Electromagnetic Compatibility

Véronique Beauvois
2016-2017

Université de Liège

Electromagnetic Compatibility Introduction

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The electromagnetic compatibility (EMC) course is divided in 4 parts/lessons. The first one is an introduction to EMC, some reminders on basic notions and the legal part based on European directive and standards.

The other parts are: (2) coupling, disturbances and Power Quality, (3) measurements and (4) the design of electric and electronic circuits.

0. Introduction - ULg

- EMC activities are included in research unit ACE (Applied and Computational Electromagnetics)

- Prof. Christophe Geuzaine

- Véronique Beauvois

- 1 PhD, 2 technicians.

- EMC Laboratory

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In ULg, EMC activities are included in the research unit called ACE (Applied and Computational Electromagnetics) of Professor Christophe Geuzaine.

EMC team is Prof. Geuzaine, Véronique Beauvois, Engineer and responsible of this activity, 1 PhD and 2 operators.

This research unit is also concerned by electromagnetic modelling and simulations, with 10 PhD and post-Doc.

The activity is also based on laboratories and many measuring equipment.

Université de Liège ULg

0. Introduction - ULg

1996

-Directive 89/336/CEE

-Walloon companies (especially SMEs) are searching for an EMC laboratory (competent, nearby, independent, accredited)

-Funding: Europe & Walloon Region

I would like to give you a short history of the EMC activities in ULg.

In 1996, a European directive (89/336/EEC) was published and mandatory, and transposed in the Belgian law; this directive concerns EMC.

Since then all Belgian companies who want to put in the market new electronic products should be in conformity with this new legislation.

For Walloon companies, and especially SMEs, there were no EMC lab in the region and many of them would like to find EMC competence, advices and measurements, as well as an independent and accredited laboratory.

It ‘s why the Walloon Region government had invested at ULg to build an EMC laboratory, with some European Funds (Feder Objectif II).

This is in the building mentioned on the picture (white circle) where the lab was built (this high building was previously a high voltage lab, for which the activities were very law).

0. Introduction - ULg

July 1997

-Building of a semi-anechoic chamber 9 x 6 x 6 m

-Instrumentation

-Budget ~ 1.500.000 €

March 1998

Official opening

2003

Initial Accreditation BELAC

After a public market, during summer 1997, the installation of a semi anechoic chamber (9 x 6 x 6 m) and a control room to perform EMC measurements.

Moreover some instrumentation was installed.

The total budget was around 1.5 million of euros.

An official opening was organised in March 1998 with the minister and many companies of the region.

Due to the pressure of the competition and the requests of the potential industrial customers, the laboratory get an accreditation in 2003 by BELAC (Belgium Accreditation) based on ISO 17025 standard, showing the competence of testing laboratories.

0. Introduction - ULg

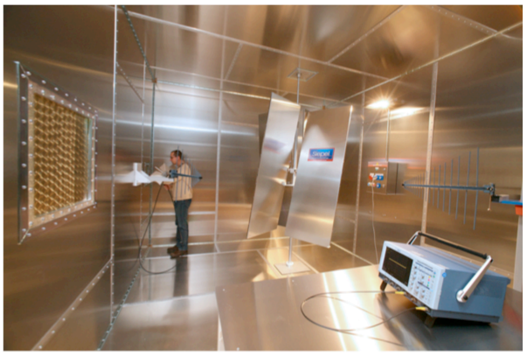
In almost 20 years, more than 100 companies

This slide is showing some pictures of our laboratories and the name of some companies from Wallonia and Brussels region who trust us since many years. These companies are active in different domains as power electronic (ABB), telecommunications (Gillam Fei), railway applications (Alstom), image processing (EVS, Euresys, Deltatec) and product certification (SGS, one of the most important product and system certification company in the world, we work with his office in Brussels).

0. Introduction - ULg

2009

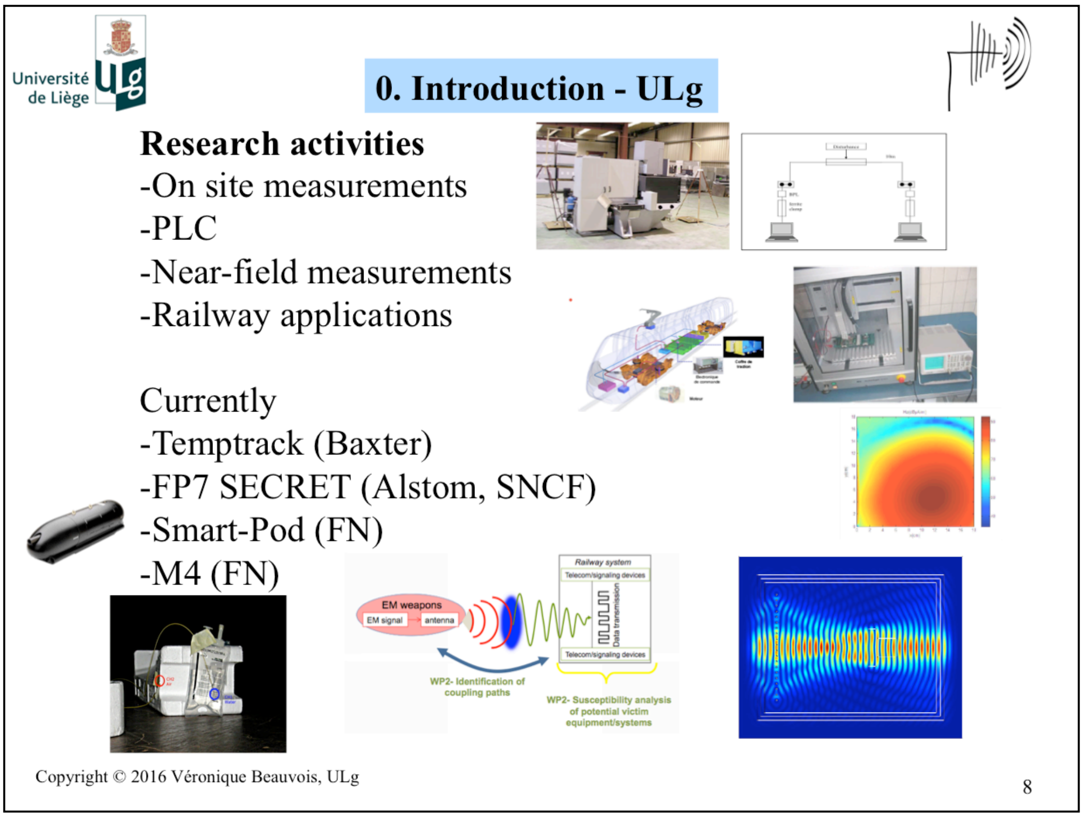
-New needs for military & spatial applications
-Reverberating Chamber
-High electric fields and larger frequency band
-Budget ~ 1.600.000 € (SPW)



After around 10 years, and according to the evolution of EMC, we had detected that we could not answer to all requests, in particular in the military and space applications, which are very important in Wallonia (see also CSL - Centre Spatial de Liège from ULg at 5 minutes by car of our laboratory).

Therefore we had requested a new budget to the Walloon region, and we get 1.6 millions of euros, 50% financed for research and 50% for industry services.

With the amount, we built a reverberating chamber and bought many equipment to generate higher electric fields or to cover larger frequency band (up to 18 GHz).



