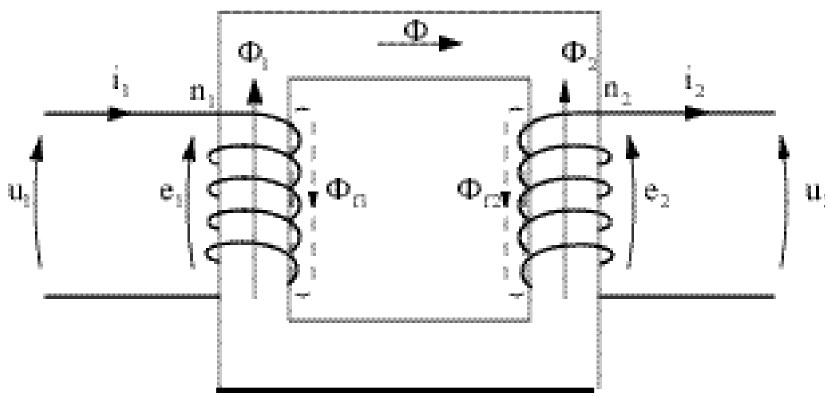
Single-phase transformer

2 windings (primary & secondary) around a magnetic core (magnetic coupling)

Faraday's law

A time-dependent (varying) magnetic flux density in the magnetic core induces (time-dependent) electromotive forces (e.m.f.s) in each winding

The voltage in each winding (~) is proportional to the number of turns



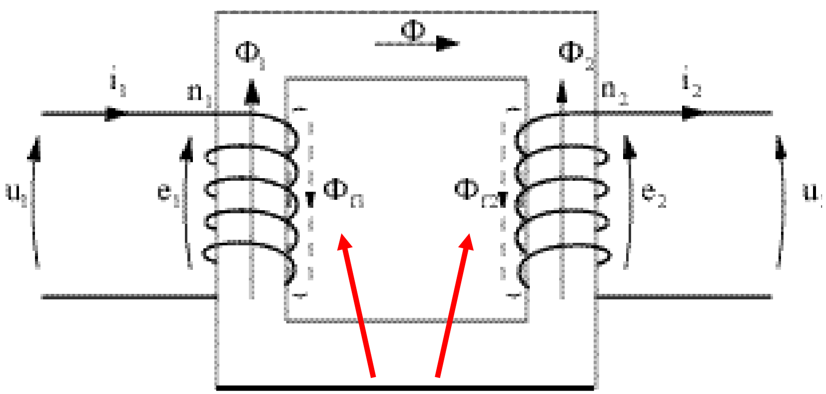
Open or closed secondary …

Electric energy conversion from one voltage level to another

General transformer relations

Useful flux Φ

$$\Phi_1 = \Phi + \Phi_{f1} \qquad \Phi_2 = -\Phi + \Phi_{f2}$$



Leakage flux

$$\Phi_{f1} = \frac{n_1 i_1}{R_{f1}} \quad \text{et} \quad \Phi_{f2} = \frac{n_2 i_2}{R_{f2}}$$

Leakage reluctances

$$\frac{e_1}{e_2} = \frac{n_1}{n_2} = n$$

Transformation ratio

Primary winding equation

$$u_1 = R_1 i_1 + n_1 \partial_t \Phi_1 = R_1 i_1 + n_1 \partial_t \Phi_{f1} + n_1 \partial_t \Phi$$

$$u_1 = R_1 i_1 + \lambda_1 \partial_t i_1 + e_1$$

e.m.f in 1 due to Φ

Secondary winding equation

$$u_2 = -R_2 i_2 - n_2 \partial_t \Phi_2 = -R_2 i_2 - n_2 \partial_t \Phi_{f2} + n_2 \partial_t \Phi$$

$$u_2 = -R_2 i_2 - \lambda_2 \partial_t i_2 + e_2$$

e.m.f. in 2 due to Φ

Coupling equation

$$\Phi = \frac{n_1 i_1 - n_2 i_2}{R} \Rightarrow e_1 = \frac{n_1^2}{R}\left(\partial_t i_1 - \frac{n_2}{n_1}\partial_t i_2\right)$$

