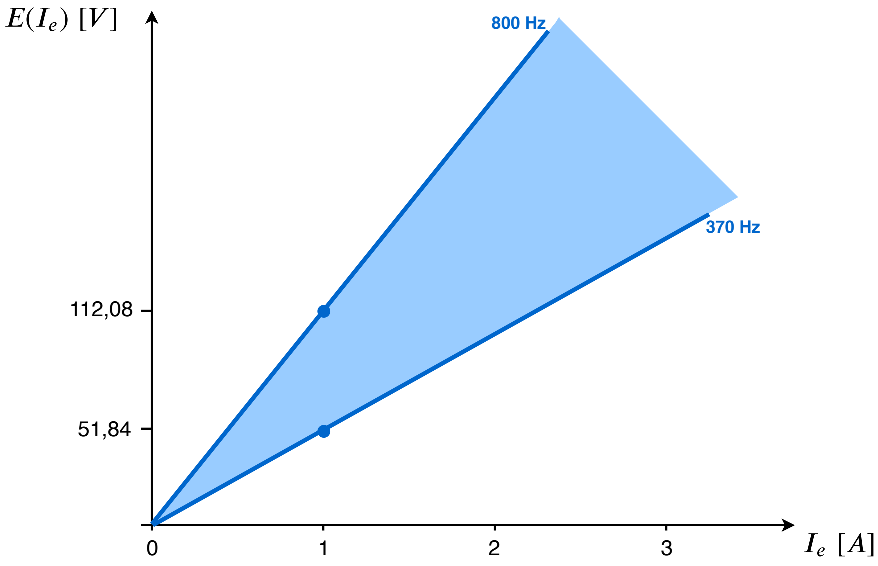


Figure 1: Emf as a function of the excitation current.

Also, in the linear range,

$$\Phi_M = \frac{\Phi_{M0}}{I_{e0}} I_e \tag{17}$$

Then,

$$E = k_b\ N_s\ \sqrt{2}\ \pi\ f\ \frac{\Phi_{M0}}{I_{e0}}\ I_e \tag{18}$$

$$= 0.85\ 16\ \sqrt{2}\ \pi\ 370\ \frac{0,00684}{2,95}\ 1 = 51,84\, V\ \ (\text{for } f_{min}\text{=370 Hz}) \tag{19}$$

$$= 0.85\ 16\ \sqrt{2}\ \pi\ 800\ \frac{0,00684}{2,95}\ 1 = 112,08\, V\ \ (\text{for } f_{max}\text{=800 Hz}) \tag{20}$$

8. Total fluxes $\Psi_{a,b,c}$

$$\Psi_a = L_s\ i_a + M_s\ i_b + M_s\ i_c + M_{af}\ I_e \tag{21}$$

$$= L_s\ i_a + M_s\ i_b + M_s\ i_c + M\ \cos(p\,\theta)\ I_e \tag{22}$$

$$\tag{23}$$

$$\Psi_b = M_s\ i_a + L_s\ i_b + M_s\ i_c + M_{bf}\ I_e \tag{24}$$

$$= M_s\ i_a + L_s\ i_b + M_s\ i_c + M\ \cos(p\,\theta - \frac{2\pi}{3})\ I_e \tag{25}$$

$$\tag{26}$$

$$\Psi_c = M_s\ i_a + M_s\ i_b + L_s\ i_c + M_{cf}\ I_e \tag{27}$$

$$= M_s\ i_a + M_s\ i_b + L_s\ i_c + M\ \cos(p\,\theta - \frac{4\pi}{3})\ I_e \tag{28}$$

9. $\nu_{a,b,c}$

$$\nu_a = -R_s\ i_a\ - \frac{d\Psi_a}{dt}\quad ;\quad \nu_b = -R_s\ i_b\ - \frac{d\Psi_b}{dt}\quad ;\quad \nu_c = -R_s\ i_c\ - \frac{d\Psi_c}{dt} \tag{29}$$

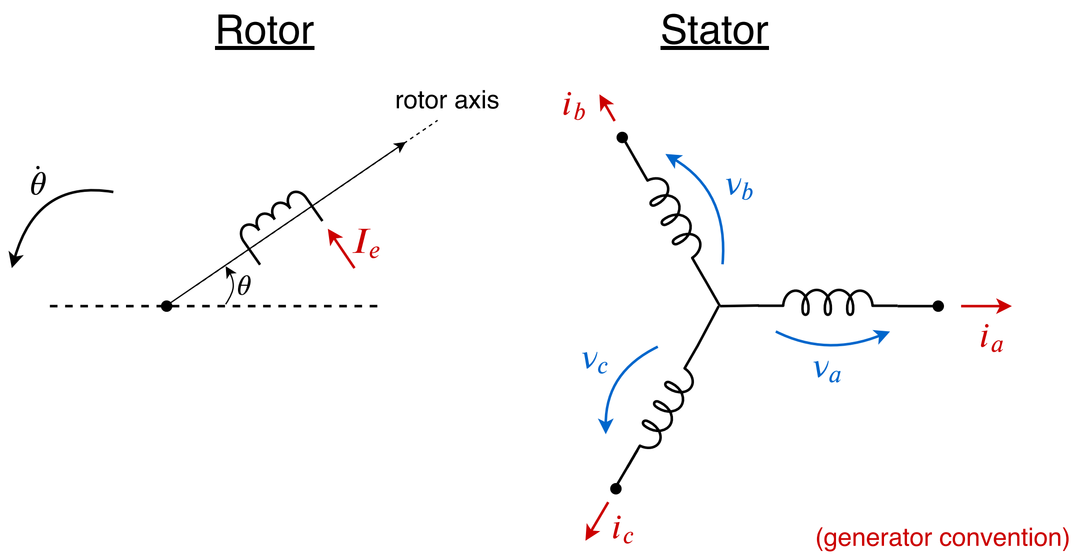


Figure 2: Rotor and stator windings.

With R_s the resistance of each phase and $\Psi_{a,b,c}$ the flux linkage of each phase.

10. Show that ...

$$\Psi_a(t) = L_s\, i_a(t) + M_s\, i_b(t) + M_s\, i_c(t) + M\ \cos(p\,\dot{\theta}\,t)\ I_e \tag{30}$$
$$= L_s\, i_a(t) + M_s\,\big(i_b(t) + i_c(t)\big) + M\ \cos(p\,\dot{\theta}\,t)\ I_e \tag{31}$$

With $\quad i_b(t) + i_c(t) = -i_a(t) \quad <=> \quad i_a(t) + i_b(t) + i_c(t) = 0$

Then,

$$\Psi_a(t) = (L_s - M_s)\, i_a(t) + M\ \cos(p\,\dot{\theta}\,t)\ I_e \tag{32}$$
$$= \quad \mathcal{L} \quad i_a(t) + M\ \cos(p\,\dot{\theta}\,t)\ I_e \tag{33}$$
$$\tag{34}$$
$$-\frac{d\Psi_a(t)}{dt} = -\mathcal{L}\,\frac{di_a(t)}{dt} + p\,\dot{\theta}\,M\ \sin(p\,\dot{\theta}\,t)\ I_e \tag{35}$$
$$-\frac{d\Psi_a(t)}{dt} = -\mathcal{L}\,\frac{di_a(t)}{dt} + e_a \tag{36}$$

is obtained by defining, $\mathcal{L}$: the cyclic impedance and e_a : the internal emf of phase a. Finally, one can get,

$$\nu_a = -R_s\, i_a(t)\ - \frac{d\Psi_a(t)}{dt} = e_a\ - R_s\, i_a(t) - \mathcal{L}\,\frac{di_a(t)}{dt} \tag{37}$$

The same derivation remains valid for phases b and c.