Regarding our research activities, we had participated or participate to different research projects, e.g.

- Research on alternate measuring methods for on site measurements (with KUL)
- EMC measurements on PLC systems (PLC for Power Line Communication – transfer of HF signals on power lines
- Near field electromagnetic field measurement over PCB (with ISIL)
- Measurements for railways applications (Alstom, SNCB)

Currently:

- We just finished of project called Temptrack (financed by Walloon Region) on effects of electromagnetic fields emitted by RFID systems on Baxter products, liquid bags.

-We just finished a European project called SECRET with ISTTAR, Alstom and SNCF on electromagnetic attacks of railway systems.

-We are involved in two projects with FN Herstal (financed by Walloon Region) on the development of a new Pod and a new electronic machine gun.

Université de Liège ULg

1. Introduction

Brief historical introduction

- •Beginning of the 30s: radio communication
- •First radio interferences problems (especially related to electrical motors sparks).
- •Germany 1924: High-Frequency Committee from VDE.
- •Netherlands 1931: Radiostoringscommissie.
- •England 1933: IEE creates a RFI committee.
- •1933: Creation of CISPR (International Special Committee on Radio Interference) by IEC (International Electrotechnical Commission) to develop standards to limit interferences.
- •2nd World War: Electronic and radio communication equipment (radio, navigation, radar) developments increase and the number of reported interferences problem also (e.g. air navigation).
- •CISPR activities: technical publications with measurement procedures and emission limits. Some European countries adopt these recommendations.

After this introduction on our activities, we had now an introduction of EMC.

Let's start with a brief history.

At the beginning of years 30s, radio communications appeared and very quickly problems of interferences appeared, especially related to sparks generated by dc motors (brush).

In reaction in some countries some standardisation committees have been created on radio frequency fields, as in Germany with VDE (Verband der Elektrotechnik Elektronik Informationstechnik e.V., in english Association for Electrical, Electronic &Information Technologies) in 1924, in Netherlands in 1931, in England in 1993 with the initiative of IEE (Institution of Electrical Engineers).

On the international level, in 1933, IEC (International Electrotechnical Commission) creted a special committee called CISPR (International Special Committee on Radio Interference) to develop standards to limit electromagnetic interferences.

We should know that wars are always promoting new technologies, and during the second world war, the development of electronic and radio communications exploded, and this indeuced many probelms related to interferences.

Therefore CISPR had continued to develop new publications on measurement methods and emission limits, some European countries had transposed this in their own legislation.

1. Introduction

Brief historical introduction (2)

Electronic evolution: transistors, integrated circuits, high density components, microprocessors, ... 

Enlarging frequency spectrum to increase information transfer capacity.

Electronic circuits susceptibility is increasing.

1979: FCC (*Federal Communications Commission)* – emission limits for all electronic applications.

1996: All products to be on the European market should be in conformity with emission and susceptibility requirements, in order to protect communication systems.

CE

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Let's continue with the brief history.

In electronics, some developments had an important impact on EMC as transistors, integrated circuits, the increasing of components density, microprocessors, … but also the increasing of the bandwidth of the frequency band to transmit more information.

Those developments had increased the susceptibility of the electronic circuits and they are more vulnerable.

In the history, we can add that United States in 1979 with the FCC (Federal Communications Commision) had created emission limits for all electronic appliances.

Finalyy in Europe, in 1996, all electronic appliances that we would like to introduce on the European market (even if they are produced out of Europe) should be in conformity with a European directive (89/336/EEC) concerning emission and immunity, especially to protect communication and telecommunication systems from interferences.

Since that they had to get the CE marking.

1. Introduction

•ABS development/electronic on board of automotive vehicles
•Mobile phones or electronic equipment on planes

•Mobile phones in hospitals
•Pacemakers, hearing aids

•1967: aircraft carrier Forrestal was destroyed during Vietnam war. An on-board radar disturbed the firing system of rockets under a plane, the rocket was launched accidently, hit a plane which exploded and set on fire the deck.

•1982: HMS Sheffield missile destroyer was destroyed by an Exocet missile – the antimissile detection system interfered with the satellite communication system (Falklands war – Argentina vs United Kingdom).
http://en.wikipedia.org/wiki/HMS_Sheffield_%28D80%29

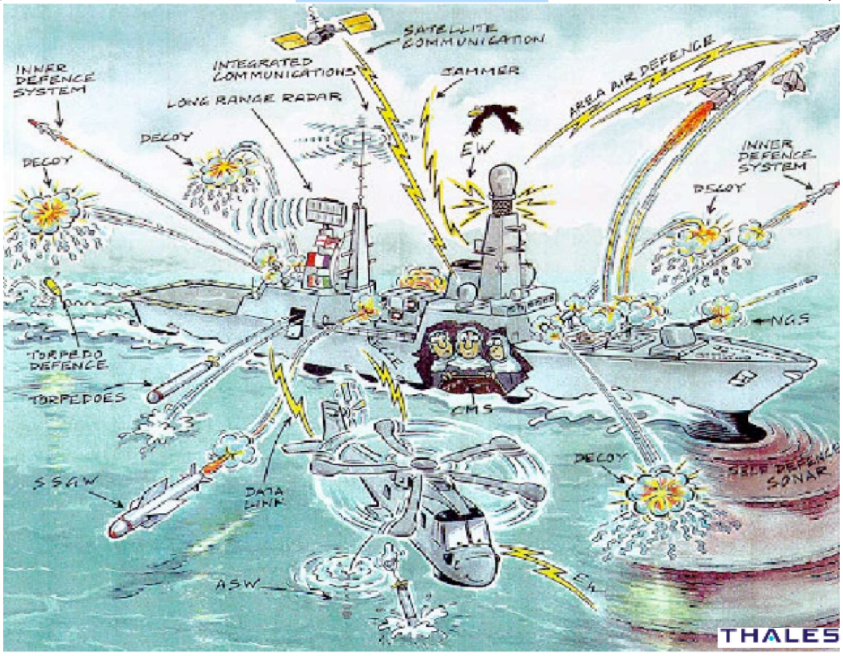
Some examples of interferences or critical situations should exist in your everyday life :

- The developments in electronics for automotive applications had given a lot of problems, e.g. with the development of ABS (in German Antiblockiersystem, in English Anti-lock braking system) (a car blocked suddenly on the highway because it passes near a radar installation on Germany).
- You should not use electronic appliances or mobile phones in planes (even if there is an evolution on that forbidding.
- You should not use your mobile phone in hospitals or nearby some places in hospitals.
- Some restrictions exist on active implants (as pacemakers, hearing implants).

In some books on EMC, you can find some historical examples as:

- In 1967, aircraft carrier Forrestal was destroyed during Vietnam war. An on-board radar disturbed the firing system of rockets under a plane, the rocket was launched accidently, hit a plane which exploded and set on fire the deck.
- In 1982, HMS Sheffield missile destroyer was destroyed by an Exocet missile – the antimissile detection system interfered with the satellite communication system (Falklands war – Argentina vs United Kingdom).