Magnetic circuit reluctance

magnetizing inductance λ_μ

Transformer equivalent circuit

$$u_1 = R_1 i_1 + \lambda_1 \partial_t i_1 + e_1$$ **(a)**

$$u_2 = -R_2 i_2 - \lambda_2 \partial_t i_2 + e_2$$

$$\frac{e_1}{e_2} = \frac{n_1}{n_2} = n$$

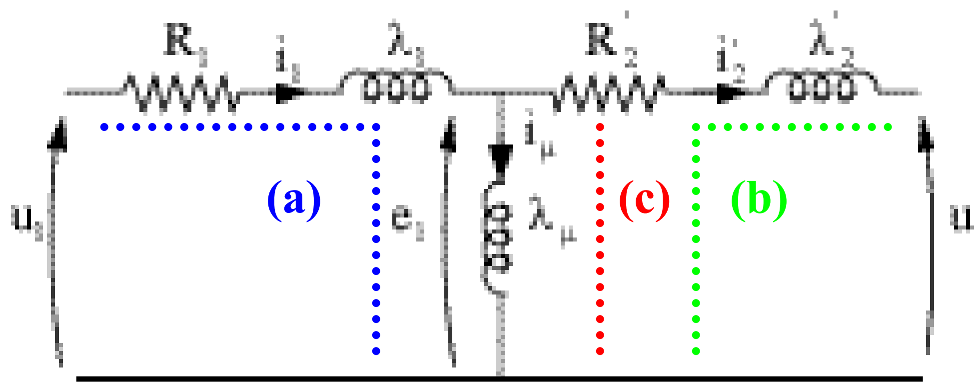
$$\rightarrow \quad e_1 = n u_2 + n^2 R_2 \frac{1}{n} i_2 + n^2 \lambda_2 \frac{1}{n} \partial_t i_2$$

Secondary quantities as seen from the primary

$$e_1 = u'_2 + R'_2 i'_2 + \lambda'_2 \partial_t i'_2$$ **(b)**

$$e_1 = \lambda_\mu \left(\partial_t i_1 - \frac{n_2}{n_1} \partial_t i_2 \right)$$

$$\rightarrow \quad e_1 = \lambda_\mu (\partial_t i_1 - \partial_t i'_2)$$ **(c)**



Complex formalism: phasors

Complex representation of sinusoidal quantities

$$a(t) = A\cos(\omega t + \varphi)$$

Sinusoidal quantity (voltage, current, ...)

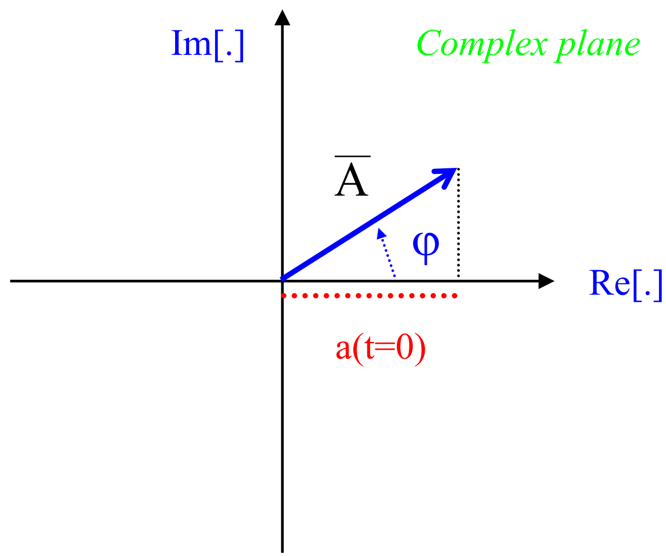
A maximum value (amplitude) of $a(t)$ [V, A, ...]
ω pulsation of $a(t)$ ($2\pi f$, f = frequency) [rad/s]
φ phase of $a(t)$ at $t = 0$ [rad]

$$a(t) = \mathrm{Re}\left(A\, e^{j\varphi}\, e^{j\omega t}\right)$$

with $$e^{j\varphi} = \cos\varphi + j\sin\varphi$$

$$\overline{A} = A\, e^{j\varphi}$$

Quantity as a ***phasor***

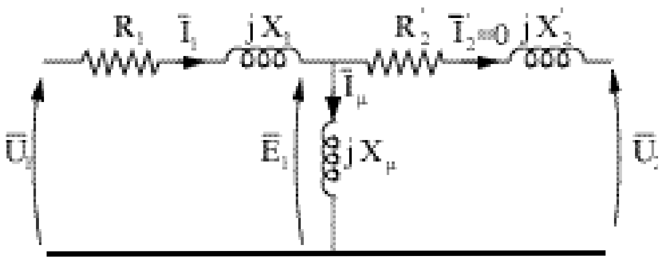


$$\begin{aligned}\mathrm{Re}\left(A\, e^{j\varphi}\, e^{j\omega t}\right) = \mathrm{Re}\left(A\, e^{j\varphi + j\omega t}\right) = \mathrm{Re}\left(A\, e^{j(\omega t + \varphi)}\right) \\ = \mathrm{Re}\left(A\cos(\omega t + \varphi) + j\sin(\omega t + \varphi)\right) \\ = A\cos(\omega t + \varphi)\end{aligned}$$