11. Compute λ for $I_e = 0{,}4$; 3 and 5,4 A

$$\omega = 2\pi\, f = 2\,\pi\Big(\frac{2\times 11000}{60}\Big) = 2303,83\ \text{rad/s} \tag{38}$$

I_e	E	λ
0,4	21,2	0,02300512
3	137	0,01982202
5,4	148	0,01189643

Table 1: λ parameter computation.

$$\lambda = \frac{E}{\omega\, I_e} \tag{39}$$

12. E wrt I_r for $f_{min} = 370$ Hz and $f_{max} = 770$ Hz

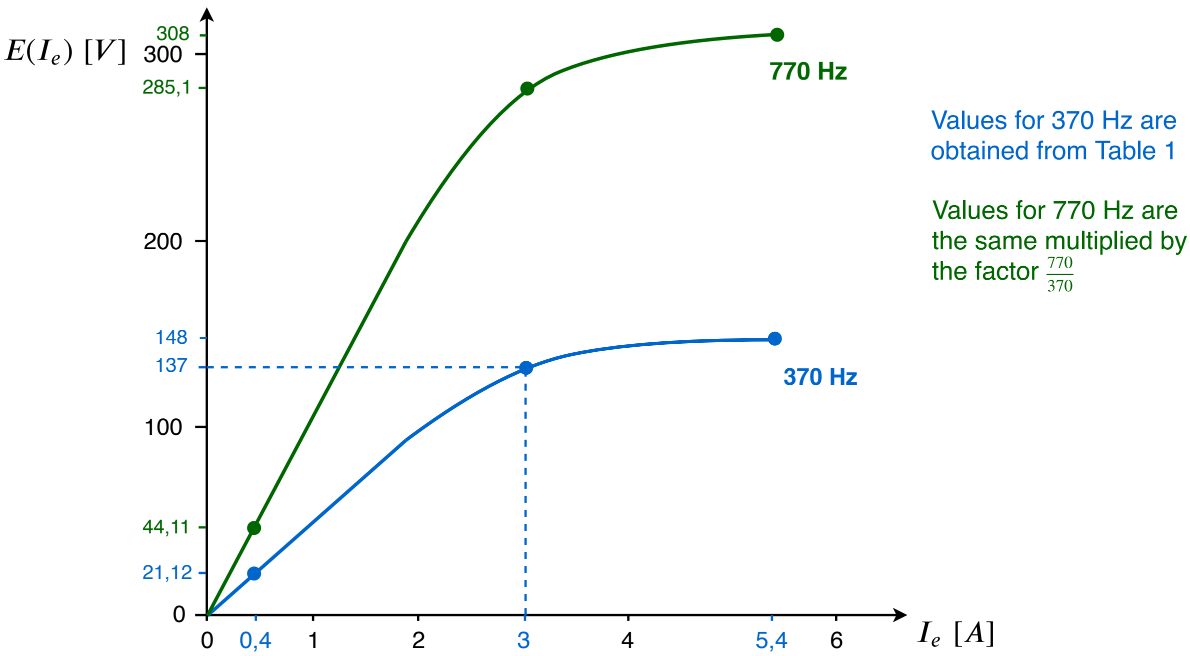


Figure 3: Emf as a function of the excitation current in the non linear case.

13. Calculate the synchronous reactance X_s for the linear part of the curve

The curve is linear for $I_e \in [0; 2]$. One can take the values corresponding to $I_e = 1,6$ A in order to minimize the measurement error and still remain in the linear part.

For $I_e = 1,6$ A, the no load voltage is $E = 84{,}8$ V and the short circuit current is $I_s = 379$ A.

$$Z_s = R_s + j\, X_s \tag{40}$$

$$Z_s = \frac{E}{I_s} = \frac{84,8}{379} = 0,2237\Omega \tag{41}$$

Taking the winding resistance $R_s = 0{,}4$ mΩ into account.

$$X_s = \sqrt{Z_s^2 - R_s^2} = 0,223699\Omega \tag{42}$$

Then, L_s = 96,22 μH and it is clear that the resistance R_s can be neglected in the electrical model of the machine.

14. Plot I_s wrt I_e for f_{min} = 370 Hz and f_{max} = 770 Hz

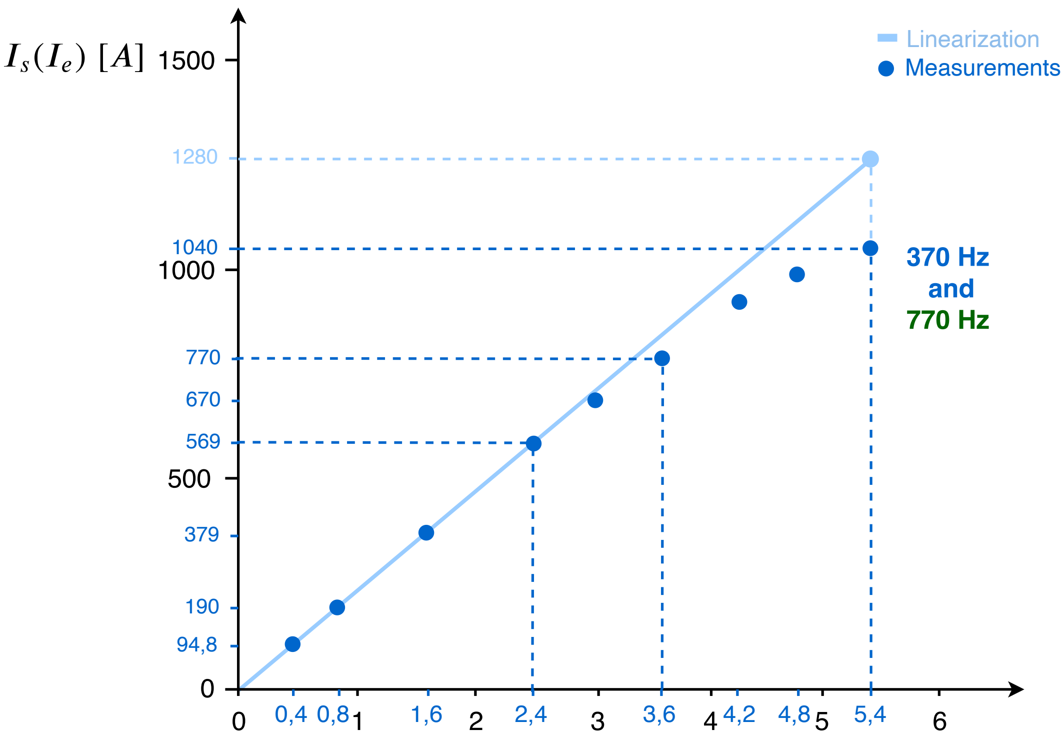


Figure 4: Short circuit current as a function of the excitation current.

As,

$$E = \lambda \, \omega \, I_e \qquad \text{(obtained from question 11)} \tag{43}$$

and

$$I_s = \frac{E}{X_s} \qquad \text{(obtained from question 13)} \tag{44}$$

One can deduce that :

$$I_s = \frac{E}{X_s} = \frac{\lambda \, \omega \, I_e}{\omega \, L_s} = \frac{\lambda}{L_s} \, I_e \tag{45}$$

Therefore, the short circuit current I_s is proportionnal to the excitation current I_e (less true out of the linear zone, but considered linear as λ does not vary much) and the short circuit current I_s does not depend of the frequency.

15. Resisitve load at 500 Hz with I_e = 2 A

Table 1 of the statement provides an emf of 106 V for I_e = 2 A and a frequency of 370 Hz. At 500 Hz, the emf becomes E = 106 $\frac{500}{370}$ = 143,24 V.

(a)

$$\overline{I} = \frac{\overline{E}}{R_L + j\,X_s} \tag{46}$$

$$I = \frac{E}{\sqrt{R_L^2 + X_s^2}} = \frac{143,24}{\sqrt{0,5^2 + (2\pi\,500\,96,22 \times 10^{-6})^2}} = 245,17 \text{ A} \tag{47}$$

$$V = R_L\,I = 0,5\;245,17 = 122,59 \text{ V} \tag{48}$$

(b)

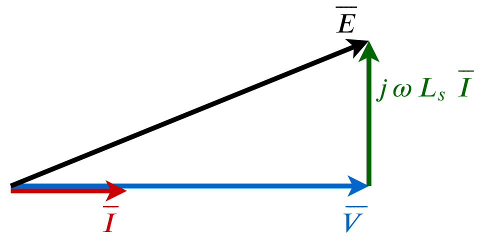


Figure 5: Phasor diagram for the purely resistive load.

(c) The load current is proportionnal to the frequency. Then, if the frequency increases, both the output voltage and current increase.