1. Introduction



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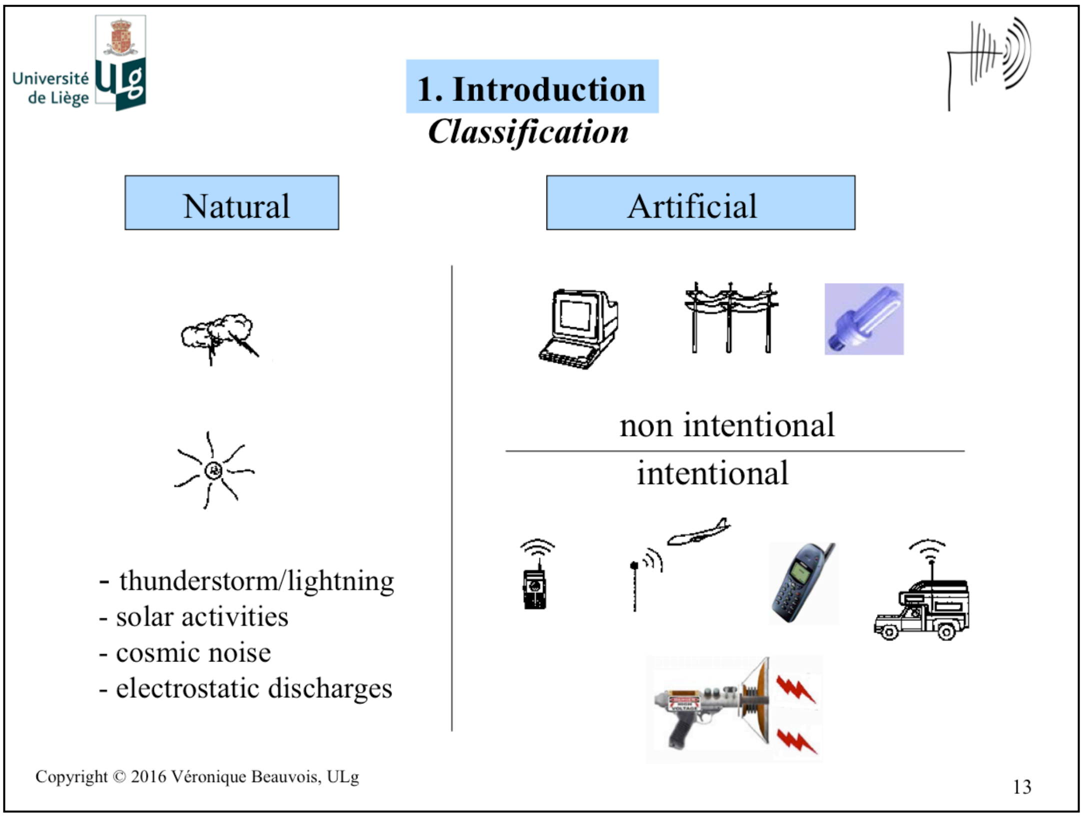
This is an illustration provided by Thales (Netherlands) who is in charge of the delopment of the fire ststem control and the radar systems, as well as war control systems for a lot of ships in the world. In the Thales group, Thales Nethermands is the Excellence Centre for radar technologies.



Since the beginning, we are talking about interferences. They can be classified in different categories and in different ways. The following classification is well known : natural (those generated by storms, lighting, solar activities, cosmic noise and electrostatic discharges) and artificial (generated by men activities). In the artificial type, you can use also the following classification : unintentional (generated by the normal functioning of an electronic equipment) and intentional (as the signal created by a specific action, as an electromagnetic jammer).

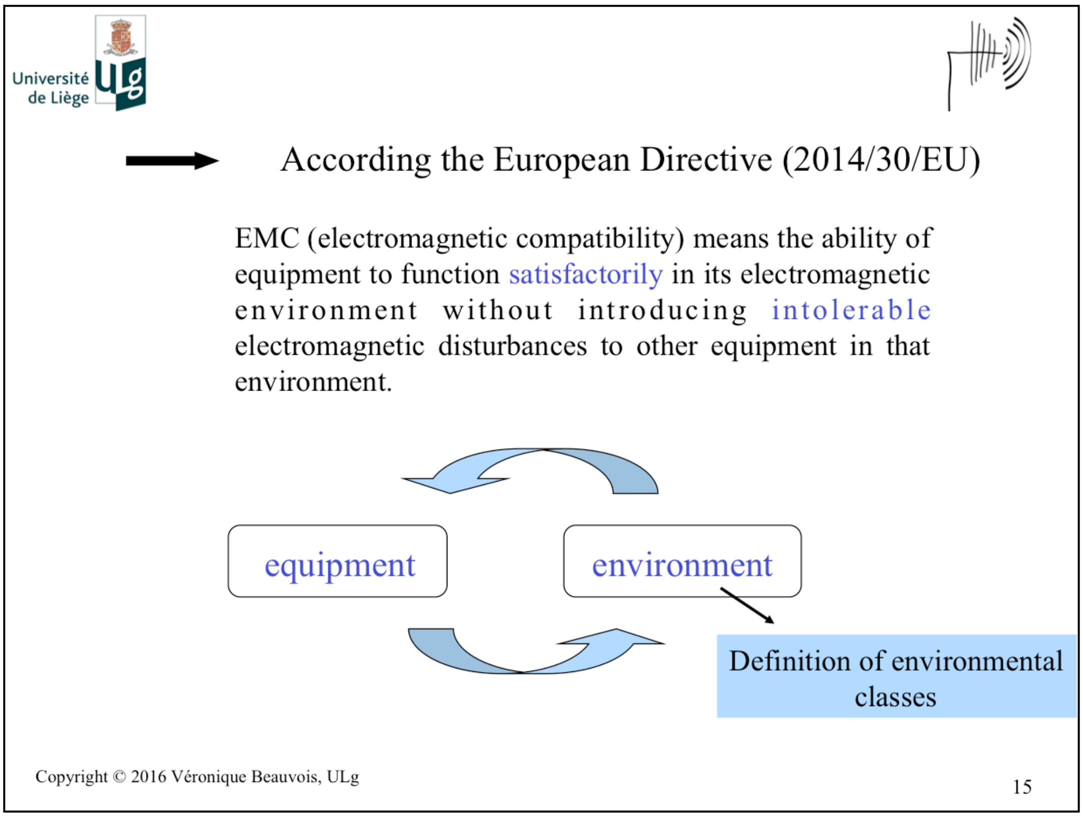
1. Introduction

Electromagnetic interferences

→ Electromagnetic Compatibility (EMC)

→ What is EMC?

And now what is electromagnetic compatibility ? As we have talked about electromagnetic interferences since now.