Limit cases with phasors

No-load case

$$\bar{I}'_2 = 0 \Rightarrow \bar{U}'_2 = \bar{E}_1 = \frac{jX_\mu}{R_1 + jX_1 + jX_\mu}\bar{U}_1$$



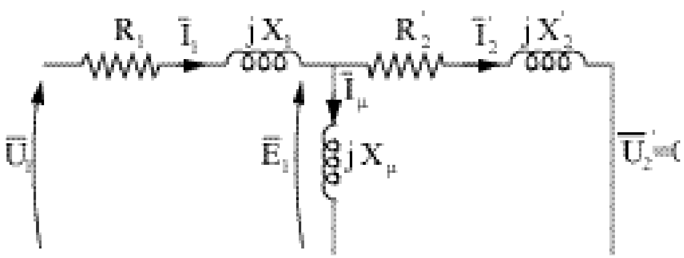
$R_1, X_1 << X_\mu$ **(ratios ~ 400 X, 4000 R)**

$$U'_2 \approx U_1 \Rightarrow \frac{U_1}{U_2} = n$$

$$I_1 = I_\mu << I_{nominal}$$

Short-circuit case

$$\bar{U}'_2 = 0 \Rightarrow \frac{\bar{I}_1}{\bar{I}'_2} = \frac{R'_2 + jX'_2 + jX_\mu}{jX_\mu}$$



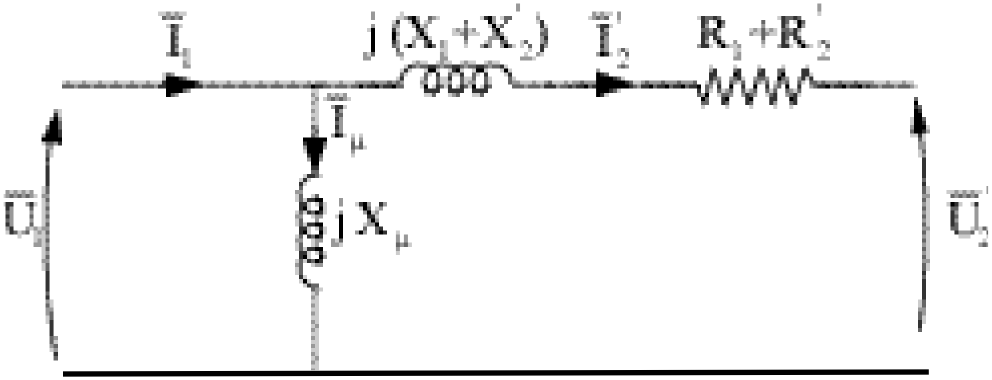
$R'_2, X'_2 << X_\mu$ **(ratios ~ 400 X, 4000 R)**

