Synchronous machines

**Synchronous generator (alternator)**: transforms mechanical energy into electric energy; designed to generate sinusoidal voltages and currents; used in most power plants, for car alternators, etc.

**Synchronous motor**: transforms electric energy into mechanical energy; used for high-power applications (ships, original TGV...)

**Rotor (inductor)**: 2p poles with excitation windings carrying DC current; non-laminated magnetic material

\[ \theta = \frac{\omega}{p} \]

**Stator**: polyphase (e.g. 3-phase) winding in slots; laminated magnetic material
No-load characteristic

Evolution of the voltage $E_v$ in a stator phase vs. intensity of the excitation current $I_e$, for a given rotation speed and with no generated stator current

- The rotor winding, carrying the DC current $I_e$ and rotating at speed $\omega/p$, produces in the airgap a sliding m.m.f. $F_e$ (as seen from the stator).
- $F_e$ generates a magnetic flux density $B_r$ (with the same phase) in the airgap, which induces sinusoidal e.m.f.s $E_v$ in the stator windings, with a phase lag of $\pi/2$.

$$E_v = f(I_e) \quad \text{with} \quad \begin{cases} \text{speed} & \dot{\theta} = \text{constant} \\ I = 0 \end{cases}$$

$$E_v = k_F \dot{\theta} \Phi_v(I_e)$$

Magnetic flux produced by the inductor and seen by the stator winding

Non-linearity with hysteresis
(1) The rotor winding, carrying the DC current $I_e$ and rotating at speed $\omega/p$, produces in the airgap a sliding m.m.f. $F_e$ (as seen from the stator).

(2) The polyphase current $I$ in the stator winding produces a sliding m.m.f. $F_I$ (in phase with $I$).

(3) The resulting m.m.f. is $F_r = F_e + F_I$.

(4) $F_r$ generates a magnetic flux density $B_r$ (with the same phase) in the airgap, which induces sinusoidal e.m.f.s in the stator windings, with a phase lag of $\pi/2$. 

Synchronous machines
Vector diagram with load

**Stator leakage flux**
(see by the stator but not coming from the rotor)

**Slot leakage flux**

**End-winding leakage flux**

**Diagrams**

\[ U = \overline{E}_r - j X_f \overline{I} - R \overline{I} \]

**Resistance of stator winding**

**Equivalent excitation current**

**Synchronous machines**

**E\textsubscript{r} \equiv no-load e.m.f. produced by I\textsubscript{r}**

**e.m.f. induced by leakage flux**

\[ e_\lambda(t) = -\lambda \partial_t i_1 - \lambda_m \partial_t i_2 - \lambda_m \partial_t i_3 \]

\[ = - (\lambda - \lambda_m) \partial_t i_1 \]

because \[ i_1 + i_2 + i_3 = 0 \]

\[ \overline{E}_\lambda = - j X_f \overline{I} \]

**with** \[ X_f = \omega (\lambda - \lambda_m) \]

**Stator leakage reactance**

**total e.m.f.**

\[ \overline{E}_t = \overline{E}_r - j X_f \overline{I} \]

\[ \overline{F}_e = \gamma \overline{I}_e \]

and \[ \overline{F}_r = \delta \overline{I} \]

\[ \overline{I}_r = \frac{\overline{F}_r}{\gamma} = \overline{I}_e + \frac{\delta}{\gamma} \overline{I} \]

\[ \text{def} \quad \overline{F}_r = \overline{I}_e \]

**End-winding leakage flux**

**Slot leakage flux**
‘Which excitation current $I_e$ should one impose in the synchronous machine to reach the functioning point corresponding to a given voltage $U$ and current $I$ in the stator, with a phase shift of $\varphi$ between $U$ and $I$?’

\[ E_r = \text{no-load e.m.f produced by the equivalent current } I_r \]

\[ I_e = I_r - \frac{\delta}{\gamma} \]

\[ E_e = U + R I + j X_f I \]
Reaction

**Demagnetizing reaction**

The m.m.f. is smaller than the no-load m.m.f. ($I_r < I_e$)

**Magnetizing reaction**

The m.m.f. is larger than the no-load m.m.f. ($I_r > I_e$)

Inductive behaviour of the load
(I lagging behind U)

Capacitive behaviour of the load
(I in front of U)

Synchronous machines
**Zero power factor characteristic**

Evolution of the stator voltage $U$ as a function of the excitation current $I_e$, for a given rotation speed and stator current, with a zero power factor.

\[ U = f(I_e) \text{ with } \begin{cases} \text{speed } \dot{\theta} = \text{constant} \\ I = \text{constant} \\ \cos \varphi = 0 \ (\varphi = \pm \pi/2) \end{cases} \]

Example: $\varphi = \pi/2$

\[ I_e \approx I_r + \frac{\delta}{\gamma} I \]

\[ U \approx E_r - X_f I \]

