

# ELEC0431 Electromagnetic Power Conversion Exercises

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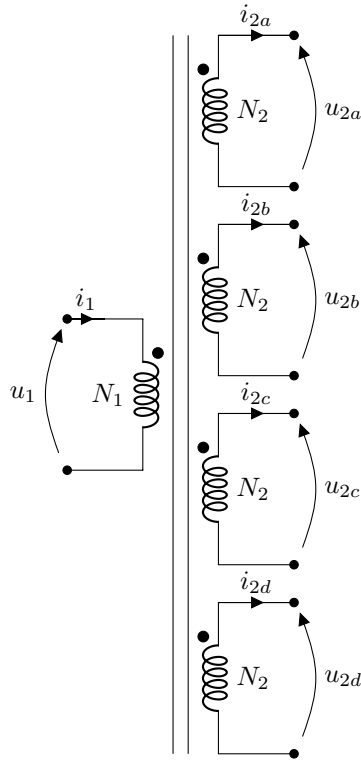


Figure 1: Four secondaries single-phase transformer.

## 1 Transformers

### 1.1 Exercise 1: Four secondaries single-phase transformer

In Belgium, most of the railways are powered using Direct Current (DC) 3 kV voltage. High speed train lines are however supplied with Alternating Current (AC) 25 kV 50 Hz (single-phase) voltage, requiring the use of high power single phase transformers. In this exercise, an input transformer of a locomotive is considered. It contains four secondary identical windings, as detailed in Fig. 1. A nominal Root Mean Square (RMS) voltage of  $U_{1n} = 25$  kV with nominal frequency  $f = 50$  Hz is supplied to the primary winding with an apparent power  $S_n = 5.6$  MVA.

To characterize the transformer two tests have been performed:

- Using open secondary windings, the transformer generates a voltage  $U_{2o} = 1.36$  kV at one secondary winding, for a current drawn at the primary  $I_{1o} = 1.25$  A, and an active consumed power  $P_{1o} = 6.8$  kW;
- Using short-circuited secondary windings, the transformer consumes an active power  $P_{1s} = 25$  kW, considering that a reduced voltage of 37.1 % of  $U_{1n}$  was applied to the primary winding to maintain the secondary winding currents to their nominal values  $I_{2n}$ .

1. Calculate the transformer ratio  $m$  (for a single secondary winding);
2. Determine the nominal RMS secondary current  $I_{2n}$  and primary current  $I_{1n}$ ;
3. Compute the power factor  $\cos \phi_{1o}$  for the first test (open secondary windings) and deduce the phase shift  $\phi_{1o}$  of the current at the primary winding with respect to the primary winding voltage;
4. Give the reactive power  $Q_{1o}$  for the first test (open secondary windings);
5. Considering a simplified circuit of the transformer model (leak resistance and inductance moved to the secondary windings), calculate the leakage resistance  $R_l$  and the leakage inductance  $L_m$ ;
6. Compute the RMS current intensity  $I_{2s}$  in a secondary winding for the second test (shorted secondary windings), compute the primary winding voltage  $U_{1s}$ , and calculate the values of the resistance  $R_s$  and of the inductance  $L_s$  of a secondary winding in the equivalent model;
7. Considering that  $L_s$  is chosen to large enough to provide sufficient smoothing at the input of single-phase rectifiers, compare the values of  $R_s$  and  $X_s = L_s\omega$  (for  $\omega$  the angular frequency corresponding to  $f$ ) and propose a simplified version of the equivalent model of the transformer.

The nominal regime is now considered by applying the nominal voltage  $U_{1n}$  at the primary winding and connecting a load at each secondary winding, drawing a RMS current  $I_2 = 1.03$  kA with a power factor  $\cos \phi_2$ , the current being ahead on the voltage. The currents  $i_{2k}$  in each secondary winding  $k$  is aimed to be in phase with the voltage  $u_{2ko}$  of the considered secondary winding.

8. Build the corresponding Fresnel diagram, clearly identifying the load voltage  $u_{2k}$ ;
9. Compute the phase shift  $\phi_2$  of the current  $i_{2k}$  with respect to  $u_{2k}$ , and deduce the load power factor  $\cos \phi_2$ ;
10. Compute the RMS voltage value  $U_2$  appearing at each secondary winding;
11. Compute the reactive power  $Q_2$  drawn by the load at each secondary winding, and the reactive power  $Q_1$  at the primary winding;
12. Compute the active power  $P_2$  drawn by the load at each secondary winding, and the active power  $P_1$  at the primary winding;
13. Check that the power factor of the primary winding is almost equal to 1;
14. Compute the transformer efficiency  $\eta$ .

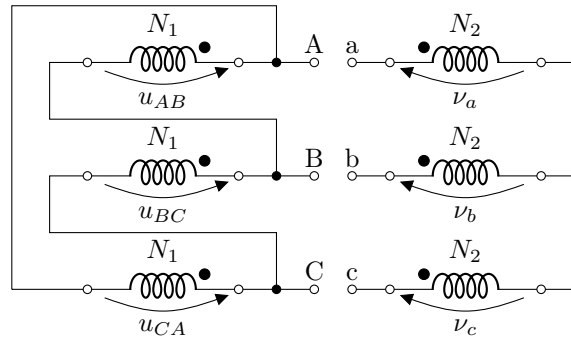


Figure 2: Three-phase transformer.

## 1.2 Exercise 2: Three-phase transformer

Three-phase power transformers are commonly used to adapt power line voltages and to provide some galvanic insulation between two parts of an electrical grid. The three-phase transformer, described by the normalized scheme in Fig. 2, is connected to a balanced three-phase network of composed voltages  $u_{AB}$ ,  $u_{BC}$ ,  $u_{CA}$  of RMS voltage  $U_1$  on the primary side, whereas on the secondary side, a three-phase balanced system of composed voltages  $u_{ab}$ ,  $u_{bc}$ ,  $u_{ca}$  of RMS voltage  $U_2$  is obtained.