16. Resistive-inductive load

(a)

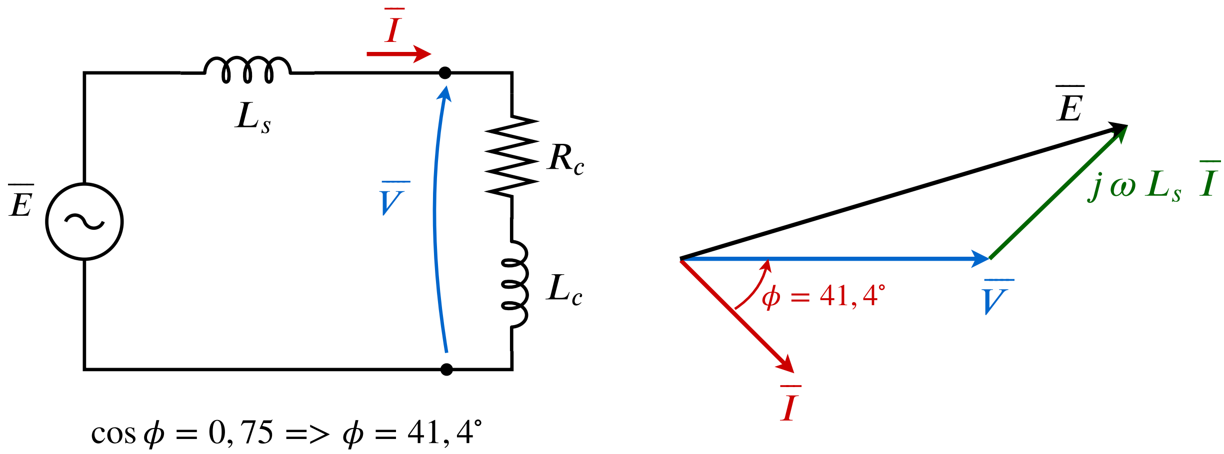


Figure 6: Phasor diagram for the resistive-inductive load.

(b)

Use

$$E = \omega \, \lambda \, I_e \tag{49}$$

$$I = \frac{E}{\sqrt{R_c^2 + \Big(\omega(L_c + L_s)\Big)^2}} \tag{50}$$

$$V = \sqrt{R_c^2 + (\omega \, L_c)^2} \; I \tag{51}$$

To expresss

$$I = \frac{\omega \, \lambda \, I_e}{\sqrt{R_c^2 + \Big(\omega(L_c + L_s)\Big)^2}} \tag{52}$$

$$V = \sqrt{R_c^2 + (\omega \, L_c)^2} \; \frac{\omega \, \lambda \, I_e}{\sqrt{R_c^2 + \Big(\omega(L_c + L_s)\Big)^2}} \tag{53}$$

(c)
With

$$P_{3\phi} = C_r \, \dot{\theta} \tag{54}$$

$$\dot{\theta} = \frac{\omega}{p} \tag{55}$$

$$P_{3\phi} = 3 \; P_{1\phi} \tag{56}$$

$$P_{1\phi} = R_c \; I^2 = R_c \; \frac{(\omega \, \lambda \, I_e)^2}{R_c^2 + \Big(\omega(L_c + L_s)\Big)^2} \tag{57}$$

One finally obtain

$$C_r = 3 \, p \; R_c \; \frac{(\omega \, \lambda \, I_e)^2}{R_c^2 + \Big(\omega(L_c + L_s)\Big)^2} \tag{58}$$

(d)
$R_c = 0{,}5 \; \Omega$, $L_c = 150 \; \mu$H and $L_s = 96{,}224 \; \mu$H.

	I_e	λ	E	I	V	C_r
$f_{min} = 370$ Hz	0,4	0,02300512	21,2	27,89	17	1
$\omega_{min} = 2324{,}8$ rad/s	3	0,01982202	137	180,5	109,88	41,92
	5,4	0,01189643	148	194,73	118,7	48,93
$f_{max} = 800$ Hz	0,4	0,02300512	45,84	34,34	31	0,7
$\omega_{min} = 5026{,}5$ rad/s	3	0,01982202	296,22	221,9	200	29,39
	5,4	0,01189643	320	239,7	216,8	34,3

Table 2: λ parameter computation.

(e)
A higher frequency means a lower power factor.

$$PF = \frac{R_c}{\sqrt{R_c^2 + \omega^2 \, L_c^2}} \tag{59}$$

Exercise 11: Three-phase turbo-alternator

Exercise 12: Alternator and synchronous condenser

1) $\phi = 27°$; $I = 4.16$ kA
2) $Q =$ -66 kvar
3) Graph

5 AC asynchronous machines

Exercise 13: Asynchronous motor 1

1. Synchronous speed $\dot{\theta}_s$, number of pairs of poles p, nominal slip g_n

The synchronous speed $\dot{\theta}_s$ is close to the nominal rotation speed $\dot{\theta}_n$ = 1 460 rpm. Then, $\dot{\theta}_s$ = 1 500 rpm.

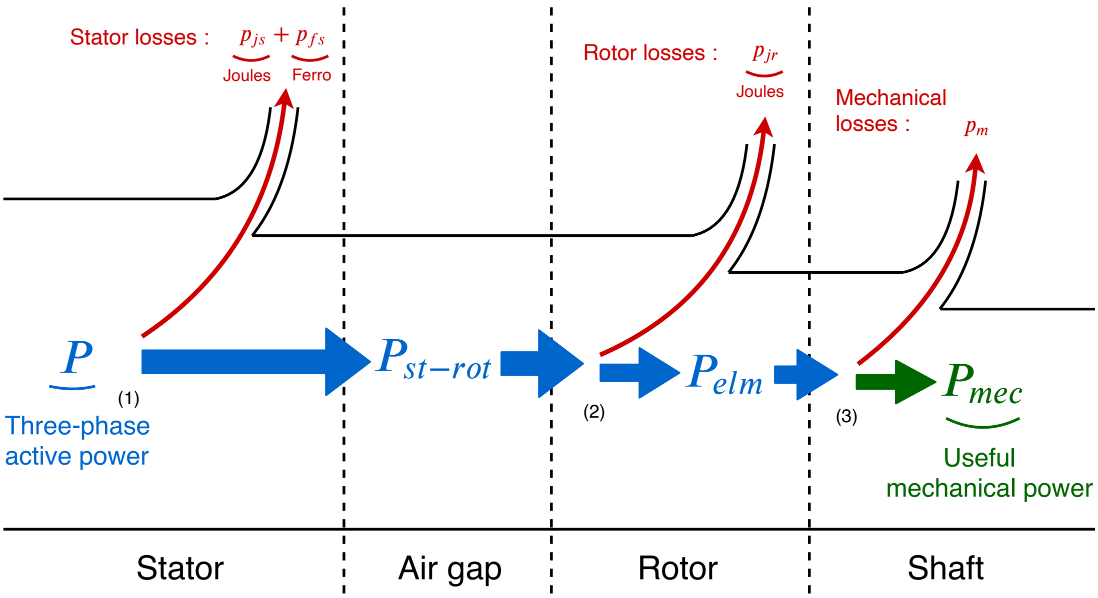
$$\dot{\theta}_s = \frac{1500}{60} = 25s^{-1} \tag{60}$$

$$p = \frac{f}{\dot{\theta}_s} = \frac{50}{25} = 2 \tag{61}$$

The slip corresponds to the relative difference between the synchronous speed $\dot{\theta}_s$ and the rotation speed $\dot{\theta}_n$ such that

$$\boxed{g_n = \frac{\dot{\theta}_s - \dot{\theta}_n}{\dot{\theta}_s} = \frac{1500 - 1460}{1500} = 2,67\%} \tag{62}$$

Recall of the power balance for an asynchronous motor



(1) $P_{st-rot} = P - p_{js} - p_{fs}$

(2) $P_{elm} = P_{st-rot} - p_{jr}$

(3) $P_{mec} = P_{elm} - p_m$

↑ The shaft output power (here 5.5 kW)

Figure 7: Power balance of the asynchronous motor.

The ferromagnetic losses in the rotor can ben neglected since the frequency of the rotoric currents is much smaller than the grid frequency (*i.e.* the frequency of the statoric currents) : $g_n\, f << f$ and $p_{fr} \simeq 0$ (good approximation for small slip).

