Asynchronous machines

**Stator:** magnetic circuit and polyphased winding (usually 3-phase), with \( p \) pairs of poles, with polyphased currents

**Rotor:** magnetic circuit and electric circuit...

... or made of copper or aluminum bars, short-circuited at each extremity of the rotor → *squirrel-cage rotor*

... either made of a polyphased winding, (with star connection in order to avoid currents between rotor phases) connected to short-circuit rings on the rotor axis; brushes connect these rings to external resistances → *wound rotor*
General principle

When 3-phase currents of pulsation $\omega$ flow in the stator windings they create a rotating magnetic flux density with $p$ pairs of poles, rotating with angular speed $\omega/p$.

This field induces 3-phased e.m.f.s of pulsation $\omega$ in the (stationary) rotor windings, and thus 3-phase currents of pulsation $\omega$.

The interaction between the rotating stator flux density and the induced currents in the rotor creates a torque, which puts the rotor into movement.

(asynchronous machine, or induction machine)

$\text{Wound rotor machine initially at rest ...}$

$g = 1$

$\text{def}$

$g = \text{slip}$

$0 \leq g \leq 1$

$\dot{\theta} = (1 - g) \frac{\omega}{p}$

Angular speed of the stator rotating field with respect to the rotor

$\frac{\omega}{p} - \dot{\theta} = \frac{g \omega}{p}$

e.m.f.s and induced currents with pulsation $g \omega$, and torque

$g = 0$

Synchronous speed

$\dot{\theta}_s = \frac{\omega}{p}$

no e.m.f., no current, no torque
General equations

Stator equation

For one phase

Rotating magnetic flux density in the airgap leads to e.m.f. $E_1$ in stator winding, with pulsation $\omega$

$U_1 = E_1 + (R_1 + j X_1) \bar{I}_1$

$X_1 = \text{Leakage reactance of one stator phase}$
$R_1 = \text{Resistance of a stator phase}$

$E_1 = -j \omega k_1 \Phi_r$
$E_2 = -j g \omega k_2 \Phi_r$

Link equation

$k_1/k_2 = k'_1/k'_2 = n_{tr}$

resulting m.m.f.

$\bar{F}_r = \bar{F}_1 + \bar{F}_2 = -k'_1 \bar{I}_1 + k'_2 \bar{I}_2$

with

$\Phi_r = \frac{\bar{F}_r}{R_{ep}}$

resulting flux

$E_1 = j X_\mu \left( \bar{I}_1 - \frac{\bar{I}_2}{n_{tr}} \right)$

$X_\mu = \text{Magnetizing reactance}$

Rotor equation

Rotating magnetic flux density in the airgap leads to e.m.f. $E_2$ in rotor winding, with pulsation $g\omega$.

$E_2 = (R_2 + j g X_2) \bar{I}_2$

Asynchronous machines
Equivalent circuit

\[ U_1 = E_1 + (R_1 + jX_1)I_1 \] \hspace{1cm} (a)

\[ \frac{E_2}{g} = \left( \frac{R_2}{g} + jX_2 \right)I_2 \]

\[ \frac{E_1}{E_2} = \frac{n_{tr}}{g} \]

\[ E_1 = jX_\mu \left( I_1 - \frac{I_2}{n_{tr}} \right) \]

\[ E_1 = \left( \frac{n_r^2 R_2}{g} + j n_r^2 X_2 \right) \frac{I_2}{n_r} \] \hspace{1cm} (b)

| \[ E_1 = \left( \frac{R_2'}{g} + jX_2' \right) \bar{I}_2' \] \hspace{1cm} (c) magnetizing current
| \[ I_\mu = \bar{I}_1 - \bar{I}_2' \] |

Simplified equivalent circuit

\[ R_1, R_2', X_1, X_2' \ll X_\mu \] (ratios \(~40\times, 600 \times R\) )