The line current intensities in the primary and secondary windings are respectively denoted  $I_1$  and  $I_2$ . The transformer has the following characteristics:

- Apparent nominal power  $S_n = 250$  kVA;
- Composed primary winding RMS voltages  $U_{1n} = 5.2$  kV;
- Nominal frequency  $f_n = 50$  Hz;

and ferromagnetic losses are neglected. To characterize the transformer two tests have been performed:

- Using open secondary windings, the transformer generates a composed voltage of RMS value  $U_{2o} = 400$  V at each secondary winding, for an applied composed nominal voltage of RMS value  $U_{1n}$ ;
- Using short-circuited secondary windings, a composed voltage of RMS value  $U_{1s} = 600$  V is applied at each primary winding for a total primary power  $P = 7.35$  kW, producing line current of RMS intensity  $I_{2s} = 350$  A.

1. Calculate the transformer ratio  $m$ ;
2. For the first test condition (open secondary windings), draw a Fresnel diagram including the primary composed voltages  $u_{AB}$ ,  $u_{BC}$ ,  $u_{CA}$ , the direct secondary voltage  $\nu_a$ ,  $\nu_b$ ,  $\nu_c$  and the secondary composed voltages  $u_{ab}$ ,  $u_{bc}$ ,  $u_{ca}$ ;
3. Express and compute the ‘single-phase’ transformer ratio  $m_c = \frac{N_2}{N_1}$  according to  $m$ ;

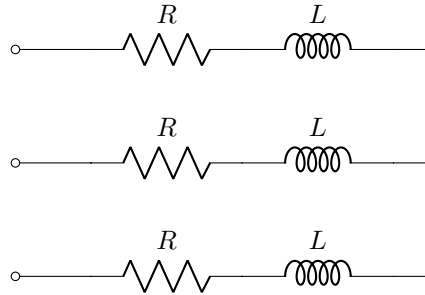


Figure 3: First load connected to the secondary side of the three-phase transformer.

4. Given that the transformer is composed of 3 cores of section  $S = 5 \text{ dm}^2$ , and that the magnetic field amplitude is  $B_m = 1.2 \text{ T}$ , compute the number of turns  $N_1$  of each primary winding and deduce the value of the number of turns of each winding  $N_2$ ;
5. Using a simple single-phase equivalent model (leak resistance and inductance moved to the secondary windings), provide the Thévenin's model seen from a secondary winding and calculate the resistance  $R_s$  and the reactance  $X_s$  of this model;

The nominal regime is now considered by applying the composed nominal voltage  $U_{1n}$  at the primary windings and connecting a three-phase balanced load on the secondary side (detailed in Fig. 3). Each branch is composed of a resistor of value  $R = 554 \text{ m}\Omega$  in serial with a coil of value  $L = 3.05 \text{ mH}$ .

6. Calculate the power factor  $\cos \phi_2$  of this load;
7. Draw the Fresnel diagram corresponding to the balanced single-phase equivalent model. Using an appropriate approximation, deduce the RMS values of the current intensities  $I_2$  and the composed voltages  $U_2$ ;
8. Compute the power  $P_2$  flowing from the transformer to the load;
9. Calculate the transformer efficiency  $\eta$ ;
10. Another load is used (Fig. 4), compute the value of the resistance  $R'$  and the inductance  $L'$  such that this load is equivalent to the one detailed in Fig. 3.

### 1.3 Exercise 3: Single-phase autotransformer

When a galvanic insulation is not required, due to its better efficiency, reduced cost and smaller size, the autotransformer is an interesting alternative to the classical transformer. Autotransformers are also known to have larger short circuit currents which is not always suitable. Two tests are performed on the transformer, illustrated in Fig. 5:

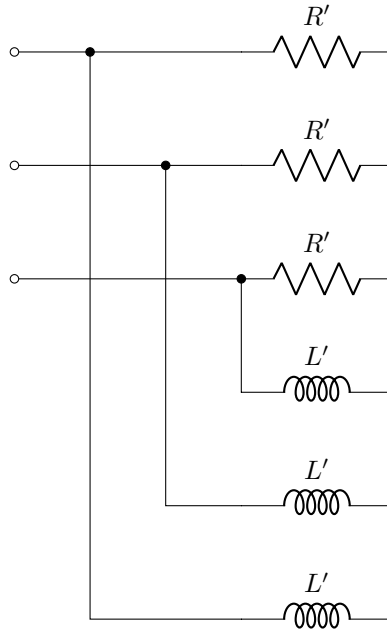


Figure 4: Second load connected to the secondary side of the three-phase transformer.

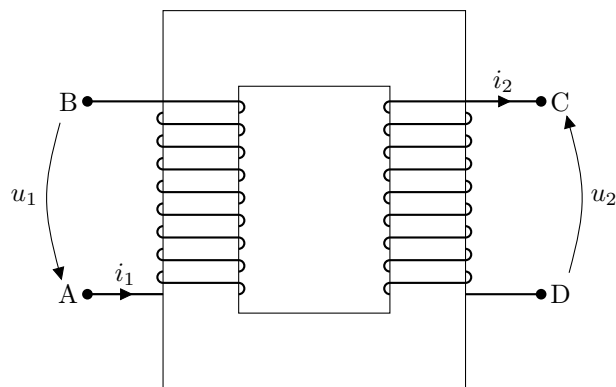


Figure 5: Single-phase transformer.