Synchronous machines
Short-circuit characteristic

Evolution of the stator current as a function of the excitation current $I_e$, for a given rotation speed and with the stator windings in short-circuit.

\[ E_r = U + R I + j X_f I \]

For a non-saturated machine:

\[ E_r \approx X_f I \]

with

\[ I_e \approx I_r + \frac{\delta}{\gamma} I \]

\[ I \approx \frac{\beta}{X_f + \beta \frac{\delta}{\gamma}} I_e \]

\[ I = f(I_e) \quad \text{with} \begin{cases} \text{speed } \dot{\theta} = \text{constant} \\ U = 0 \end{cases} \]

Synchronous machines
When the magnetic materials are not saturated, the combined effect of the reaction and of stator leakage fluxes can be taken into account thanks to a single parameter: the synchronous reactance $X_s$.

$$\frac{E_v}{I_e} = \frac{E_r}{I_r} = \text{constant}$$

Equal angles OAB and A’OB’

Similar triangles OAB and OA’B’

A, B and C colinear

$$\bar{E}_v = \bar{U} + R \bar{I} + jX_s \bar{I}$$
Experimental determination of $X_s$

Behn-Eschenburg’s method – Synchronous reactance $X_s$

$U = 0 \implies E_v = (R + jX_s)I_{cc}$

$\implies R + jX_s = \frac{E_v}{I_{cc}}$ with $R << X_s$

$X_s \approx \frac{E_v(I_e)}{I_{cc}(I_e)}$

Approximation when magnetic materials are saturated!
Evolution of the voltage $U$ on a given stator phase as a function of the current $I_{in}$ in this phase, when the alternator drives a load characterized by a constant power factor, at constant speed and excitation.

$$U = f(I) \quad \text{with} \quad \begin{cases} \text{speed } \dot{\theta} = \text{constant} \\ I_e = \text{constant} \\ \cos \varphi = \text{constant} \end{cases}$$
Network connection

Need for interconnection of electric power plants

Economical organization of power production + Stability of the network despite local defects

Synchronization of an alternator on an ideal (infinitely powerful) AC network

Large number of production units in parallel ⇒ constant voltage and frequency

The current should be zero when the connection is made ⇒ 4 conditions

1. same pulsation \( \omega \) (correct rotation speed)
2. same amplitudes for \( E_v \) and \( U \) (adjusting \( I_e \))
3. no phase shift between \( E_v \) and \( U \)
4. identical phase ordering (in a 3-phase system)
Behaviour with load

**Electromagnetic power**

\[ P_{elm} \approx P = 3 \, U \, I \cos \varphi \]

**Active electric power**

\[ C = \frac{P}{\omega / p} = \frac{3 \, p}{\omega} \, U \, I \cos \varphi \]

**Torque**

\[ C = \frac{3 \, p}{\omega \, X_s} \, U \, E_v \sin \delta_{int} \]

**Internal angle**

\[ X_s \, I \cos \varphi = E_v \sin \delta_{int} \]

After network synchronization, there is no exchanged current. Then:

- If mechanical power is provided to the alternator, \( E_v \) gets ahead of \( U \) \( \Rightarrow \delta_{int} \) increases (until the equilibrium of the electromagnetic and mechanical torques)
- If a braking torque is applied to the alternator, \( E_v \) gets behind \( U \) \( \Rightarrow \delta_{int} \) decreases (negative torque)

The variations of the rotor mechanical angle \( \Delta \delta_{mec} \) are proportional to the variations of the internal (electric) angle \( \Delta \delta_{int} \)

\[ \Delta \delta_{mec} = \frac{\Delta \delta_{int}}{p} \]
Behaviour with load

**Static stability**

Increasing the mechanical (breaking) torque leads to an increase of the absolute value of $\delta_{\text{int}}$ and a decrease in the absolute value of $C_{\text{elm}} \Rightarrow \text{unstable}$.

Increasing the mechanical (breaking) torque leads to an increase of $\delta_{\text{int}}$ and $C_{\text{elm}} \Rightarrow \text{stable}$.

The equilibrium is reached when the two torques are equal.

Synchronous machines
Behaviour with load

Power diagram

\[ MB = X_s I \cos \varphi = \frac{X_s}{3} U I \cos \varphi = \frac{X_s}{3} U P = \frac{X_s}{3} P_{\text{elm}} \]

Active power \( P \)

\[ O'B = X_s I \sin \varphi = \frac{X_s}{3} U I \sin \varphi = \frac{X_s}{3} U Q \]

Reactive power \( Q \)

Alternator

Motor

Synchronous machines
**V-curves (Mordey curves)**

Evolution of the stator current $I$ as a function of the excitation current $I_e$ of a synchronous machine connected to an ideal network, at constant active power.

- **Constant active power**
- **Under-excited machine**
- **Over-excited machine**

$\phi > 0$ ...

Network = inductive load
Machine = capacitif system

Static stability limit

Synchronous machines