In order to define EMC, we have taken the definition from the current European directive (2014/30/EU), in application since April 20 2016, which is:

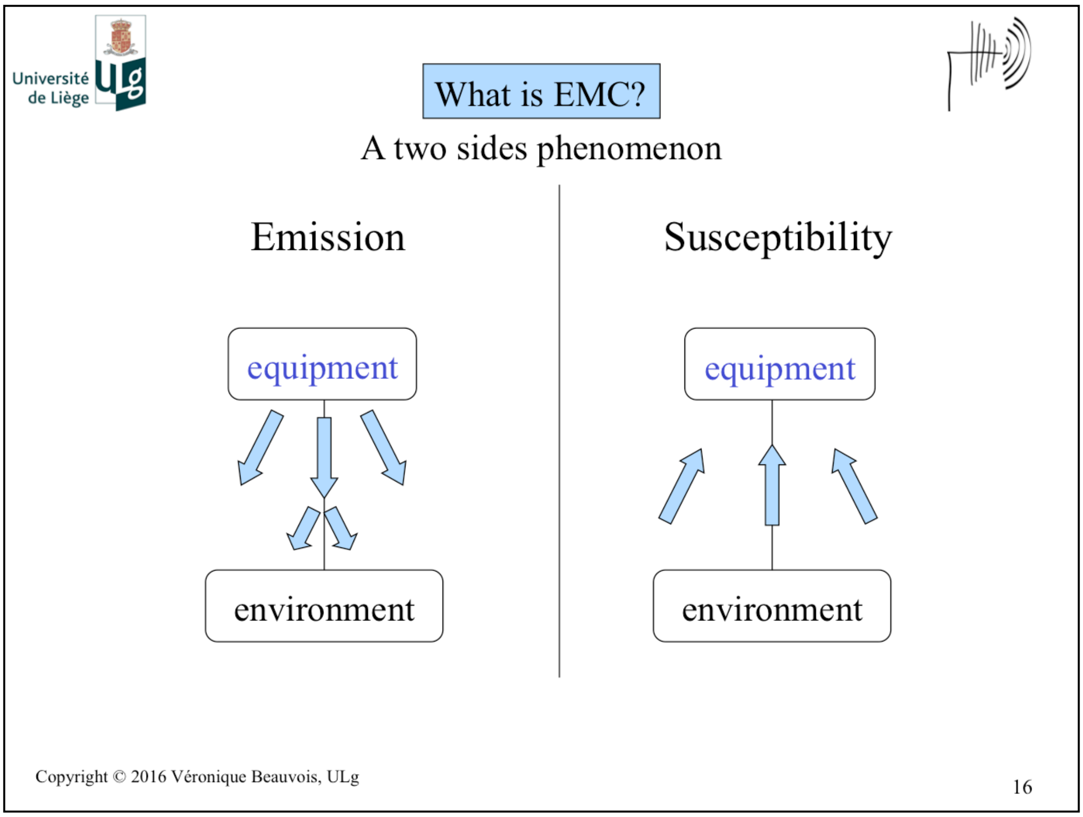
the ability of equipment to function satisfactorily in its electromagnetic environment without introducing intolerable electromagnetic disturbances to other equipment in that environment.

On the scheme, we had represented an electronic equipment and its environment, and we had shown that EMC concerns the impact of the equipment on environment (and therefore all other equipment present in this environment) and also the impact of the environment on the equipment.

We can have different classes of environment on an electromagnetic point of view, and we can imagine that the electromagnetic environment of an office, a satellite, a railway station or a heavy industry is not similar at all.

In the definition, we have highlighted 2 words as « satisfactory » and « intolerable », these words are very indefinite. Indeed what is satisfactory to you is not necessary the case for me (someone could consider that a salary of 1500 EUR per month is satisfactory as another one could consider the contrary).

As the definition is indefinite, we should be helped by numbers to take decisions (those numbers are given in the standards, as we will see in the next part of this first lesson).



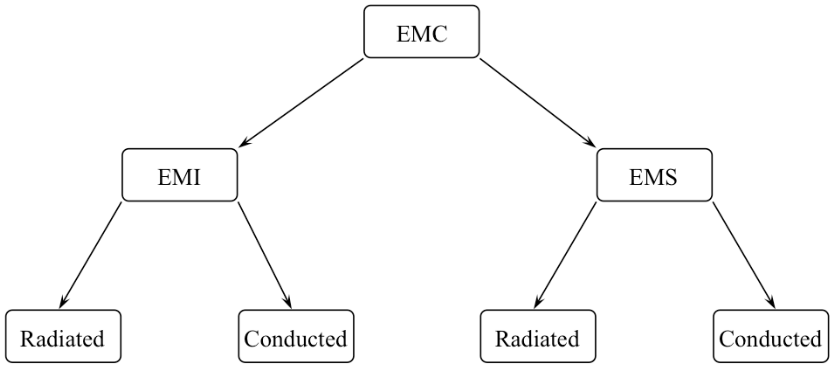
As we have seen on the previous scheme, the arrows between equipment and environment are bidirectionals.

Therefore, EMC has two sides :

- What we called emission, means the effect of the equipment on the environment, as the signals emitted by the equipment though lines (signal or power) or in the space throuhg electromagnetic fields,
- What we called immunity or susceptibility, as the effect of the environment on the equipment (by lines or air).

What is EMC?

A two sides phenomenon



EMI = ElectroMagnetic Interference
EMS = ElectroMagnetic Susceptibility

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The scheme is now completed, as EMC is emission and susceptibility, and in both cases, we have conducted and radiated phenomena.

In French, « émission » et « immunité ».

In English, EMI for Electromagnetic Interference and EMS for Electromagnetic Susceptibility.

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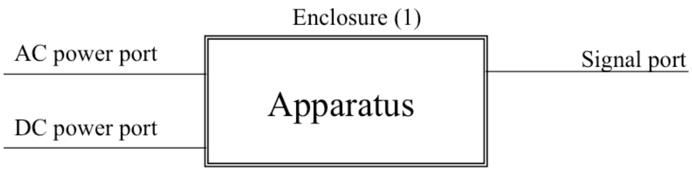
What is EMC?

Equipment - Ports



(1) physical boundary of the apparatus which electromagnetic fields may radiate through or impinge on.

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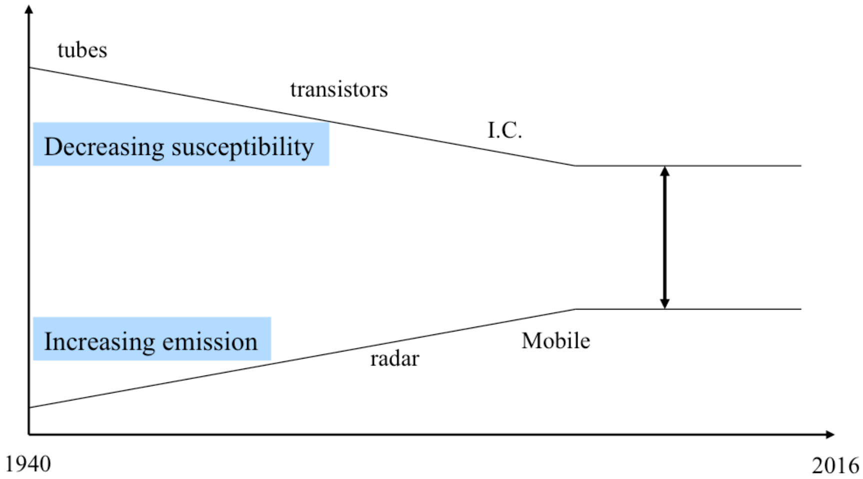
The conducted and radiated phenomena are concerned, so in the standards they define what des called « ports » (accès in French) of equipment/apparatus:

- Envelop / enclosure, which interacts through fields,
- Power ports (AC : DC),
- Signal ports.

What is EMC?

Electromagnetic compatibility gap



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How EMC is evolving in time?

What is EMC gap?