Understanding $\frac{R_2'}{g}$

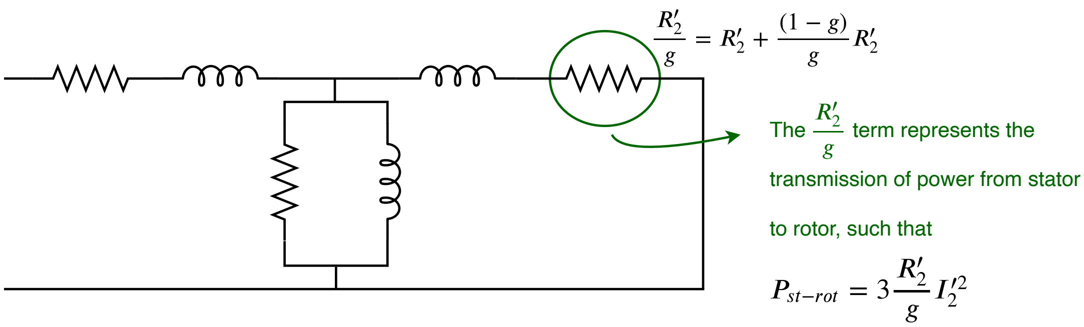


Figure 8: Power transmission between the stator and the rotor of the asynchronous motor.

$$P_{st-rot} = 3\,\frac{R_2'}{g}\,I_2'^2 \tag{63}$$
$$p_{jr} = 3\,R_2'\,I_2'^2 \tag{64}$$
$$P_{elm} = P_{st-rot} - p_{jr} = 3\,\frac{R_2'}{g}\,I_2'^2\,(1-g) \tag{65}$$
$$\tag{66}$$

Which leads to

$$\boxed{P_{elm} = (1-g)\,P_{st-rot}} \tag{67}$$

$$\boxed{p_{jr} = g\,P_{st-rot}} \tag{68}$$

2. Stator resistance R_s

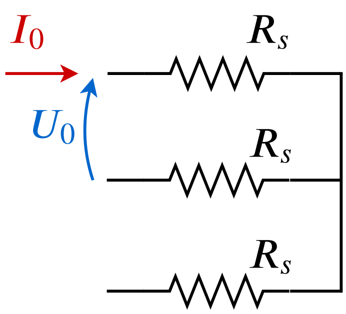


Figure 9: Stator model (star-shaped).

$$R_s = \frac{U_0}{2\,I_0} = \frac{20.6}{2 \cdot 10} = 1,03\ \Omega$$

3. R_{HF} and X_μ

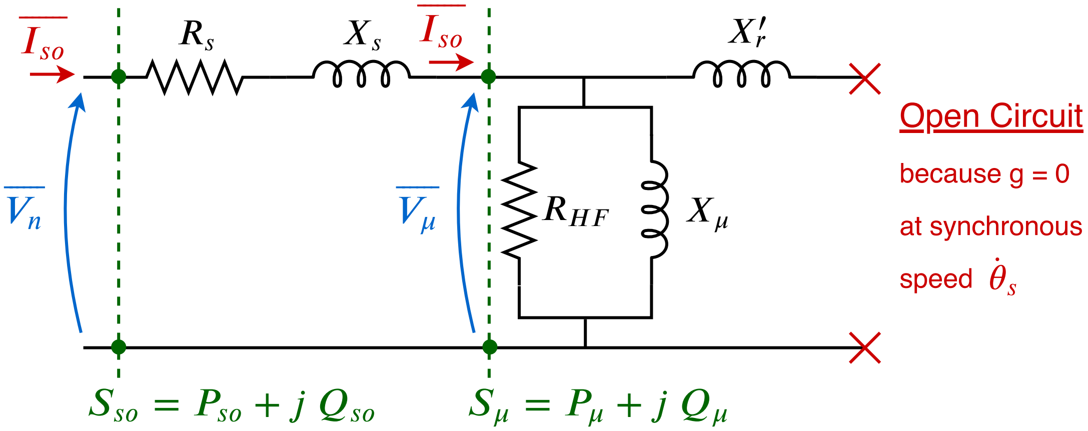


Figure 10: Equivalent circuit of the asynchronous motor running at synchronous speed.

$$I_{so} = 3,07\ \text{ A} \tag{69}$$
$$V_n = \frac{U_n}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230,94\ \text{ V} \tag{70}$$
$$P_{so} = \frac{245}{3} = 81,667\ \text{ W} \tag{71}$$
$$S_{so} = V_n\ I_{so} = 708,986\ \text{VA} \tag{72}$$
$$Q_{so} = \sqrt{S_{so}^2 - P_{so}^2} = 704,27\text{var} \tag{73}$$
$$P_\mu = P_{so} - R_s\,I_{so}^2 = 81,667 - 1,03 \cdot 3,07^2 = 71,96\ \text{W} \tag{74}$$
$$Q_\mu = Q_{so} - X_s\,I_{so}^2 = 704,27 - 1,03 \cdot 3,07^2 = 694,56\ \text{var} \tag{75}$$
$$S_\mu = \sqrt{P_\mu^2 + Q_\mu^2} = 698,28\ \text{VA} \tag{76}$$
$$V_\mu = \frac{S_\mu}{I_{so}} = 227,45\ \text{ V (relatively close to } V_n) \tag{77}$$

At the rotor, the losses can be decomposed as

$$\frac{R'_r}{g}\,I_r'^2 = \underbrace{R'_r\,I_r'^2}_{\text{rotor Joule losses}} + \underbrace{\frac{(1-g)\,R'_r}{g}\,I_r'^2}_{\text{mechanical losses}} \tag{78}$$

You are told that the ferromagnetic losses equal the mechanical losses.

Then,

$$\underbrace{\frac{V_\mu^2}{R_{HF}}}_{\text{ferromagnetic losses}} \underset{=}{+} \underbrace{\frac{(1-g)\,R'_r}{g}\,I_r'^2}_{\text{mechanical losses}} = P_\mu \tag{79}$$

$$\rightarrow 2\,\frac{V_\mu^2}{R_{HF}} = P_\mu \tag{80}$$

$$\boxed{R_{HF} = 2\,\frac{V_\mu^2}{P_\mu} = 2\,\frac{227,45^2}{71,96} = 1437,88\Omega} \tag{81}$$

$$\boxed{X_\mu = 2\,\frac{V_\mu^2}{Q_\mu} = 2\,\frac{227,45^2}{694,56} = 74,99\Omega} \tag{82}$$

The losses seen from the magnetizing (μ) branch are $P_\mu = 71,96$ W in the equivalent circuit of one phase. Half of those losses account for the mechanical losses $p_{m,1\phi} = 35,98$ W and the other half account for the ferromagnetic losses $p_{f,1\phi} = 35,98$ W. The total mechanical and ferromagnetic losses of the three-phase motor correspond to

$$p_m = 3\,p_{m,1\phi} = 3 \cdot 35,98 = 107,94 \text{ W} \tag{83}$$
$$p_f = 3\,p_{f,1\phi} = 3 \cdot 35,98 = 107,94 \text{ W} \tag{84}$$

4. Nominal operating point : P_{st-rot}, p_{js}, P

The mechanical losses are considered independent of the rotation speed. Then, $p_m(1460 \text{ rpm}) = p_m(1500 \text{ rpm}) \simeq 108$ W.