- Using open secondary winding, the transformer generates a voltage of RMS value  $U_{2o} = 100\text{ V}$  at the secondary winding, for an applied composed voltage of RMS value  $U_1 = 20\text{ V}$  for a drawn current intensity of RMS value  $I_{1o} = 3.2\text{ A}$  and a consumed power  $P_{1o} = 8\text{ W}$ ;
- Using short-circuited secondary winding, a voltage of RMS value  $U_{1s} = 0.8\text{ V}$  for a total power of  $P_{1o} = 24\text{ W}$  is measured, causing a current flow of RMS value  $I_{2s} = 10\text{ A}$  through the secondary winding.

Considering a simplified equivalent model of the transformer (leak resistance and inductance moved to the secondary windings):

1. Calculate the transformer ratio  $m$ ;
2. Calculate the leakage resistance  $R_l$  and the leakage inductance  $L_m$ ;
3. Compute the values of the resistance  $R_s$  and the reactance  $X_s$  corresponding to the inductance  $L_s$  of the secondary winding.

Using the transformer connected to a load on the secondary side drawing a current of RMS value  $I_2 = 12\text{ A}$  with a power factor  $\cos \phi_2 = 0.8$  (the current is lagging the voltage), a RMS voltage of  $U_1 = 20\text{ V}$  is applied to the primary winding.

4. Calculate the RMS voltage  $U_2$  appearing across the secondary winding by using a wise approximation of the voltage dropout  $\Delta U_2$  and justify that the approximation is relevant;
5. Deduce the active power  $P_2$  provided to the load;
6. Calculate the RMS current  $I_1$  on the primary side;
7. Compute the transformer efficiency  $\eta$ .

To turn the transformer into an autotransformer, the terminals  $A$  and  $D$  are connected together, such that the primary winding is located between  $C$  and  $B$  and the secondary between  $C$  and  $D$  (Fig. 6).

8. Compute the RMS voltage  $U'_1$  to be applied on the primary winding to reach a RMS voltage value of  $U_{2o} = 100\text{ V}$  at the terminals of the secondary winding;
9. Calculate the RMS current  $I'_{1o}$  drawn at the primary winding in the case of an open secondary winding;
10. Neglecting the open circuit magnetomotive force, provide the Thévenin's model of the secondary winding including the electromotive force as well as the impedance components namely  $R'_s$  and  $X'_s$ ;
11. The autotransformer is now loaded by a resistor  $R$  in parallel with a capacitor  $C$ , calculate the value of the product  $RC$  required to maintain the open circuit voltage RMS value;
12. In the same situation, determine the resistor value  $R_m$  maximizing the autotransformer efficiency and compute the resulting efficiency  $\eta_m$ .

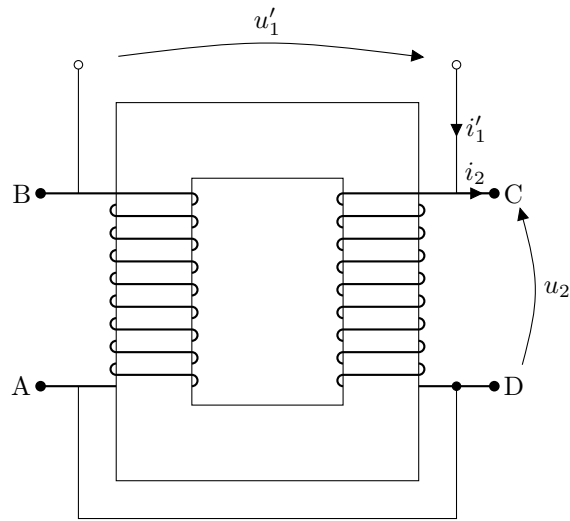


Figure 6: Single-phase autotransformer.

## 2 AC synchronous machines

### 2.1 Exercise 4: Constant air gap alternator

Several alternators are used on airplanes which, coupled to the reactors, feed all the necessary onboard electrical grids. Those alternators are characterized by the higher frequency of the generated voltage and currents compared to alternators coupled to 50 Hz or 60 Hz electrical grids. Moreover, due to the variable speed of the airplane reactors, the delivered frequency is not constant. The considered constant air gap three-phase alternator and its rotor winding are coupled following a star shape. Magnetic leakage, saturation, hysteresis and Eddy currents will be neglected.

For a rotating speed of the alternator shaft of  $n = 11\,100$  RPM, the frequency of the delivered voltages and currents is  $f = 370$  Hz for a nominal apparent power  $S_n = 150$  kVA and a direct voltage of RMS value  $V_n = 115$  V. The rotation speed of the reactor  $n_e$  varies from 4160 RPM to 9000 RPM. The alternator is therefore coupled to the reactor through a gear box of ratio  $k_m = \frac{n}{n_e} = 2.67$ .

The excitation of the alternator is controlled such that the output voltage of the alternator is 115 V (direct voltage or 200 V for the composed voltage). This excitation consists of an inversed alternator coupled with a permanent magnet alternator (Fig. 7).

1. Explain how the excitation system works. What are the main advantages of such a system?
2. Express the frequency of the generated voltages and currents  $f$  with respect to the rotation speed of the reactor  $n_e$ , the gear box ratio  $k_m$  and the number of pairs of poles of the alternator  $p$ ;
3. Deduce the number of pair of poles, as well as the minimal and maximal values  $f_{\min}$ ,  $f_{\max}$  of the generated voltages and currents;