Asynchronous machines
Equivalent circuit parameters

Experimental determination of the parameters

**No load test**

*Under nominal voltage and at synchronous speed*

\[ I_1 << \Rightarrow P_{\text{Joule primaire}} << \]

\[ P_v = P_{\text{mag}} \quad Q_v = 3 \frac{U_1^2}{X_\mu} \]

magnetic losses in the stator laminations

**Short-circuit test**

*Under reduced voltage at zero speed (stationary rotor)*

\[ U_1 << \Rightarrow I_\mu \text{ et } P_{\text{mag}} << \]

\[ P_{cc} = 3 \left( R_1 + R'_2 \right) I_1^2 \quad Q_{cc} = 3 \left( X_1 + X'_2 \right) I_1^2 \]

in the resistances of the stator and rotor windings

in the leakage reactances of the stator and the rotor

Asynchronous machines
Power, torque and efficiency

**Electric power**

\[ P = 3 \, U_1 \, I_1 \, \cos \varphi = 3 \left( R_1 \, I_1^2 + \frac{R'_2}{g} \, I_2^2 \right) \]

Absorbed power

Stator

Power transmitted from stator to rotor

Airgap

Electromagnetic power

Rotor

Electromagnetic torque

Asynchronous machines

**Stator efficiency (1)**

\[ \eta_{\text{stat}} = \frac{P_{\text{st} \rightarrow \text{rot}}}{P} = \frac{3 \, R_1 \, I_1^2}{3 \, R_1 \, I_1^2} = 1 \]

**Rotor efficiency (2)**

\[ \eta_{\text{rot}} = \frac{P_{\text{elm}}}{P_{\text{st} \rightarrow \text{rot}}} = \frac{\frac{3 \, (1-g) \, R'_2 \, I_2^2}{g}}{\frac{3 \, \frac{1-g}{g} \, R'_2 \, I_2^2}{g}} = 1 - g \]

**Mechanical efficiency (3)**

\[ \eta_{\text{mec}} = \frac{P_{\text{elm}}}{P_{\text{st} \rightarrow \text{rot}}} = \frac{3 \, \frac{1-g}{g} \, R'_2 \, I_2^2}{g} \]

**Asynchronous motor efficiency**

\[ \eta = (1-g) \, \eta_{\text{stat}} \, \eta_{\text{mec}} < 1-g \]
Mechanical characteristic

Evolution of the electromagnetic torque $C$ as a function of the slip $g$ (or the rotation speed), for a given stator voltage $U_1$ and pulsation $\omega$

$$C = \frac{3}{\omega} \frac{R'_2}{g} I'^2_2$$

with

$$I'^2_2 = \frac{E_1}{\sqrt{\left(\frac{R'_2}{g}\right)^2 + X'^2_2}} \approx \frac{U_1}{\sqrt{\left(\frac{R'_2}{g}\right)^2 + X'^2_2}}$$

$$C = f(g) \quad \text{with} \quad \begin{cases} U_1 = \text{constant} \\ \omega = \text{constant} \end{cases}$$

Maximum torque $\neq f(R'_2)$

$$C_{g<<} = \frac{3}{\omega} \frac{g U^2_1}{R'_2}$$

$$C_{\text{max}} = \frac{3}{\omega} \frac{U^2_1}{2 X'_2}$$

Nonzero startup torque (at $g = \pm \frac{R'_2}{X'_2}$)

Zero torque at synchronous speed ($g = 0$)

Nonzero startup torque (at $g = 1$)

above synchronous speed ($g < 0$)

Asynchronous machines
A decrease (resp. increase) in speed, or an increase (resp. decrease) in the slip \( g \), leads to an increase (resp. decrease) of the motor torque, from point \( M \rightarrow M' \) (resp. \( M \rightarrow M'' \)). The machine will thus accelerate (resp. decelerate) to reach back point \( M \), where the motor and resistant torques match 
\[ \Rightarrow \text{stable}. \]