From the equation (3) of the power balance

$$P_{mec} = P_{elm} - p_m \tag{85}$$
$$P_{elm} = P_{mec} + p_m = 5\,500 + 108 = 5\,608 \text{ W} \tag{86}$$

From (67)

$$P_{elm} = P_{st-rot}\,(1-g) \tag{87}$$
$$\boxed{P_{st-rot} = \frac{P_{elm}}{1-g} = \frac{5\,608}{1-0,02667} = 5\,761,6 \text{ W}} \tag{88}$$

The Joule losses in the stator are

$$\boxed{p_{js} = 3\,R_s\,I_{sn}^2 = 3 \cdot 1,03 \cdot 11^2 = 373,89 \text{ W}} \tag{89}$$

The ferromagnetic losses are kept constant

$$\boxed{p_f = 108 \text{ W}} \tag{90}$$

Finally, the three-phase active power can be computed as

$$\boxed{P = P_{st-rot} + p_{js} + p_f = 5761,6 + 373,4 + 108 = 6\,243 \text{ W}} \tag{91}$$

5. <u>Find R'_r and L'_r (seen from the stator)</u>

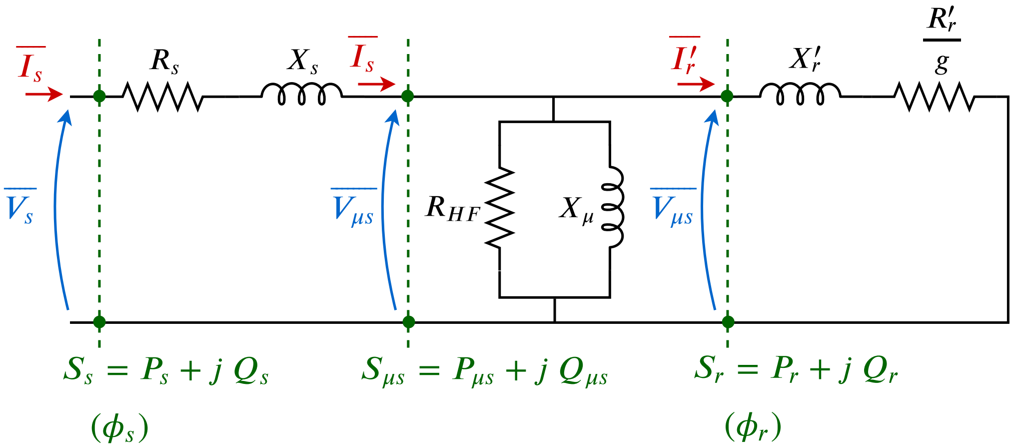


Figure 11: Equivalent circuit of the asynchronous motor in operation.

In order to determine R'_r and X'_r, find P_r, Q_r and I'_r and deduce

$$R'_r = g\frac{P_r}{I'^2_r} \tag{92}$$

$$X'_r = \frac{Q_r}{I'^2_r} \tag{93}$$

Then,

$$P_s = \frac{P}{3} = 2\,081 \text{ W} \tag{94}$$

$$S_s = V_s\, I_s = 230,94 \cdot 11 = 2\,540,34 \text{ VA} \tag{95}$$

$$Q_s = \sqrt{S_s^2 - P_s^2} = 1\,457 \text{ var} \tag{96}$$

$$\phi_s = \arccos\left(\frac{P_s}{S_s}\right) = 35° \tag{97}$$

Accross the magnetizing branch,

$$P_{\mu s} = P_s - R_s\, I_s^2 = 2\,081 - 1,03 \cdot 11^2 = 1\,956,37 \text{ W} \tag{98}$$

$$Q_{\mu s} = Q_s - X_s\, I_s^2 = 1\,457 - 1.03 \cdot 11^2 = 1\,332,37 \text{ W} \tag{99}$$

$$S_{\mu s} = \sqrt{P_\mu^2 + Q_\mu^2} = 2\,366,9 \text{ VA} \tag{100}$$

$$V_{\mu s} = \frac{S_{\mu s}}{I_s} = \frac{2\,366,9}{11} = 215,2 \text{ V} \tag{101}$$

Then, on the rotor side of the equivalent circuit,

$$P_r = P_{\mu s} - \frac{V_{\mu s}^2}{R_{HF}} = 1\,956,37 - \frac{215,2^2}{1\,437,88} = 1\,923,2 \text{ W} \tag{102}$$

$$Q_r = Q_{\mu s} - \frac{V_{\mu s}^2}{X_\mu} = 1\,332,37 - \frac{215,2^2}{74,99} = 714,8 \text{ var} \tag{103}$$

$$S_r = \sqrt{P_r^2 + Q_r^2} = 2\,051,7\text{VA} \tag{104}$$

$$\phi_r = \arccos\left(\frac{P_r}{S_r}\right) = 20,43° \tag{105}$$

$$I'_r = \frac{S_r}{V_{\mu s}} = \frac{2\,052,24}{215,2} = 9,537 \text{ A} \tag{106}$$

Finally,

$$\boxed{R'_r = g\,\frac{P_r}{I'^2_r} = 0,02667\,\frac{1\,923,2}{9,537^2} = 0,564\ \Omega} \tag{107}$$

$$X'_r = \frac{Q_r}{I'^2_r} = \frac{714,8}{9,537^2} = 7,86\ \Omega \tag{108}$$

$$\boxed{L'_r = 25 \text{ mH}} \tag{109}$$

6. C_{mec}, C_{elm}, $\cos\phi_n$ and η_n (for the nominal operating point)

$$P_{mec} = C_{mec,n}\,\dot{\theta}_n \tag{110}$$

Then, the output torque is given by

$$C_{mec,n} = \frac{P_{mec}}{\dot{\theta}_n} = \frac{5\,500}{1\,460 \cdot \frac{2\,\pi}{60}} = 35,97 \text{ Nm} \tag{111}$$

The electromagnetic torque is

$$C_{elm,n} = \frac{P_{elm}}{\dot{\theta}_n} = \frac{5\,608}{1\,460 \cdot \frac{2\pi}{60}} = 36,7 \text{ Nm} \tag{112}$$

The $\cos\phi_n$ can be computed by two ways,

$$\cos\phi_n = \frac{P}{S_n} = \frac{6\,243}{\sqrt{3} \cdot 400 \cdot 11} = 0,818 \tag{113}$$

or

$$\phi_s = \phi_n \Rightarrow \cos\phi_n = 0,818 \tag{114}$$

Finally, the efficiency during nominal operation is

$$\eta_n = \frac{P_{mec}}{P} = \frac{5\,500}{6\,242} = 88,1\% \tag{115}$$

7. I_s and $\cos\phi$ for $\dot{\theta} = 0$ rpm

At 0 rpm, the motor is stalled, the slip becomes $g = 1$, $\frac{R'_r}{g} = R'_r$ and the equivalent circuit of the asynchronous motor becomes

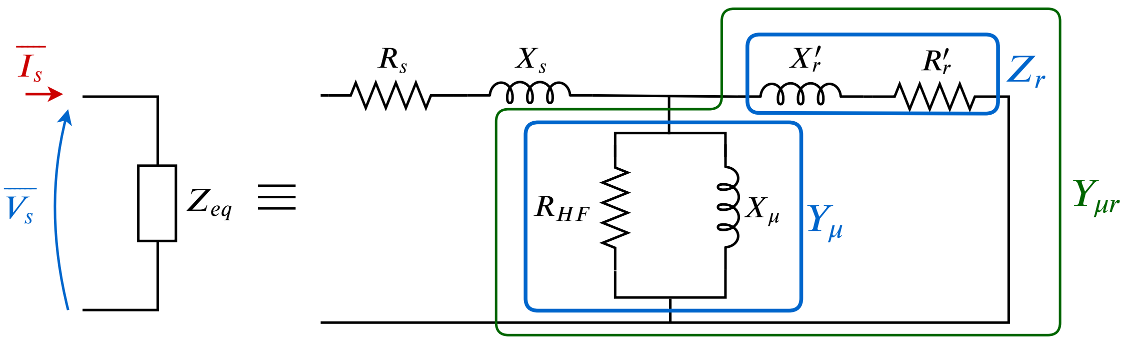


Figure 12: Equivalent circuit of the asynchronous motor when stalled.

$$Z_r = R_r' + j\ X_r' \tag{116}$$

$$Y_r = \frac{1}{Z_r} = \frac{1}{R_r' + j\ X_r'} \tag{117}$$

$$Y_\mu = \frac{1}{R_{HF}} + j\ \frac{1}{X_\mu} \tag{118}$$

$$Y_{\mu r} = Y_\mu + Y_r = \frac{1}{R_{HF}} + j\ \frac{1}{X_\mu} + \frac{1}{R_r' + j\ X_r'} \tag{119}$$

$$Z_{\mu r} = \frac{1}{Y_{\mu r}} = \frac{1}{\frac{1}{R_{HF}} + j\ \frac{1}{X_\mu} + \frac{1}{R_r'+j\ X_r'}} \tag{120}$$

$$Z_{eq} = R_s + j\ X_s + Z_{\mu r} = R_s + j\ X_s + \frac{1}{\frac{1}{R_{HF}} + j\ \frac{1}{X_\mu} + \frac{1}{R_r'+j\ X_r'}} \tag{121}$$

$$Z_{eq} = 1,03 + j\ 1,03 + \frac{1}{\frac{1}{1\,437{,}8} + j\ \frac{1}{75} + \frac{1}{0{,}564+j\ 7{,}87}} = 9,969\angle 79,67^\circ \tag{122}$$

$$\boxed{\overline{I_s} = \frac{\overline{V_s}}{Z_{eq}} = \frac{230}{9,969\angle 79,67^\circ} = 23,07\angle \underbrace{-79,67^\circ}_{\phi}} \tag{123}$$

$$\boxed{\cos\phi = \cos(-79,67^\circ) = 0,179} \tag{124}$$

Exercise 14: Asynchronous motor 2

1. Explain the nameplate

Nominal active power : $P_n = 4,4$ kW

RMS voltages : $\underbrace{230}_{\Delta} \ / \ \underbrace{400}_{Y}$ V

Line currents : $\underbrace{15,5}_{\Delta} \ / \ \underbrace{9}_{Y}$ A

Nominal frequency : $f_n = 50$ Hz

Number of pairs of poles : 4 poles $\rightarrow$ 2 pairs of poles $\rightarrow p = 2$

2. Which coupling for a 230 V network ?

230 V network means that the RMS value of thez composed voltages is $U = 230$ V. Then, the armature (stator) of the machine should be connected in Δ.