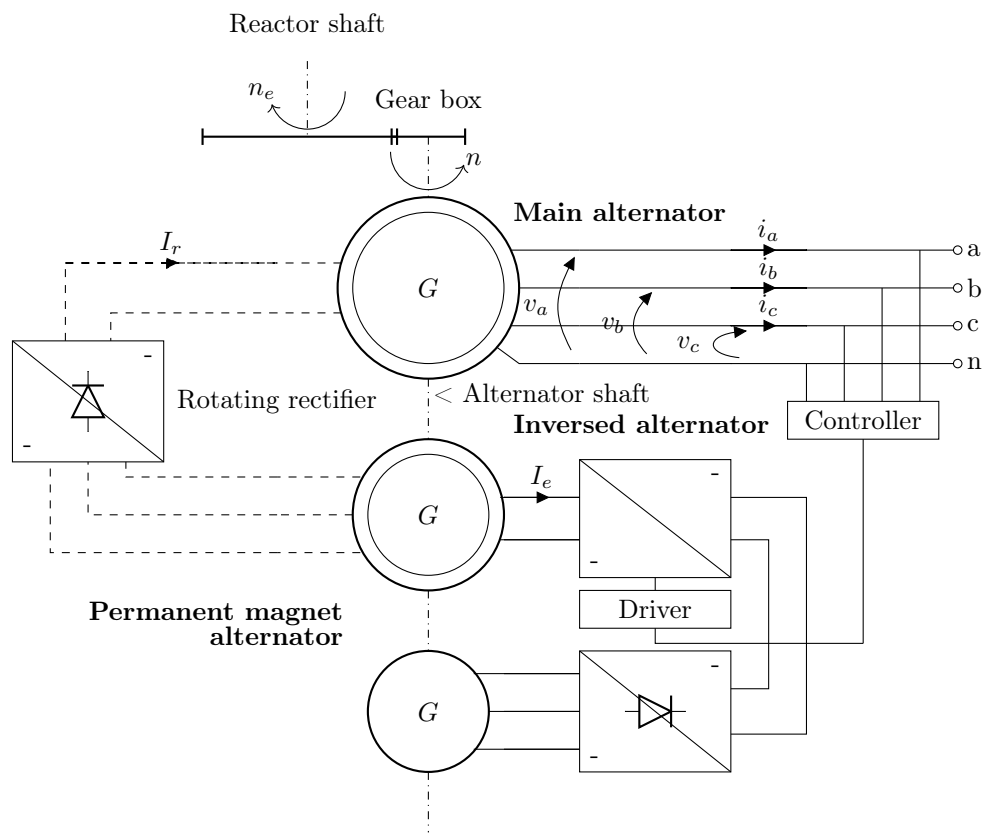


Figure 7: Excitation system of the alternator.

4. For an airplane, justify the relevance of a system working at a variable frequency in the targeted range;
5. Calculate the nominal RMS current  $I_{sn}$  of the line currents of the alternator;
6. The flux generated by a pole is:

$$\phi = \Phi_M \cos(p(\Omega t - \theta_0)),$$

where  $\Phi_M$  is the flux amplitude,  $p$  the number of pairs of poles,  $\Omega$  the speed of rotation,  $t$  the time variable and  $\theta_0$  the initial angular position of the rotor. Express the electromotive force  $e_s$  induced in a single turn of the rotor with respect to  $\Phi_M$ ,  $f$ ,  $t$  and  $\theta_0$ . Deduce the RMS value  $E_s$  of  $e_s$  with respect to  $\Phi_m$  and  $f$ ;

7. The RMS value  $E$  of the induced electromotive force in a phase is  $E = k_b N_s E_s$  where  $k_b = 0.850$  is the coil factor and  $N_s = 16$  is the number of turn per phase. The magnetic circuit is built using laminations allowing to reach a maximal magnetic field corresponding to a flux amplitude  $\Phi_{M0} = 6.84$  mWb and a current  $I_{r0} = 2.95$  A. Where does the coil factor comes from? Express the RMS value  $E$  of the electromotive force induced in each phase with respect to  $k_b$ ,  $N_s$ ,  $\Phi_{M0}$ ,  $I_{r0}$ ,  $I_r$  and  $f$ . Plot  $E$  with respect to  $I_r$  in the range between  $f_{\min}$  and  $f_{\max}$  and conclude.

The stator of the machine is composed of three-phase windings whose phases are noted  $a$ ,  $b$  and  $c$ , while the rotor is composed of a inductor winding  $f$  (Fig. 8). Each phase has an impedance composed of a resistance  $R_s$ , a self inductance  $L_s$  and a mutual inductance  $M_s$  with respect to each other phase. The mutual inductances between each phase and the inductor phase have a sinusoidal pulsation with respect to the rotation angle  $\theta$ :

$$\begin{aligned} M_{af} &= M \cos(p\theta) \\ M_{bf} &= M \cos(p\theta - \frac{2\pi}{3}) \\ M_{cf} &= M \cos(p\theta + \frac{2\pi}{3}) \end{aligned}$$

8. Express the total fluxes  $\Psi_a$ ,  $\Psi_b$  and  $\Psi_c$  crossing the phase windings  $a$ ,  $b$  and  $c$  with respect to the flowing current intensities  $i_a$ ,  $i_b$  and  $i_c$ , the excitation current intensity  $I_r$ , the self inductance  $L_s$ , the mutual inductance  $M_s$ ,  $M$  and the angle  $p\theta$ ;
9. Express the voltages  $\nu_a$ ,  $\nu_b$  and  $\nu_c$  across the phase windings  $a$ ,  $b$ ,  $c$  with respect to  $i_a$ ,  $i_b$  and  $i_c$ , the total flux derivatives  $\Psi_a$ ,  $\Psi_b$  and  $\Psi_c$  and  $R_s$ ;
10. Show that the direct voltages of the stator can be written:

$$\begin{aligned} \nu_a &= e_a - R_s i_a - \mathcal{L} \frac{di_a}{dt} \\ \nu_b &= e_b - R_s i_b - \mathcal{L} \frac{di_b}{dt} \\ \nu_c &= e_c - R_s i_c - \mathcal{L} \frac{di_c}{dt} \end{aligned}$$