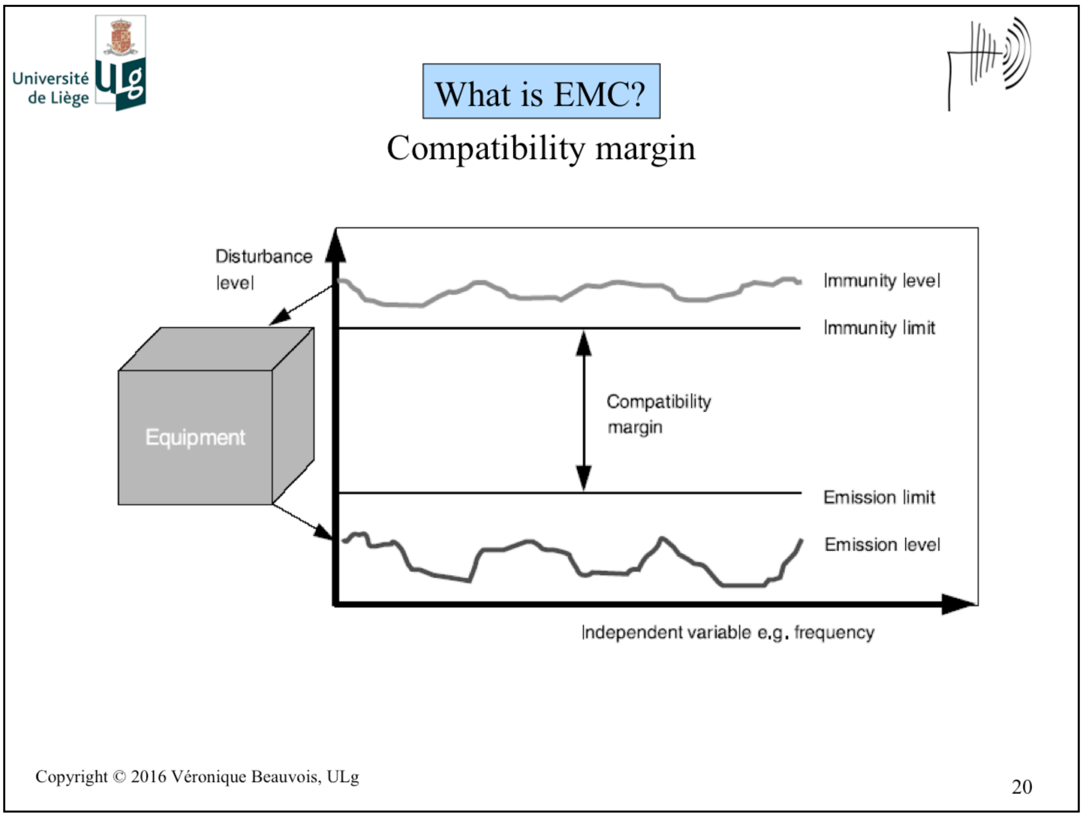
The sensitivity to electromagnetic interferences of electronic apparatus is increasing in parallel with an increasing of the electromagnetic pollution of the environment.

This sensitivity is related to the very high density of the components integration (VLSI, Very Large Scale Integration) and also with the development of microprocessors, to do many tasks and to replace some tasks done previously by analogue or electromechanical means.

The decreasing of the energy level susceptible to provoke disturbances is also impacting this sensitivity.

Furthermore, there are more and more radio communications, more transmitters and the increasing of field level.

The high number of applications using digital electronic and wireless systems is increasing emissions which affect radiocommunications, creating a real electromagnetic smog.



What is compatibility margin in EMC?

You can see on the scheme that any equipment emits a certain level of emission (Emission level) and that this level should be lower that emission limit.

On the other side, this equipment is susceptible to a certain immunity level which should be higher that that the immunity limit.

Between the two limits, there is an important margin, meaning that emission limit is significantly lower than immunity limit.

Indeed, we have to take into account the fact that there are a lot of apparatus in the environment, each of them emitting; in the worst case, these emissions add up to each other.

Typical example: for IT equipment, emission limit is 50 dBmicroV/m and immunity level is 3 V/m.

What is Power Quality?

According to I.E.E.E. Std 1100-1999
Recommended Practice for Powering and Grounding Electronic Equipment:
The concept of powering and grounding electronic equipment in a manner that is suitable to the operation of that equipment and compatible with the premise wiring system and other connected equipment.
Harmonics, sag, transients, voltage variations and interruptions, …

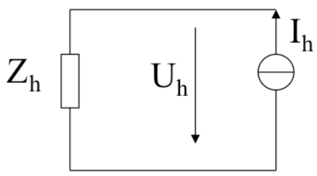
Frequently Power Quality and EMC are associated, but what is exactly Power Quality and what are the differences?

What is Power Quality?

All these phenomena concern mainly the **voltage** provided to the user. Two cases should be considered:

(1) **no disturbing load**: the current may be modified according to the voltage.

(2) **disturbing load**: the load modifies the current, which disturbs the voltage (according to Z).



What is Power Quality?

Power Quality = Voltage Quality

> Power Quality = **Voltage** Quality **+** **Reliability** of Supply (in order to take into account the concept of **fiability**, according to the number of interruptions and their duration).

What is Power Quality vs. EMC?

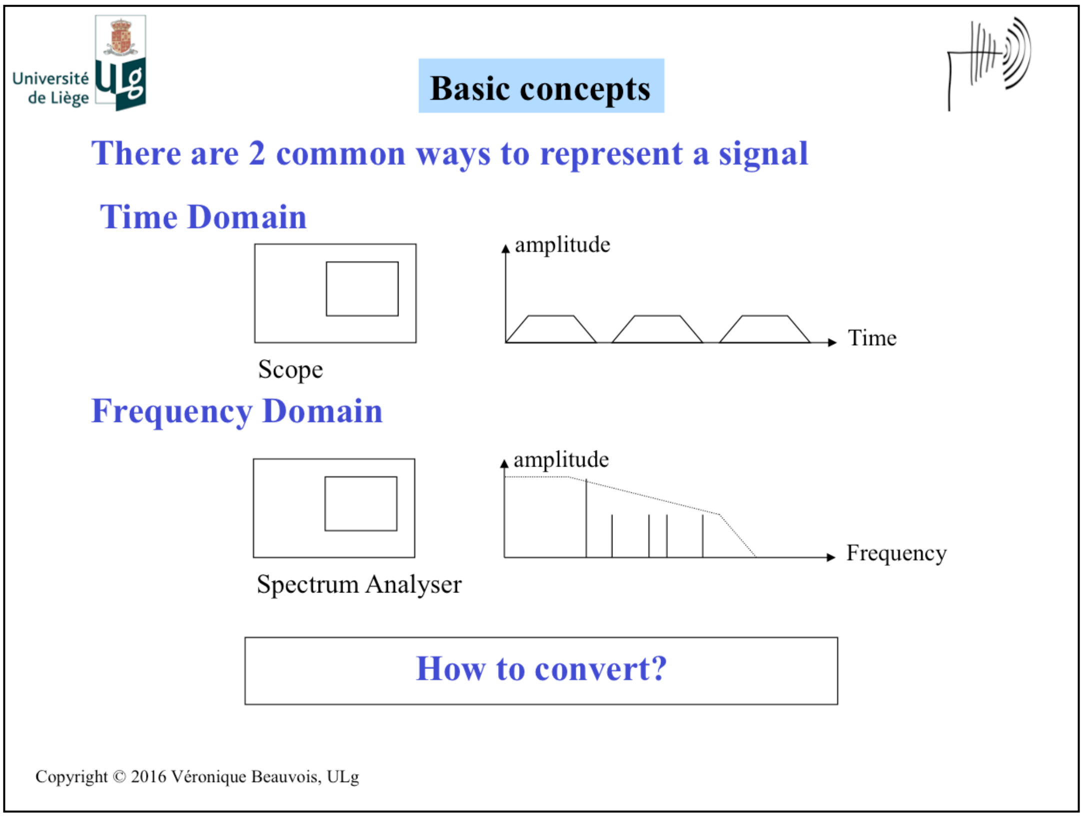
According to the definition:
EMC (electromagnetic compatibility) means the ability of equipment to function satisfactorily in its electromagnetic environment without introducing intolerable electromagnetic disturbances to other equipment in that environment.