3. Synchronous speed $\dot{\theta}_s$

$$\dot{\theta}_s = \frac{f_n}{p} = \frac{50}{2} = 25 \text{ Hz} = 1500 \text{ rpm} = 157 \text{ rad/s} \tag{125}$$

4. Stator resistance R_s

R_a corresponds to the total resitance measured between two terminals of the stator. Then, R_s, the resistance of one phase is two times smaller than $R_a = \frac{U_0}{I_0}$.

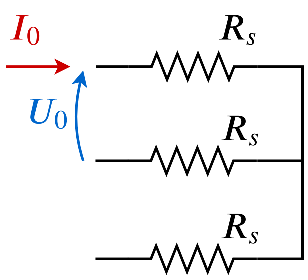


Figure 13: Stator model (star-shaped).

$$R_s = \frac{R_a}{2} = \frac{0,654}{2} = 0,327 \ \Omega$$

5. <u>Mechanical losses at synchronous speed</u>

The asynchronous machine is put into potion by an external motor rotating at $\dot{\theta}_s$ and remains unpowered at the stator. Therefore, the mechanical power provided by the external potor exactly compensate the mechanical losses of the asynchronous machine : $p_m = 86$ W.

The mechanical losses depends on the rotation speed. The nominal rotation speed is close to the synchronous rotation speed (low slip), that is why the variation in mechanical losses can be neglected.

6. <u>Determine R_{HF} and L_μ</u>

During the no load test, the active power in the stator P_{so} is

$$P_{so} = \underbrace{p_{js}}_{\text{stator joule losses}} + \underbrace{p_f}_{\text{ferromagnetic losses}} + \underbrace{p_m}_{\text{mechanical losses}} \tag{126}$$

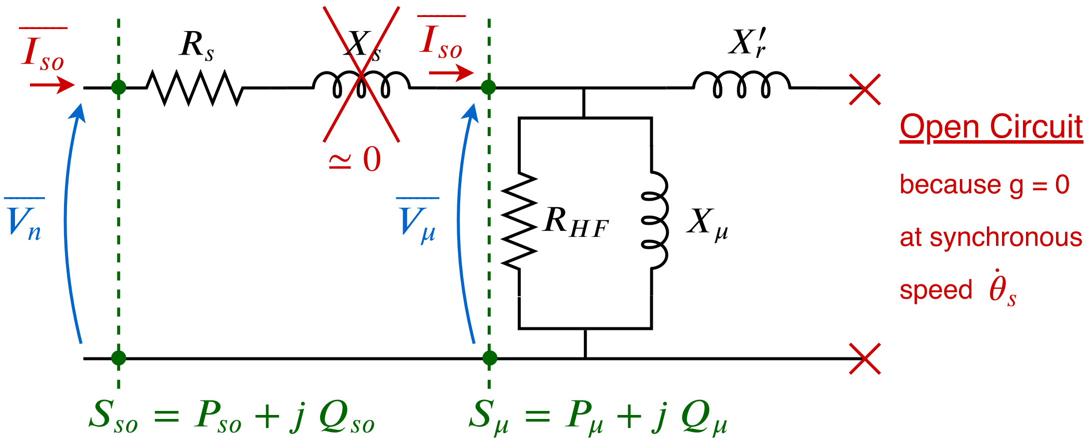


Figure 14: Equivalent circuit of the asynchronous motor running at synchronous speed.

$$I_{so} = 3,82 \ \text{ A} \tag{127}$$
$$V_n = \frac{U_n}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132,79 \ \text{ V} \tag{128}$$
$$P_{so} = \frac{300}{3} = 100 \ \text{ W} \tag{129}$$
$$S_{so} = V_n\ I_{so} = 507,26 \text{ VA} \tag{130}$$
$$Q_{so} = \sqrt{S_{so}^2 - P_{so}^2} = 497,3\text{var} \tag{131}$$
$$P_\mu = P_{so} - R_s\,I_{so}^2 = 100 - 0,327 \cdot 3,82^2 = 95,23 \text{ W} \tag{132}$$
$$Q_\mu = Q_{so} - \underbrace{X_s}_{\simeq 0}\ I_{so}^2 = Q_{so} = 497,3 \text{ var} \tag{133}$$
$$S_\mu = \sqrt{P_\mu^2 + Q_\mu^2} = 506,34 \text{ VA} \tag{134}$$
$$V_\mu = \frac{S_\mu}{I_{so}} = \frac{506,34}{3,82} = 132,55 \ \text{ V (relatively close to } V_n) \tag{135}$$

At this point, the mechanical losses are still taken into account and must be subtracted.

Then, the ferromagnetic losses correspond to

$$p_f = P_\mu - \frac{p_m}{3} = 95,23 - \frac{86}{3} = 66,56 \text{ W} \tag{136}$$

$$\boxed{R_{HF} = \frac{V_\mu^2}{P_f} = \frac{132,55^2}{66,56} = 264\ \Omega} \tag{137}$$

$$X_\mu = \frac{V_\mu^2}{Q_\mu} = \frac{132,55^2}{497,3} = 35,2\ \Omega \tag{138}$$

$$\boxed{L_\mu = \frac{X_\mu}{100\,\pi} = 0,112 \text{ H}} \tag{139}$$

7. Stalled rotor test

Stalled rotor $\rightarrow g = 1 \rightarrow \frac{R'_r}{g} = R'_r$

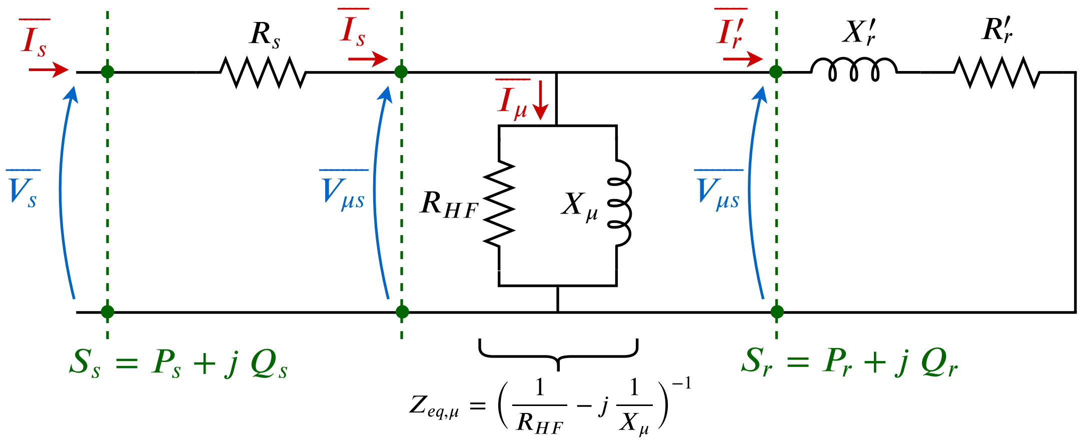


Figure 15: Equivalent circuit of the stalled asynchronous motor.