A decrease (resp. increase) in speed, or an increase (resp. decrease) in the slip \( g \), leads to a decrease (resp. increase) of the motor torque, from point \( N \rightarrow N' \) (resp. \( N \rightarrow N'' \)). The machine will thus decelerate (resp. accelerate), further increasing the mismatch between the resistant and motor torques (resp. leading to an evolution towards the stable stable \( N'' \rightarrow M \)) 
\[ \Rightarrow \text{unstable}. \]
Speed control

Modification of the rotor resistance $R'_2$

Reduction of speed at the cost of efficiency ($\eta < 1-g$)

Modification of the voltage $U_1$

Small variation range

Fixed limit

Modification of the frequency

Interest in keeping the magnetic flux (hence $U_1/\omega$) constant...

$\Phi_t = \lambda_\mu I_\mu = \frac{X_\mu I_\mu}{\omega} = \frac{E_1}{\omega} \approx \frac{U_1}{\omega}$

Asynchronous machines
Asynchronous motor startup

Startup torque (g = 1)

\[ C = \frac{3p}{\omega} R'_2 \frac{g U_1^2}{R_2'^2 + X'_2^2} \]

Startup current

\[ I_d = \frac{U_1}{\sqrt{(R_1 + R'_2)^2 + (X_1 + X'_2)^2}} \]

usually quite small, since \( R'_2 \ll \)

very large, should be limited...

Squirrel cage motor

... thanks to reduced voltage
(transformer, electronic converter, transient star coupling)

Startup current

... resistances in series with the rotor winding (progressively removed)

Startup possible only for small resistant torque at low speed (e.g. pumps, ventilators, air conditioners)

Wound rotor

Startup torque divided by 3

Startup torque increases and startup current decreases

Asynchronous machines
Special asynchronous motors

Double cage rotor

- Exterior (e) and interior (i)
  - \( R'_e \gg R'_i \)
  - \( X'_e \ll X'_i \)

Skin effect rotor

- Elongated rectangular rotor conductors
  - same behavior as double cage

Startup (\( g=1 \)):
  - \( R'_{i,e} \ll gX'_e \ll gX'_i \)
  - \( C_e \gg C_i \)

Nominal speed (\( g \ll \)):
  - \( R'_e \gg R'_i \gg gX'_{i,e} \)
  - \( C_i \gg C_e \)

Skin effect rotor

- Material magnetic (rotor)

Startup (\( g=1 \)):
  - high apparent resistance
  - \( C_e \gg C_i \)

Nominal speed (\( g \ll \)):
  - low apparent resistance

Asynchronous machines
Circle diagram

\[ \bar{U}_1 = j X \bar{I}_2 + \left( R_1 + R'_2 / g \right) \bar{I}_2 \]

\[ \frac{\bar{U}_1}{jX} = \bar{I}_2 + \frac{R_1 + R'_2 / g}{jX} \bar{I}_2 \]

\[ \Rightarrow \text{Locus of } \bar{I}_2 \text{ is a circle of diameter } U_1/X \]

Active power
\[ P = 3 U_1 I_1 \cos \varphi = 3 U_1 \text{ AM} \]

\[ \text{AM} = \frac{P}{3 U_1} \]  (5)

Reactive power
\[ Q = 3 U_1 I_1 \sin \varphi = 3 U_1 \text{ O'A} \]

\[ O'A = \frac{Q}{3 U_1} \]  (7) = (7a) + (7b)

Reactive power in \( X_\mu \) et \( X \)

Asynchronous machines
Operating modes

The machine
• absorbs electric power \((A_1M_1 \geq 0)\);
• produces mechanical power \((C_1M_1 \geq 0)\).

Asynchronous motor

The machine
• absorbs electric power \((A_2M_2 \leq 0)\);
• absorbs mechanical power \((C_2M_2 \leq 0)\).

Asynchronous generator

The machine
• absorbs electric power \((A_3M_3 \geq 0)\);
• absorbs mechanical power \((C_3M_3 \leq 0)\).

Radiator

Asynchronous machines