EMC is more general and concerns emission and immunity, LF and HF, all ports (power, signal, enclosure, …), conducted and radiated modes, …

Electromagnetic Compatibility
Basic concepts

Véronique Beauvois
2016-2017

We will continue with some basic concepts, useful for the EMC lesson.



There are two ways of representing a signal:

- In the time domain, amplitude versus time. In that case, we use a scope to measure the signal.
- In the frequency domain, amplitude versus frequency. We use a spectrum analyser.

- How to convert them?

Mathematical Conversion Time vs Frequency

Periodic signal – Fourier Serie

s(t) period T : $s(t) = s(t+kT)$ $\forall k$ integer

$s(t) = \sum c_n \exp^{j2\pi nt/T}$ linear comb. of complex exponential functions

where $c_n = 1/T \int s(t).\exp^{-j2\pi nt/T}.dt$

Examples

*A cos(ωt): A/2 @ +f and -f

*Rectangular signal amplitude A, duty cycle ½ and period T:

2A/nπ (n odd) @frequencies multiple of 1/T

Mathematical Conversion Time vs Frequency

Non periodic signals – Fourier Transform

Non periodic signal = periodic signal with T -> ∞

Discrete spectrum -> continuous spectrum (gap -> 0)

$$f(t) = \int F(f) \exp^{j2\pi ft} df$$

$$F(f) = \int f(t) \exp^{-j2\pi ft} dt$$

Basic concepts

Examples

*** Pulse i(t) width τ and amplitude A: spectrum in sinx/x (BW = $1/\tau$)**

*** Dirac pulse d(t) (limit of i(t) when $\tau \to 0$) amplitude A:**

Continuous spectrum amplitude A

*** Single pulse ESD (τ_r 1ns / τ 60ns):**

Continuous spectrum ($1/\pi\tau$ @ -20dB/dec and $1/\pi\tau_r$ @ -40dB/dec)

*** …**

Basic concepts

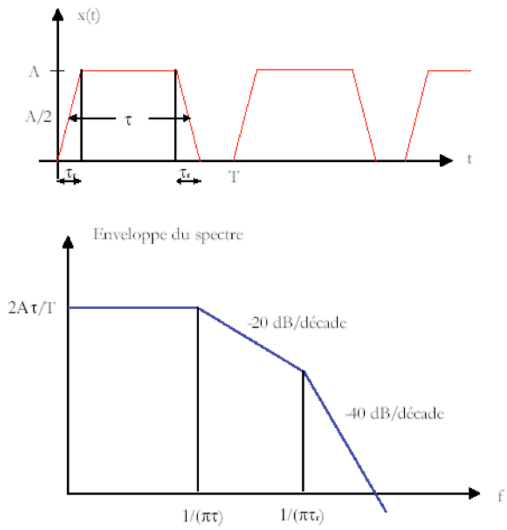
µP clock 8MHz, tr = 5ns, τ=62,5ns (duty cycle τ/T)

fc1 = 1/πτ = 5,1MHz,

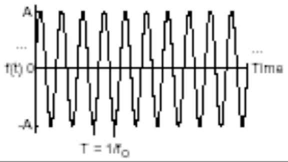
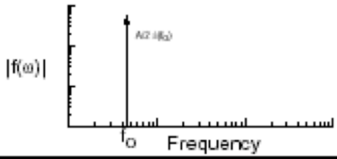
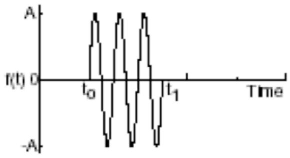
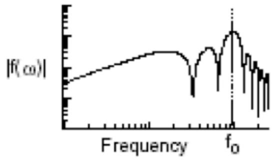
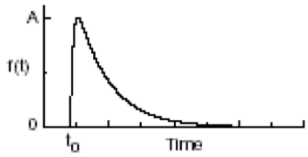
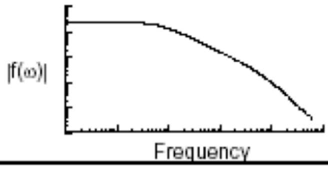
then -20dB/dec.

fc2 = 1/πtr = 63,7MHz,

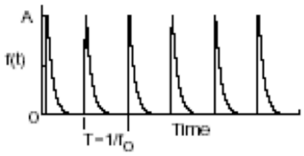
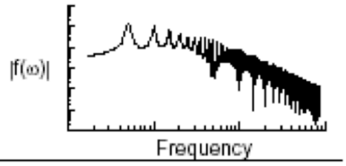
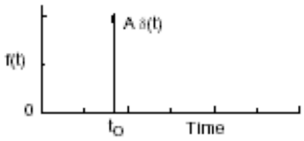
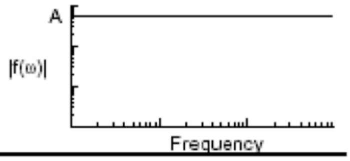
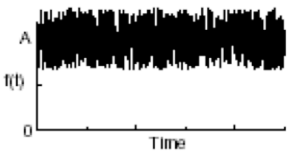
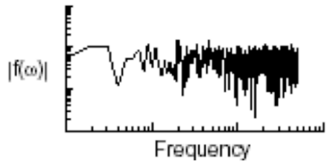
then -40dB/dec.



Basic concepts

Signal	Forme d'onde	Spectre
1. Sinusoïde		
2. Impulsion sinusoïdale		
3. Impulsion unique		

Basic concepts

Signal	Forme d'onde	Spectre
4. Train d'impulsions		
5. Impulsion de Dirac		
6. Bruit avec une composante DC		

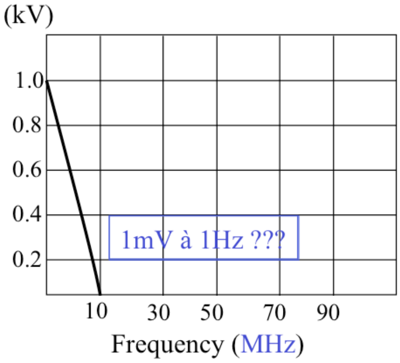
Basic concepts

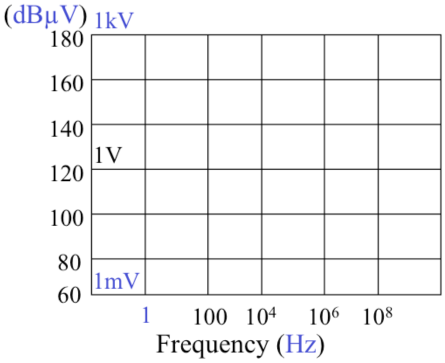
Units in EMC? Why dB and logarithmic scales?