$$P_s = \frac{374}{3} = 124,67 \ \text{ W} \tag{140}$$
$$Q_s = \frac{1090}{3} = 363,3 \text{ var} \tag{141}$$
$$S_s = \sqrt{P_s^2 + Q_s^2} = 384,1 \text{ VA} \tag{142}$$
$$V_s = \frac{U_s}{\sqrt{3}} = \frac{57,5}{\sqrt{3}} = 33,2 \ \text{ V} \tag{143}$$
$$\overline{I_s} = \frac{P_s - j\,Q_s}{V_s} = \frac{124,67 - j\;363,3}{33,2} = 11,57\angle - 71,06° \tag{144}$$
$$I_s = 11,57 \ \text{ A} \tag{145}$$
$$V_\mu = V_s - R_s\,I_s = 33,2 - 0,327 \cdot 11,57 = 29,42 \text{ V} \tag{146}$$
$$\overline{I_\mu} = \frac{\overline{V_\mu}}{Z_{eq,\mu}} = \frac{\overline{V_\mu}}{\left(\frac{1}{R_{HF}} - j\,\frac{1}{X_\mu}\right)^{-1}} = \frac{29,42}{\left(\frac{1}{264} - j\,\frac{1}{35,2}\right)^{-1}} = 0,843\angle - 82° \tag{147}$$
$$\overline{I'_r} = \overline{I_s} - \overline{I_\mu} = (11,57\angle - 71,06°) \ - (0,843\angle - 82°) = 10,74\angle - 70,21° \tag{148}$$
$$I'_r = 10,74 \text{ A} \tag{149}$$
$$P_r = P_s - R_s\,I_s^2 - \frac{V_\mu^2}{R_{HF}} = 124,67 - 0,327 \cdot 11,57^2 - \frac{29,42^2}{264} = 77,62 \text{ W} \tag{150}$$
$$Q_r = Q_s - \frac{V_\mu^2}{X_\mu} = 363,3 - \frac{29,42^2}{35,2} = 338,71 \text{ var} \tag{151}$$

$$\boxed{R'_r = \frac{P_r}{I'^2_r} = \frac{77,62}{10,74^2} = 0,673\ \Omega} \tag{152}$$

$$\boxed{X'_r = \frac{Q_r}{I'^2_r} = \frac{338,71}{10,74^2} = 2,93\ \Omega} \tag{153}$$

8. <u>Express I_s in terms of V_s, R_s, R'_r, g and X'_r.</u>

Neglecting the magnetizing brnahc and the magnetizing current I_μ,

$$\overline{I_s} = \frac{\overline{V_s}}{R_s + \frac{R'_r}{g} + j\,X'_r} \tag{154}$$

$$\boxed{I_s = \frac{V_s}{\sqrt{\left(R_s + \frac{R'_r}{g}\right)^2 + X'^2_r}}} \tag{155}$$

9. <u>Transmitted power from stator to rotor.</u>

$$P_{st-rot} = 3\,\frac{R'_r}{g}\;I_s^2 = 3\,\frac{R'_r}{g}\;\frac{V_s^2}{\left(R_s + \frac{R'_r}{g}\right)^2 + \left(X'_r\right)^2} \tag{156}$$

10. Show that C_{elm} is maximum for $g_m$and compute C_{elm}.

As $P_{elm} = (1-g)\, P_{st-rot}$ and $\dot{\theta} = (1-g)\, \dot{\theta}_s$, the torque can be expressed as

$$C_{elm} = \frac{P_{elm}}{\dot{\theta}} = \frac{P_{st-rot}}{\dot{\theta}_s} \tag{157}$$

$$C_{elm} = \frac{P_{st-rot}}{\dot{\theta}_s} = 3\,\frac{V_s^2}{\dot{\theta}_s}\,\frac{\frac{R_r'}{g}}{\left(R_s + \frac{R_r'}{g}\right)^2 + \left(X_r'\right)^2} \tag{158}$$

$$\frac{d\,C_{elm}}{d\,g} = 3\,\frac{R_r'\,V_s^2}{\dot{\theta}_s}\left(-\frac{\frac{1}{g^2}\left(\left(R_s + \frac{R_r'}{g}\right)^2 \;+\; (X_r')^2\right) + \frac{1}{g}\,2\,\left(R_s + \frac{R_r'}{g}\right)\left(-\frac{R_r'}{g^2}\right)}{\left(\left(R_s + \frac{R_r'}{g}\right)^2 \;+\; (X_r')^2\right)^2}\right) \tag{159}$$

$$\frac{d\,C_{elm}}{d\,g} = 3\,\frac{R_r'\,V_s^2}{g^2\,\dot{\theta}_s}\left(\frac{R_s^2 - \left(\frac{R_r'}{g}\right)^2 + X_r'^2}{\left(\left(R_s + \frac{R_r'}{g}\right)^2 \;+\; (X_r')^2\right)^2}\right) \tag{160}$$

From that expression, $\frac{d\,C_{elm}}{d\,g} = 0$ if $R_s^2 - \left(\frac{R_r'}{g}\right)^2 + X_r'^2 = 0$
Meaning, for a slip such that

$$\boxed{g_{max} = +\frac{R_r'}{\sqrt{R_s^2 + X_r'^2}}} \quad \text{(positive slip for the motor)} \tag{161}$$

And for such a slip, g_{max}, the maximum electromagnetic torque is

$$\boxed{C_{elm,max} = 3\,\frac{V_s^2}{\dot{\theta}_s}\,\frac{\sqrt{R_s^2 + X_r'^2}}{\left(R_s + \sqrt{R_s^2 + X_r'^2}\right)^2 + X_r'^2} = 3\,\frac{V_s^2}{2\,\dot{\theta}_s}\,\frac{1}{R_s + \sqrt{R_s^2 + X_r'^2}}} \tag{162}$$

Neglecting the stator resistance, R_s, the result is

$$C_{elm,max} = 3\,\frac{V_s^2}{\dot{\theta}_s}\,\frac{1}{X_r'} \quad \text{(same as theory)} \tag{163}$$

11. Plot C_{elm} wrt g for $V_s = V_n$, $V_s = \frac{V_n}{\sqrt{2}}$ and $V_s = \frac{V_n}{2}$.

At $V_s = V_n = 132{,}79$ V ; the torque is

$$C_{elm,max} = 3\,\frac{132,79^2}{1\,500 \cdot \frac{2\pi}{60}}\,\frac{\sqrt{0,327^2 + 2,84^2}}{\left(0,327 + \sqrt{0,327^2 + 2,84^2}\right)^2 + 2,84^2} = 52,86 \text{ Nm} \tag{164}$$

At $V_s = \frac{V_n}{\sqrt{2}} = 93{,}9$ V ; the torque is

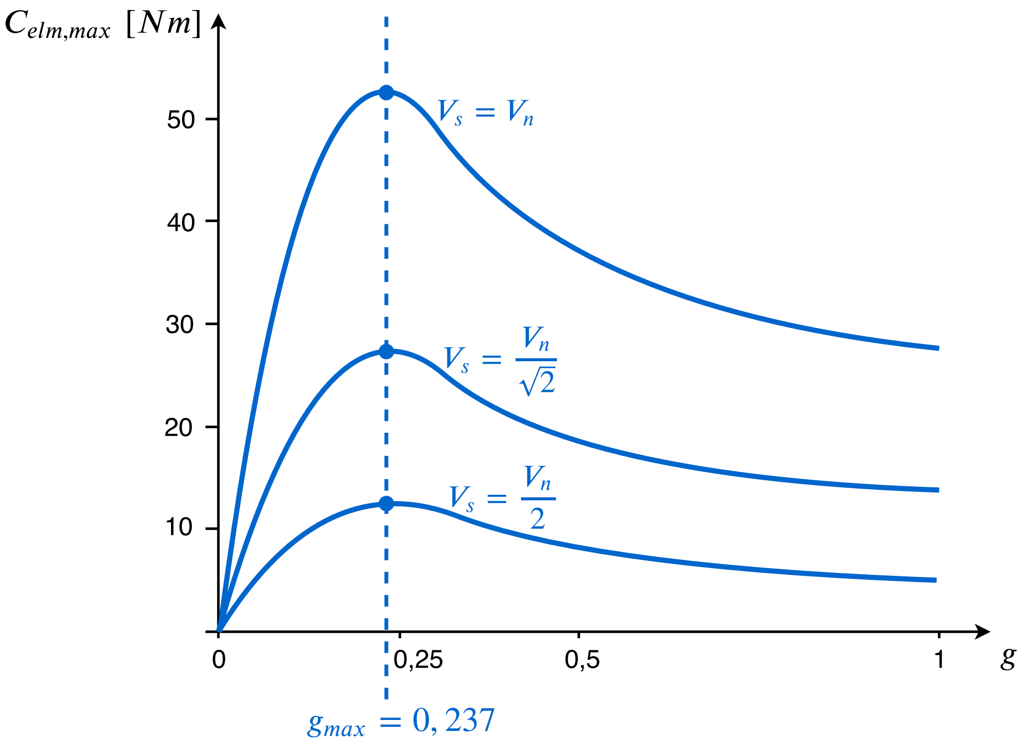
$$C_{elm,max} = 3\,\frac{93,9^2}{1\,500 \cdot \frac{2\pi}{60}}\,\frac{\sqrt{0,327^2 + 2,84^2}}{\left(0,327 + \sqrt{0,327^2 + 2,84^2}\right)^2 + 2,84^2} = 26,43 \text{ Nm} \tag{165}$$