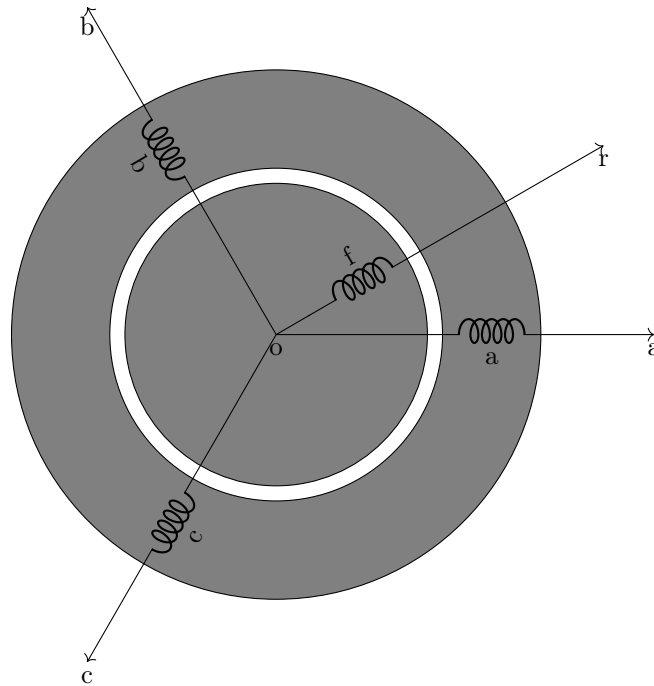


Figure 8: Electrical representation of the alternator windings.

Express the electromotive forces  $e_a$ ,  $e_b$  and  $e_c$  with respect to  $M$ ,  $I_r$ ,  $\omega$ ,  $t$ ,  $p$  and  $\theta_0$ , and their common RMS value  $E$  with respect to  $M$ ,  $I_r$  and  $\omega$ . Explain the significance of  $\mathcal{L}$  and reexpress it in terms of  $L_s$  and  $M_s$ .

The single-phase equivalent model of Behn-Eschenburg is now considered with  $R_s = 0.4 \text{ m}\Omega$ . To characterize the alternator two tests have been performed:

- Using open stator windings, at the speed of rotation  $n = 11\,000 \text{ RPM}$ , the RMS direct voltage values have been measured with respect to the RMS current intensity  $I_r$  flowing through the inductor (Table 1);
  - Using short-circuited stator windings, at the speed of rotation  $n = 11\,000 \text{ RPM}$ , the RMS current intensity  $I_c$  have been measured with respect to the RMS current intensity  $I_r$  flowing through the inductor (Table 1).
11. Knowing that  $E = \lambda\omega I_r$ , compute the value of the coefficient  $\lambda$  for  $I_r = 0.4, 3.0, 5.4 \text{ A}$ ;
  12. Plot the open stator windings curve,  $E$  with respect to  $I_r$ , for  $f_{\min} = 370 \text{ Hz}$  and  $f_{\max} = 770 \text{ Hz}$ ;
  13. Calculate the synchronous reactance  $X$  for the linear part of the curve;
  14. Plot the short-circuited stator windings curve,  $I_c$  with respect to  $I_r$ , for  $f_{\min} = 370 \text{ Hz}$  and  $f_{\max} = 770 \text{ Hz}$ ;

Table 1: Alternator open and short-circuited stator windings test measurements.

$I_r$ [A]	$E$ [V]	$I_s$ [A]
0.4	21.2	94.8
0.8	42.2	190
1.2	63.6	284
1.6	84.8	379
2	106	474
2.4	122	569
3	137	670
3.6	143	770
4.2	145	860
4.8	147	948
5.4	148	1040

15. The alternator is connected to a star-shaped load composed of 3 resistors of value  $R_L = 0.5 \Omega$  working at a frequency  $f = 500 \text{ Hz}$  for an excitation current  $I_r = 2 \text{ A}$ .
  - (a) Calculate the stator RMS current and voltage values  $I_s$  and  $V_s$ ;
  - (b) Sketch the Behn-Eschenburg diagram;
  - (c) Explain how  $I_s$  and  $V_s$  vary when the frequency increases;
16. Working at constant  $I_r$ , a balanced inductive load is now considered with a corresponding impedance  $Z_c = R_c + j\omega L_c$  for each phase.
  - (a) Sketch the Behn-Eschenburg diagram for a power factor  $\cos \phi = 0.75$ ;
  - (b) Express the stator RMS voltage  $V_s$  with respect to  $\lambda$ ,  $\omega$ ,  $R_c$ ,  $L_c$ ,  $L$  and  $I_r$ ;
  - (c) Express the resistive torque  $\Gamma_r$  with respect to  $p$ ,  $\lambda$ ,  $\omega$ ,  $R_c$ ,  $L_c$ ,  $L$  and  $I_r$ ;
  - (d) Knowing that  $R_c = 0.5 \Omega$  and  $L_c = 150 \mu\text{H}$ , compute  $I_s$ ,  $V_s$  and  $\Gamma_r$  for  $f_{\min}$  and  $f_{\max}$  for  $I_r = 0.4 \text{ A}$ ,  $3.0 \text{ A}$  and  $5.4 \text{ A}$ ;
  - (e) How does the frequency variation influence the load power factor?

## 2.2 Exercise 5: Three-phase turbo-alternator

Turbo-alternators are alternators coupled to turbines allowing to convert the mechanical power of a moving fluid (steam or liquid) to electrical power. In this exercise a turbo alternator, built with a star-shaped rotor, has the following nominal characteristics:

- Power  $P_n = 600 \text{ MW}$ ,
- Frequency  $f_n = 50 \text{ Hz}$ ,
- Speed of rotation  $n_n = 3000 \text{ RPM}$ ,
- Power factor  $\cos \phi_n = 0.9$ ,

Table 2: Alternator open circuit stator windings test measurements.