1mV @ 1 Hz and 1kV @ 10MHz on the same graph?

-Linear scale (f in MHz and V in kV)

-Log scale (f in Hz and V in dBµV)





Basic concepts

Units in EMC?

$$\log (ab) = \log a + \log b$$

$$\log (a/b) = \log a - \log b$$

$$\log (1/a) = - \log a$$

$$\log a^n = n \log a$$

Basic concepts

Units in EMC?

dB

= logarithmic division between 2 quantities (without units)

Power (initially)
dB = **10** log (P_{1Meas}/P_{2Ref})
dBW > P_2 = 1 Watt
dBm (dBmW) > P_2 = 1 mW

Voltage ($P_i = V_i^2/Z$)
dB = **20** log (V_{1Meas}/V_{2Ref})
dBV > V_2 = 1 V
dBµV > V_2 = 1 µV

	dB(P)	dB(V)
1	0	0
2	3	6
10	10	20

Exercises

Convert 50W in dBW

50W = 10 x 10 / 2 W > 10 + 10 –3 = 17 dBW

Convert 50W in dBm (1mW as reference)

50W x 1000 mW/W > (10 x 10 /2) x 10^3 > 10+10-3+(3x10)=50-3=47 dBm

Relationship V(dBμV) - P(dBm) for any value of Z and for Z 50Ω

$P=V^2/Z$

$10\log P/1W=10\log P/10^3 mW= 10\log P/1mW-30=20\log V/1V – 10\log Z$

$=20\log V/10^6\mu V-10\log Z=20\log V/1\mu V-120-10\log Z$

P(dBm)=V(dBμV)-90-10logZ

If Z=50Ω

V(dBμV)=P(dBm)+107 dB

Basic concepts

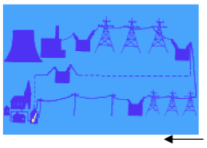
Units in EMC?

Power		Voltage	Current
W	EMS →	V	A
dBW		dBV	dBA
→ **dBm**	EMI →	**dBµV**	dBµA
dBm/kHz		dBµV/MHz	dBµA/MHz

	E	H
EMS →	**V/m**	A/m
	dBV/m	dBA/m
EMI →	**dBµV/m**	dBµA/m
	dBµV/m/MHz	dBµA/m/MHz

Frequency / Wavelength

$$\lambda = c/f$$
$$c = 3.10^8 \text{ m/s}$$



	Frequency	Wavelength
	50Hz	6000km
EMI-V	150kHz	2km
	1MHz	300m
	30MHz	10m
EMI-F	300MHz	1m
	900MHz	33.3cm
	1GHz	30cm
	2.4GHz	12cm
	18GHz	1.67cm

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Electromagnetic Models

Maxwell' s equations

- Electrostatics
- Electrokinetics
- Electrodynamics
- Magnetostatics
- Magnetodynamics
- Full Wave

Professeur C. Geuzaine.

Electromagnetic Models

- Electrostatics
  - Distribution of electric field due to static charges and/or levels of electric potential

- Electrokinetics
  - Distribution of static electric current in conductors

- Electrodynamics
  - Distribution of electric field and electric current in materials (insulating and conducting)

- Magnetostatics
  - Distribution of static magnetic field due to magnets and continuous currents

- Magnetodynamics
  - Distribution of magnetic field and eddy current due to moving magnets and time variable currents

- Full Wave
  - Propagation of electromagnetic fields

Maxwell's Equations

$$\mathbf{curl}\,\boldsymbol{h} = \boldsymbol{j} + \partial_t \boldsymbol{d}$$ Maxwell-Ampère's equation

$$\mathbf{curl}\,\boldsymbol{e} = -\partial_t \boldsymbol{b}$$ Faraday's equation

$$\left.\begin{aligned}\operatorname{div}\boldsymbol{b} &= 0\\ \operatorname{div}\boldsymbol{d} &= \rho_q\end{aligned}\right\}$$ Conservation equations

$\boldsymbol{h}$ magnetic field (A/m)

$\boldsymbol{b}$ magnetic flux density (T)

$\boldsymbol{j}$ current density (A/m^2)

$\boldsymbol{e}$ electric field (V/m)

$\boldsymbol{d}$ electric displacement (C/m^2)

ρ_q charge density (C/m^3)

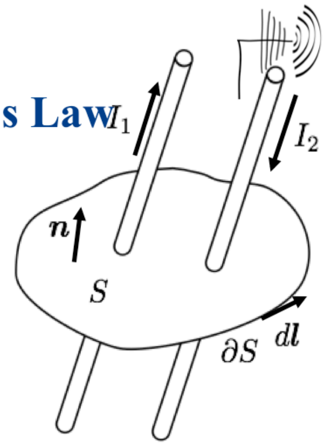
Integral form: Ampère's Law

$$\mathbf{curl}\, \boldsymbol{h} = \boldsymbol{j}$$

$$\Rightarrow \oint_{\partial S} \boldsymbol{h} \cdot d\boldsymbol{l} = I$$

Magnetomotive force (m.m.f.)

Circulation of magnetic field along closed curve equals algebraic sum of currents crossing the underlying surface



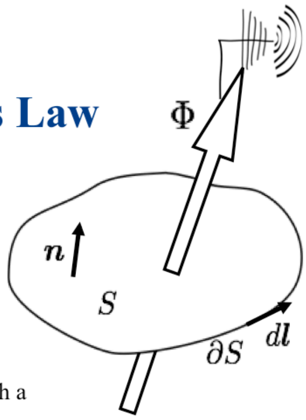
$$\oint_{\partial S} \boldsymbol{h} \cdot d\boldsymbol{l} = I_1 - I_2$$

Conservation of current: $\mathrm{div}\, \boldsymbol{j} = 0$

$$\Rightarrow \oint_{\partial V} \boldsymbol{j} \cdot \boldsymbol{n}\, ds = 0$$

Sum of currents arriving at a node is zero

Integral form: Faraday's Law



$$\mathrm{curl}\, \boldsymbol{e} = -\partial_t \boldsymbol{b}$$

$$\Rightarrow \oint_{\partial S} \boldsymbol{e} \cdot d\boldsymbol{l} = -\partial_t \Phi$$

Any variation of magnetic flux through a circuit gives rise to an electromotive force

Electromotive force (e.m.f.)

For a circuit moving at speed : $\boldsymbol{v}$

f.e.m. = $\oint_{\partial S(t)} \boldsymbol{f}/q \cdot d\boldsymbol{l} = \oint_{\partial S(t)} (\boldsymbol{e} + \boldsymbol{v} \times \boldsymbol{b}) \cdot d\boldsymbol{l} = -d_t \int_{S(t)} \boldsymbol{b} \cdot \boldsymbol{n}\, ds$

Conservation of magnetic flux density:

$$\mathrm{div}\, \boldsymbol{b} = 0$$

$$\Rightarrow \oint_{\partial V} \boldsymbol{b} \cdot \boldsymbol{n}\, ds = 0$$

Magnetic flux lines are closed

Written here for a surface S fixed (not depending of time).