$$I_1 \approx I'_2 \Rightarrow \frac{I_1}{I_2} = \frac{1}{n}$$

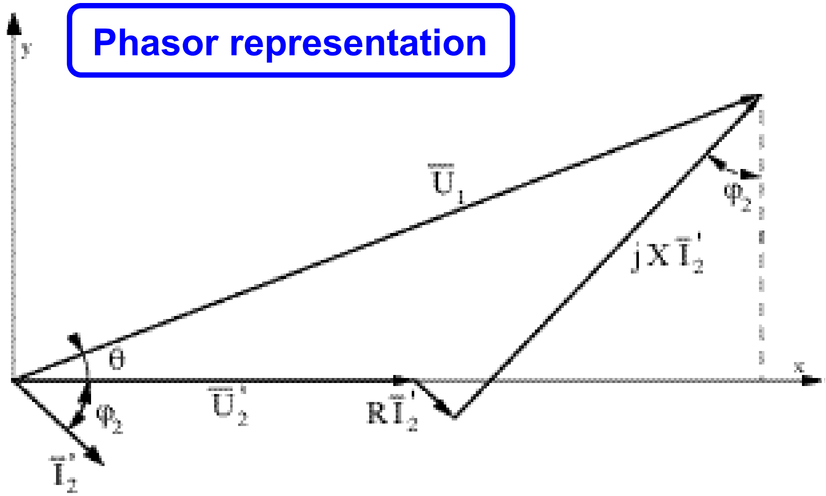
$I_1 >>$ **!**

Operating points

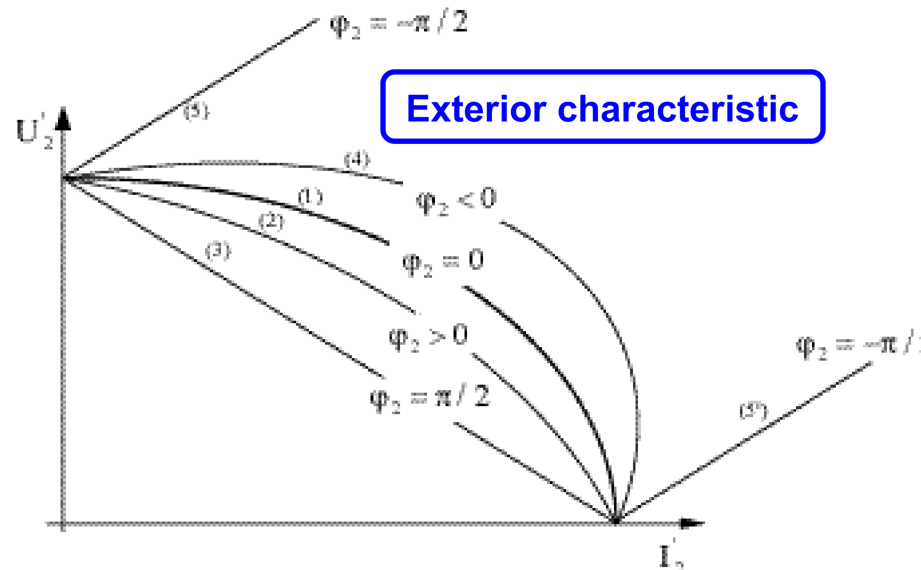
Simplified equivalent circuit



Phasor representation



Exterior characteristic

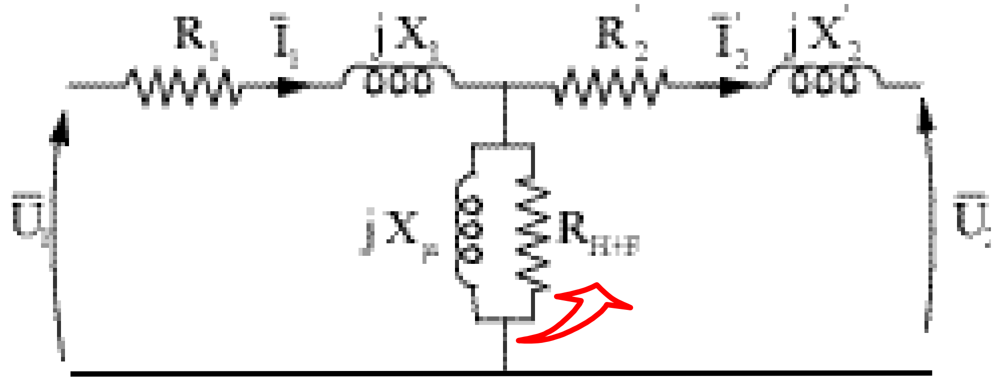


Magnetic losses and saturation

Hysteresis & eddy currents

Eddy current losses
proportional to b^2, hence to Φ^2, hence to E_1^2

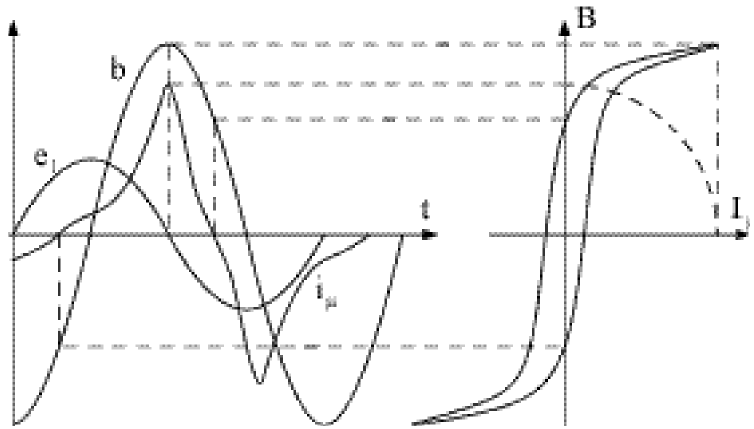
Idem for hysteresis losses (approx.)



$$p_{mag} = K_{mag}\, E_1^2 = \frac{E_1^2}{R_{H+F}}$$

(frequency dependent!)