$I_e$ [A]	$E_v$ [kV]
400	5.2
700	9.1
963	11.5
1200	13
1450	14
1900	15

- RMS value of the composed voltages  $U_n = 20$  kV,
- Ferromagnetic losses  $p_f = 543$  kW,
- Mechanical losses  $p_m = 1.35$  MW,
- Rotor resistance  $R_e = 0.17 \Omega$ ,
- Excitation system efficiency  $\eta_e = 0.92$ ,
- Stator phase resistance  $R = 2.3$  m $\Omega$ .

To characterize the turbo-alternator three tests have been performed:

- Using open stator windings, at the nominal speed of rotation  $n_n$ , the RMS direct voltage values have been measured with respect to the RMS current intensity  $I_e$  flowing through the inductor (Table 2);
- Using short-circuited stator windings, at the nominal speed of rotation  $n_n$ , using an excitation current of RMS value  $I_e = 1.18$  kA has allowed a current flow in each phase winding of the stator reaching the half of the RMS nominal value;
- Using an inductive load, an excitation current of RMS value  $I_e = 2.36$  kA has generated a voltage across each stator winding of half the RMS nominal value.

1. Calculate the nominal RMS intensity  $I_n$  of the stator currents;
2. Compute the total losses and the turbo-alternator efficiency at the nominal operating point, knowing the RMS excitation current value is  $I_e = 3.2$  kA;
3. Calculate the needed mechanical power for each of the considered test;
4. Calculate the (unsaturated) synchronous reactance  $X_d$  of the turbo-alternator.

Using Potier diagram with the experimental measurements

5. Calculate the leak partial reactance  $\lambda$  of the stator;
6. Calculate the equivalence ratio  $\alpha = \frac{I_e}{I}$  between the intensity of the current in the rotor  $I_e$  and the current in the stator  $I$ ;

7. Neglecting resistive losses in the rotor, plot Potier diagram for the nominal operating point and verify that, with a value of the resulting excitation current  $I_{er} = 1.45 \text{ kA}$ , the corresponding intensity  $I_e$  of the excitation current is equal to  $I_e = 3.2 \text{ kA}$ ;
8. Draw the internal lag  $\delta$  and give its value;
9. Compute the RMS value  $E_s$  of the synchronous electromotive force;
10. Calculate the saturated synchronous reactance  $X_d$ .

### 2.3 Exercise 6: Alternator and synchronous condenser

A synchronous condenser is a DC excited synchronous motor, whose rotating shaft is not connected to any load. By controlling its field (or the excitation current), using a voltage regulator, the condenser is able to generate or absorb reactive power as needed to adjust the voltage on the power grids, or to improve the power factor.

- Synchronous speed: 428 RPM for 14 poles,
- Star-shape coupling with a voltage  $V = 8.95 \text{ kV}$  between phase and neutral,
- Nominal intensity  $I_n = 6.33 \text{ kA}$ ,
- Apparent nominal power  $S_n = 170 \text{ MVA}$ ,
- Nominal synchronous reactance  $X_n = 1.2 \Omega$  at the nominal frequency  $f_n = 50 \text{ Hz}$ .

The machine is used as an alternator, providing active power  $P_n = 100 \text{ MW}$  and reactive power  $Q_n = 50 \text{ Mvar}$  to the power grid.

1. Calculate  $\cos \phi$ ,  $\phi$  and the lines current intensities  $I$ .

The machine is now turned into a freely spinning motor, keeping the excitation current constant and assuming  $P \approx 0 \text{ W}$ ,

2. Calculate the reactive power  $Q < 0$  provided by the motor (and thus consumed by the motor).

Going back in the alternator mode, the machine is working at a constant power  $P_n = 100 \text{ MW}$  and exchanges reactive power from  $Q_{\min} = -100 \text{ Mvar}$  to  $Q_{\max} = 100 \text{ Mvar}$ ,

3. Between  $Q_{\min}$  and  $Q_{\max}$ , plot  $I$  with respect to the electromotive force  $E$ .

## 3 AC asynchronous machines

### 3.1 Exercise 7: Asynchronous motor 1

A star-shaped asynchronous motor of a high-pressure washer cleaner has the following nominal characteristics:

- Power  $P_n = 5.5 \text{ kW}$ ,
- RMS composed voltage value  $U_{sn} = 400 \text{ V}$ ,
- Frequency  $f_n = 50 \text{ Hz}$ ,
- RMS line current intensities  $I_{sn} = 11 \text{ A}$ ,
- Speed of rotation  $n_n = 1460 \text{ RPM}$ .

Using a single-phase equivalent model of the asynchronous motor when needed,

1. Calculate the synchronous speed of rotation  $n_s$ , the number of pair of poles of the motor and the nominal slip  $g_n$ ;
2. Determine the value of the stator resistance  $R_s$  given that a current of RMS value  $I_0 = 10 \text{ A}$  flows when a voltage of RMS value  $U_0 = 20.6 \text{ V}$  is applied;
3. At the nominal operating point, without mechanical load, the motor draws a current of RMS value  $I_{sv} = 3.07 \text{ A}$  for an active power  $P_{sv} = 245 \text{ W}$ . Calculate the overall losses and calculate the resistance modelling ferromagnetic losses  $R_f$  and the statoric inductance  $L_s$ , assuming that mechanical losses equals ferromagnetic losses;
4. At the nominal operating point, calculate the transmitted power from the stator to the rotor and the Joules losses in the stator  $p_{js}$  and deduce the total consumed power  $P_s$ ;
5. Calculate the rotoric resistance  $R$  and the leak inductance  $L$  seen from the stator;
6. At the nominal operating point, calculate the mechanical torque  $\Gamma_{un}$  and the electromagnetic torque  $\Gamma_n$ , the power factor  $\cos \phi_n$  and the efficiency  $\eta_n$ ;
7. Compute the RMS value  $I_s$  of the line currents, and the power factor  $\cos \phi$  at a rotation speed of 0 RPM.