At $V_s = \frac{V_n}{2} = 66{,}4$ V ; the torque is

$$C_{elm,max} = 3\,\frac{66,4^2}{1\,500 \cdot \frac{2\pi}{60}}\,\frac{\sqrt{0,327^2 + 2,84^2}}{\left(0,327 + \sqrt{0,327^2 + 2,84^2}\right)^2 + 2,84^2} = 13,21 \text{ Nm} \tag{166}$$

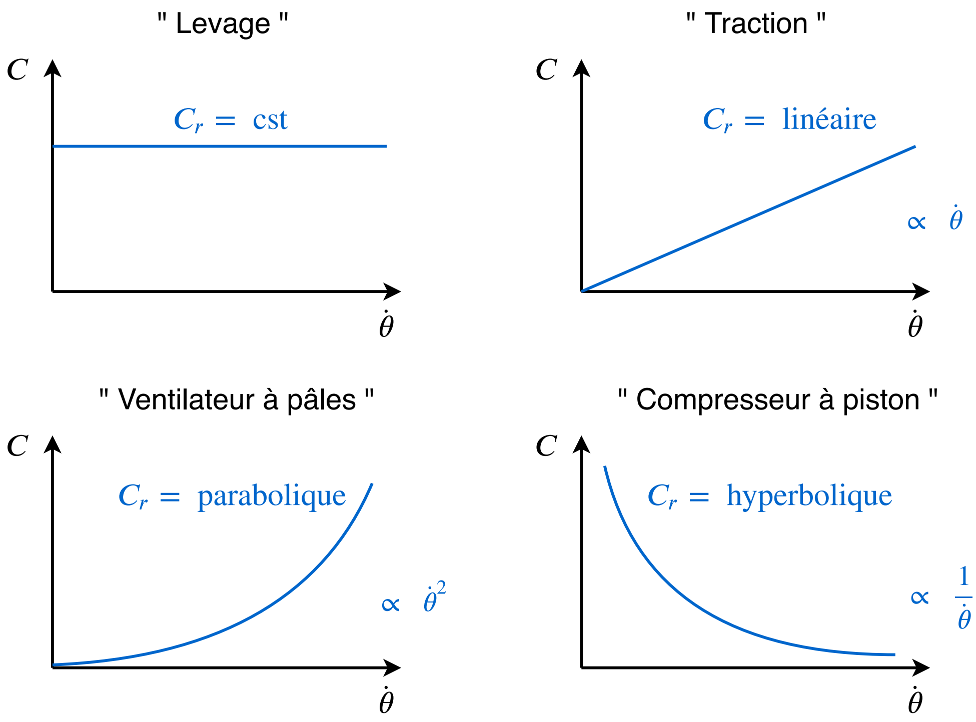
All maximum torques occur at the same slip

$$g_{max} = \frac{0,677}{\sqrt{0,327^2 + 2,84^2}} = 0,237 \tag{167}$$



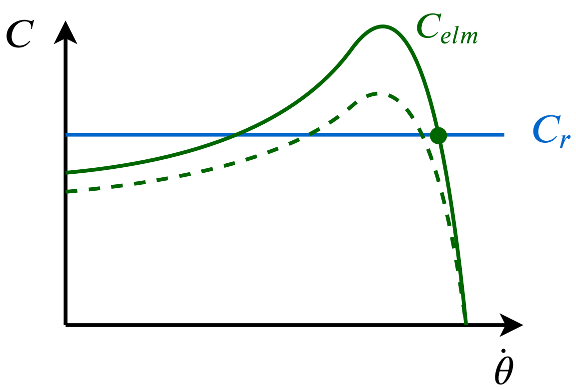
12. Why the control of the voltage is not suited for speed variations at a constant torque.

First of all, the mechanical behavior of different loads (pump, car, fan...) can be separated in 4 main categories.

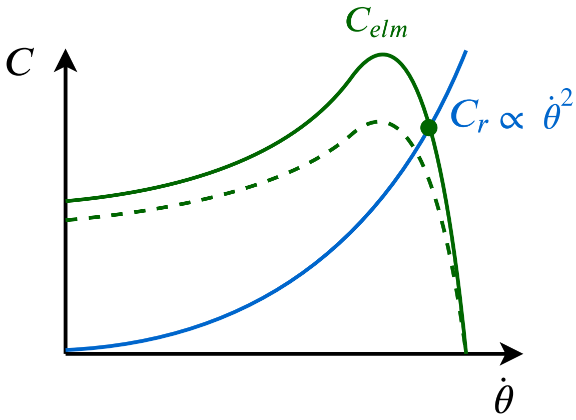


In the first case, decreasing the voltage enables to decrease the speed, but with very small impact. Moreover, the machine must remain in the stable region which is narrow.

→ The voltage control is not suited for speed variation (at a constant resistive torque).

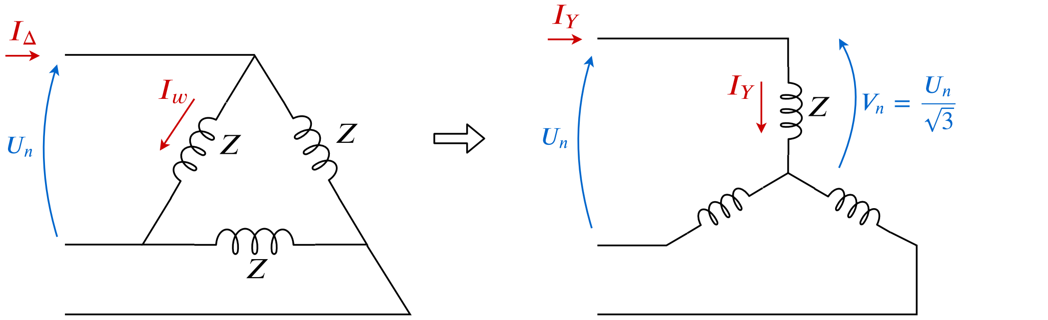


In the other case, if the load is a fan (such as in this exercise), the speed can be easily controlled only by varying the voltage.



13. Comparison without and with star/delta starter.

The use of the star/delta connection allows to reduce the inrush current by a factor of 3.



Winding current : $I_w = \frac{I_\Delta}{\sqrt{3}} = \frac{U_n}{Z}$

Winding current : $I_Y = \frac{V_n}{Z} = \frac{U_n}{\sqrt{3}\,Z} = \frac{I_\Delta}{\sqrt{3}}\frac{1}{\sqrt{3}} = \frac{I_\Delta}{3}$

Exercise 15: Wind turbine

1) $\Gamma_i = \frac{U_s^2}{R_r+R}\frac{\dot{\theta}_s-\dot{\theta}}{\dot{\theta}_s^2}$

2) $\dot{\theta}$ = 159 rad/s at 2 MW with R = 0 and g = -1.25 % ;
$\dot{\theta}$ = 172 rad/s et 4 MW with R = 9 $m\Omega$; g = -9.23 %

6 DC machines

Exercise 16: DC brushed motor

Exercise 17: Regenerative braking

Exercise 18: DC generator-motor mechanical coupling

1) $emf = 519.075$ V
2) $p = 17.11$ W
3) $P = 9603$ W
4) $C = 88.17$ Nm
5) $E_m = 502$ V ; $\dot{\theta} = 1076$ rpm
6) $I = 360$ A
7) $C = 1604$ Nm
8) Ratio = 14.2 %