Saturation



Equivalent circuit parameters

Experimental determination of the parameters from the equivalent circuit

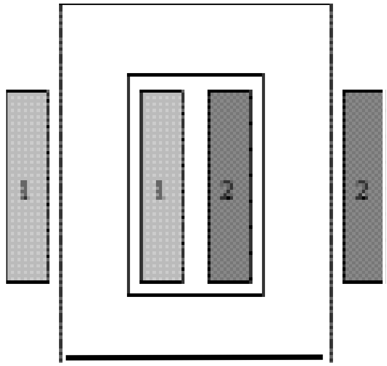
No-load test $I << \Rightarrow p_{Joule primary} <<$

$$P_v = \frac{U_1^2}{R_{H+F}} \qquad Q_v = \frac{U_1^2}{X_\mu}$$

Short-circuit test $U_1 << \Rightarrow p_{mag} <<$

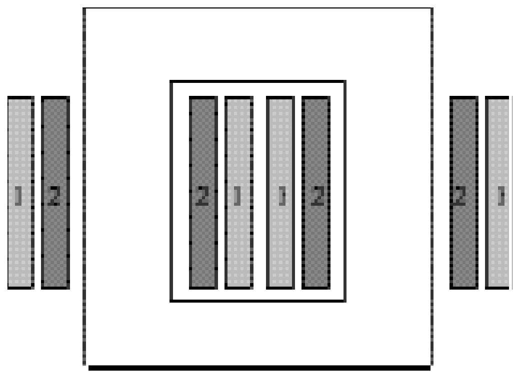
$$P_{cc} = (R_1 + R'_2)\, I_{cc}^2 \qquad Q_{cc} = (X_1 + X'_2)\, I_{cc}^2$$

Construction types



Transformer with separate columns

High leakage flux

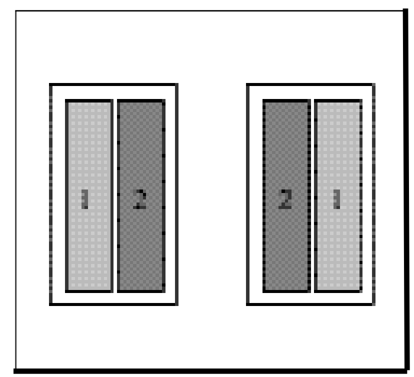


Transformer with concentric windings

Series or parallel connection of windings
1 : High Voltage, 2 : Low Voltage

Shell-type transformer (« cuirassé »)

1 : High Voltage, 2 : Low Voltage

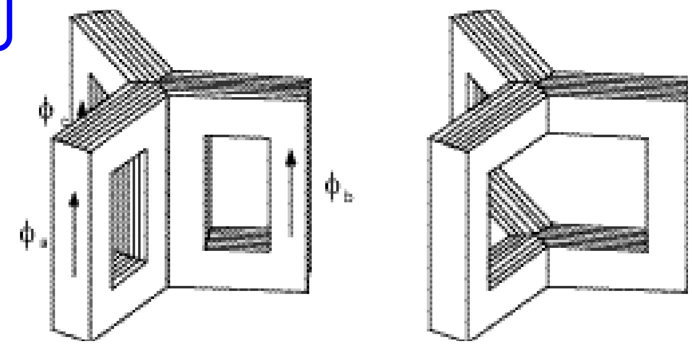


Three-phase transformer

3 single-phase transformer

advantages!

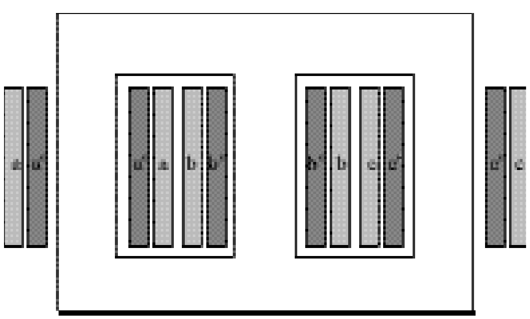
Geometric symmetry



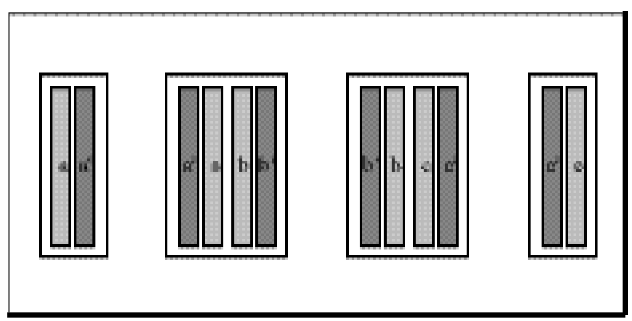
$$\Phi_a + \Phi_b + \Phi_c = 0$$

3 primary windings and 3 secondary windings with star or triangle connection

Three columns



Five columns



Shell-type