### 3.2 Exercise 8: Asynchronous motor 2

On the nameplate of an asynchronous motor of a fan used in an air handling unit, the following characteristics are read:

4.4 kW; 230/400 V; 15.5/9 A; 50 Hz; 4 poles

Using a single-phase equivalent model of the asynchronous motor:

1. Explain the meaning of each element on the nameplate;
2. The motor is used on a 230 V network, explain which winding coupling should be used for the stator;
3. Calculate the synchronous frequency of rotation  $n_s$  and the synchronous speed of rotation  $\Omega_s$ ;

4. Given that the (DC) resistance value measured between two stator terminals is  $R_a = 0.654 \Omega$ , compute the value of the statoric resistance  $R_s$  of the equivalent single-phase model;
5. A calibrated motor is used to rotate the shaft of the unpowered considered motor, upto reaching the synchronous speed, at which the calibrated motor consumes 86 W. Calculate the mechanical losses of the motor and explain why assuming that these mechanical losses remain constant is a good approximation;
6. At the nominal operating point, without mechanical load, the motor draws a current of RMS value  $I_{sv} = 3.82 \text{ A}$  for an active power  $P_{sv} = 300 \text{ W}$ . Calculate the resistance modelling ferromagnetic losses  $R_f$  and the statoric inductance  $L_s$ ;
7. The rotor shaft of the motor is stalled while a voltage of RMS value  $U_{sc} = 57.5 \text{ V}$  is applied for a consumed active power  $P_{sc} = 374 \text{ W}$  and active power  $Q_{sc} = 1.09 \text{ kvar}$ . Calculate the rotoric resistance  $R$  and the leak inductance  $L$  seen from the stator.

A direct voltage of value  $V_s$  and frequency  $f$  is applied on each phase of the motor.

8. Using single-phase equivalent model of the asynchronous motor, express the RMS current value  $I$  in terms of  $V_s$ ,  $R_s$ ,  $R$ ,  $g$  et  $X$ ;
9. Calculate the transmitted power from the stator to the rotor;
10. Calculate the electromagnetic torque  $\Gamma$  and give the maximal reachable torque  $\Gamma_{\max}$  after showing that  $\Gamma$  is maximal for a slip value  $g_m$ ;
11. Plot  $\Gamma$  with respect to  $g$  for an applied voltage  $V_s$  equal to  $V_{sn}$ ,  $\frac{V_{sn}}{\sqrt{2}}$  and  $\frac{V_{sn}}{2}$ ;
12. Explain why a control on the rotor voltages is not suitable for speed variation for load having constant resistive torque;
13. To limit the peak current when starting the motor, a star/delta starter is frequently used. Assuming that this transient mode is much more longer compared to period corresponding to the frequency  $f$  of the applied voltages, calculate the RMS current values of the line currents compared to those drawn by using a star/delta starter.

### 3.3 Exercise 9: Wind turbine

Due to the variations of wind conditions, coupling a synchronous generator, needing a constant speed of rotation, is not feasible. An asynchronous generator (hypersynchronous) is therefore generally used to allow more flexibility in the speed of rotation of the blades. A variable speed wind turbine is studied in this exercise. It is possible to tune the slip by using a wound rotor (not a squirrel cage) typically from 0% to 10% by acting on the resistances  $R$  of the rotor. During wind gust, the value  $R$  is increased, to increase the slip value (in absolute



Table 3: Electromotive force with respect to the rotation speed for an excitation current  $I_{e1} = 0.35$  A.

$n$ [RPM]	$E$ [V]
1670	240
1510	220
1380	200
1040	150
820	120
510	75
110	20
0	0

value, since it is negative) allowing thus to increase the speed of rotation and smoothen the power transmitted to the electrical grid.

The studied wind turbine has a 4-poles asynchronous generator of nominal power  $P_n = 800$  kW connected to an electrical grid having a composed voltage  $U_{sn} = 690$  V. The resistance values  $R$  of the rotor can be tuned from  $0$  m $\Omega$  to  $9$  m $\Omega$ , in addition to the resistance value of the windings  $R_R = 3$  m $\Omega$ .

1. Express the input shaft torque  $\Gamma_i$  with respect to the speed of rotation  $n$  ;
2. Calculate the operating point (speed and slip) of the generator for  $R = 0$  m $\Omega$ ,  $P = 2$  MW and  $R = 9$  m $\Omega$ ,  $P = 4$  MW.

## 4 DC machines

### 4.1 Exercise 10: DC brushed motor

The motor of a hammer drill has the following characteristics:

- independant excitation DC machine,
- 2 poles (1 pair),
- Nominal power  $p_{un} = 800$  W,
- Nominal speed of rotation  $n_n = 1500$  RPM,
- Nominal power voltage  $U_n = 220$  V,
- Nominal rotor current intensity  $I_n = 4.6$  A,
- Nominal stator current intensity  $I_{en} = 0.35$  A.

Using two different excitation currents  $I_e$ , the electromotive force has been determined for different rotation speeds (Tables 3 et 4).

1. Plot  $E$  with respect to  $n$  for  $I_{e1}$  and  $I_{e2}$  and justify the shape of the curves;
2. Show that the flux  $\Phi$  is not proportional to the excitation current intensity  $I_e$ .

Table 4: Electromotive force with respect to the rotation speed for an excitation current  $I_{e1} = 0.20$  A.

$n$ [RPM]	$E$ [V]
1800	186
1450	150
1150	120
850	90
560	60
260	30
0	0

Table 5: Electromotive force with respect to the excitation current at nominal constant rotation speed.

$I_e$ [A]	$E$ [V]
0.39	219
0.35	210
0.33	204
0.31	198
0.3	194
0.28	188
0.26	179
0.24	168
0.22	158
0.2	147
0.18	137
0.17	130
0.14	107
0.13	100
0.11	87
0.1	78
0.08	65
0.07	56

Table 6: Voltage across the rotor  $U$  and current drawn in the rotor  $I$  for different excitation currents value  $I_e$ .

$I_e$ [A]	$U$ [V]	$I$ [A]
0.4	222	0.43
0.35	213	0.44
0.3	198	0.45
0.25	176	0.48
0.2	151	0.56
0.15	120	0.66
0.1	85	0.92

Maintaining the nominal speed of rotation, the electromotive force is measured for different excitation currents  $I_e$  (Table 5).

- Plot  $E$  with respect to  $I_e$  and justify the shape of the curves.

Some measurements have allowed to quantify the stator resistance value, which is  $R_e = 512.1 \Omega$  and the rotor resistance value is  $R = 4.6 \Omega$ .

- Draw the equivalent model of the motor.

A test at constant nominal speed has been performed to measure the voltage across the rotor  $U$  and the current drawn in the rotor  $I$  for different excitation currents value  $I_e$  (Table 6).

- Plot the collective (i.e. ferromagnetic plus mechanical) losses  $p_c$  with respect to  $I_e$ ;
- For the linear part of the curve, determine the mechanical losses  $p_m$  at the nominal speed of rotation.

A hole is drilled using the drill. The nominal speed of rotation remains constant while the rotor draws a current of  $I_0 = 3 \text{ A}$  when a voltage  $U_0 = 212 \text{ V}$  is measured on the rotor terminals.

- Calculate the electromotive force and deduce the value of the excitation current  $I_{e0}$ ;
- Compute the shaft output power  $P_u$ ;
- Deduce the resistive torque  $\Gamma_r$  induced by the drilling process.

## 4.2 Exercise 11: Regenerative braking

Hybrid electric vehicles are generally provided with regenerative braking, allowing to load onboard battery when the vehicle is braking or when the vehicle acts as a driving load. In this exercise, the DC motor, having an electromotive force  $E$  and internal resistance  $R = 0.5 \Omega$  is connected (when the regenerative braking is active) to a battery delivering a current  $I$  under the voltage  $V = 100 \text{ V}$  using a chopper DC-DC converter (Fig. 9).

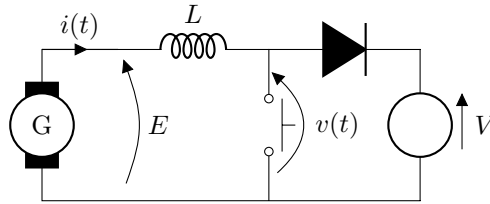


Figure 9: DC-DC converter for regenerative braking.

1. Express the voltage  $v$  with respect to  $i$ ,  $E$ ,  $R$  and  $L$  and express  $v_m$  the mean value of  $v$ ;
2. Compute the duty cycle allowing to obtain  $v_m = 60 \text{ V}$ ;
3. Compute the mean braking current  $I_m$  when the motor delivers an electro-motive force  $E = 70 \text{ V}$  for  $v_m = 60 \text{ V}$ ;
4. Calculate the braking power  $E I_m$  and the braking torque  $C_m$  if the motor speed of rotation is  $n = 955 \text{ RPM}$ .

### 4.3 Exercise 12: DC generator-motor mechanical coupling

Two identical DC machines, rated  $P_n = 200 \text{ kW}$ ,  $V_n = 520 \text{ V}$ ,  $I_n = 420 \text{ A}$ ,  $n_n = 1000 \text{ RPM}$ , are tested by coupling their shafts together, so that one will act as a generator to supply power to the other, which will act as a motor and drive the generator. This is a common test procedure to avoid the use of high-power electrical supplies and heavy loading rigs. The two rotors are connected in parallel (Fig. 10) while both machines are separately excited by manually adjusting the excitation current. To startup, the excitation current is set to its nominal value while the applied voltage  $V$  is progressively increased to  $V_n$  to reach the unloaded speed  $n_u = 1040 \text{ RPM}$ . The couple generator-motor draws  $37 \text{ A}$  from the DC source, which is mainly due the torque required to overcome windage, friction, and iron losses. Knowing the the armature resistance value of each machine is  $R = 0.05 \Omega$  and ignoring commutation losses (brush volt-drop):

1. Calculate the electromotive force at no-load;
2. Compute the power loss at no-load;
3. Compute the mechanical power losses;
4. Calculate the torque at no-load.

The excitation current of the generator is now reduced to reach the nominal value of the output current  $I_n$ .

5. Calculate the speed of rotation of each machine;
6. Calculate the current drawn by the motor;
7. Calculate the shaft output torque;

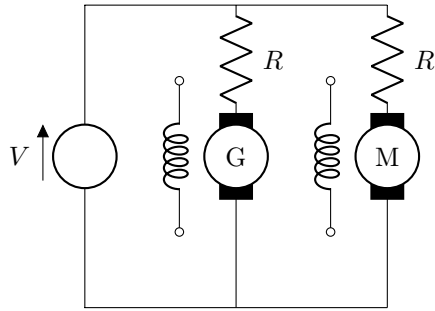


Figure 10: DC generator-motor mechanical coupling.

8. Compare the power supplied by the DC source to the nominal power of each machine and conclude about the relevance of such a test and about the load level reached during this test.