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Lorentz Force

Interaction of electromagnetic fields with a point charge moving at speed $\boldsymbol{v}$

$$\boldsymbol{f} = q(\boldsymbol{e} + \boldsymbol{v} \times \boldsymbol{b})$$

For a conductor (electrically neutral, only negative charges moving):

$$\boldsymbol{f} = \boldsymbol{j} \times \boldsymbol{b} = \mathbf{curl}\,\boldsymbol{h} \times \boldsymbol{b}$$ Laplace Force

Electromagnetic Power

Poynting vector: $\boldsymbol{s} = \boldsymbol{e} \times \boldsymbol{h}$

Power exchanged with a volume (interior normal):

$$P = \oint_{\partial V} \boldsymbol{s} \cdot \boldsymbol{n} \, ds = -\int_V \operatorname{div} \boldsymbol{s} \, dv = \int_V p \, dv$$

Power density:

$$p = -\operatorname{div} \boldsymbol{e} \times \boldsymbol{h} = -\boldsymbol{h} \cdot \mathbf{rot}\, \boldsymbol{e} + \boldsymbol{e} \cdot \mathbf{rot}\, \boldsymbol{h}$$

$$\Rightarrow p = \boldsymbol{h} \cdot \partial_t \boldsymbol{b} + \boldsymbol{e} \cdot \boldsymbol{j} + \boldsymbol{e} \cdot \partial_t \boldsymbol{d}$$

"magnetic" *"conduction"* *"electric"*

Material Constitutive Relations

$\boldsymbol{b} = \mu \boldsymbol{h}$ Magnetic relation

$\boldsymbol{d} = \varepsilon \boldsymbol{e}$ Dielectric relation

$\boldsymbol{j} = \sigma \boldsymbol{e}$ Ohm's law

Characteristics of materials:

- μ magnetic permeability (H/m)
- ε dielectric permittivity (F/m)
- σ electric conductivity (S/m)

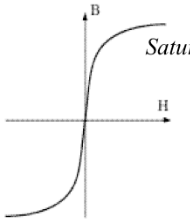
} constants (linear materials), functions of electromagnetic fields (nonlinear materials), tensorial (anisotropic materials), functions of other physical fields (temperature, ...)

Magnetic Relation

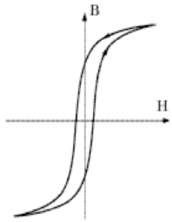
$$\boldsymbol{b} = \mu \boldsymbol{h} \qquad \mu = \mu_r \mu_0 \begin{cases} \mu_r \text{ Relative magnetic permeability} \\ \mu_0 \text{ Vacuum permeability (} 4\pi 10^{-7} \text{ H/m)} \end{cases}$$

- Diamagnetic and paramagnetic materials $\mu_r \approx 1$
  Linear materials (silver, copper, aluminium)
- Ferromagnetic materials $\mu_r \gg 1, \quad \mu_r = \mu_r(h)$
  Nonlinear materials (steel, iron)
  Ferromagnetic-paramagnetic transition for T > T_{Curie} (T_{Curie} of iron : 1043 K)

b(h) curves



Saturation



Hysteresis (non bijective law)

Energy dissipation = cycle area

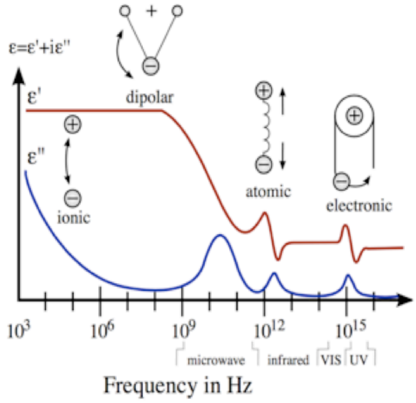
Steinmetz formula: $p_{\text{hyst}} = \omega k_h b_{\max}^{\nu}$

Dielectric Relation

$$\boldsymbol{d} = \varepsilon \boldsymbol{e} \qquad \varepsilon = \varepsilon_r \varepsilon_0 \begin{cases} \varepsilon_r & \text{Relative dielectric permittivity} \\ \varepsilon_0 = \dfrac{1}{\mu_0 c_0^2} & \text{Vacuum permittivity (8.854187817620... × 10−12 F/m)} \end{cases}$$

ε_r at room temperature for f < 1kHz

Air	1.0006
Teflon	2.1
Polyethylene	2.25
Paper	3.85
Glass	3.7 - 10
Concrete	4.5
Water	80



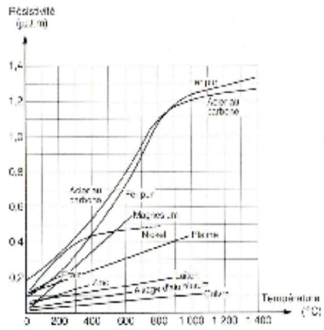
Ohm's Law

$$\boldsymbol{j} = \sigma \boldsymbol{e} \qquad \text{(Resistivity } \rho = \frac{1}{\sigma} \text{)}$$

Simple models for temperature dependency

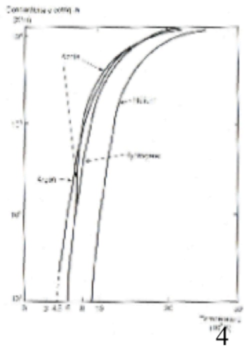
- Metals : $\rho = \rho_0(1 + \alpha(T - T_0))$

	ρ_0 ($T_0 = 20°C$) (Ωm)	α ($°C^{-1}$)
Aluminum	$2.7\ 10^{-8}$	$4\ 10^{-3}$
Copper	$1.7\ 10^{-8}$	$3.9\ 10^{-3}$
Iron	$9.6\ 10^{-8}$	$6.5\ 10^{-3}$



- Glass : $\ln \rho = A + \dfrac{B}{T}$

Common glass: $\ln \rho = -4.6 + \dfrac{7678}{T}$



- Ionic solutions : $\sigma = \sigma_0 + \alpha(T - T_0)$

Tap water: $\sigma_0 = 0.055\,\Omega^{-1}m^{-1}$

$\alpha = 1.65\,10^{-3}\,°C^{-1}\Omega^{-1}m^{-1}$

$T_0 = 20°C$

Model Choice

Maxwell's equations & constitutive relations in frequency domain, without sources:

$$\Delta \boldsymbol{e} - i\omega\sigma\mu\boldsymbol{e} + \omega^2\varepsilon\mu\boldsymbol{e} = 0$$

Using characteristic lengths

- domain size L
- skin depth $\delta = \sqrt{\dfrac{2}{\omega\sigma\mu}}$
- wavelength $\lambda = \dfrac{2\pi}{k}$

$\left\{\begin{array}{l} \text{wave number } k = \dfrac{\omega}{c} \\ \text{speed of light } c = \dfrac{1}{\sqrt{\varepsilon\mu}} \end{array}\right.$

allows to write in non-dimensional form:

$$\left(\frac{3}{L^2} - \frac{2i}{\delta^2} + \frac{4\pi}{\lambda^2}\right)\boldsymbol{e} = 0$$

Model Choice

$$\left(\frac{3}{L^2} - \frac{2i}{\delta^2} + \frac{4\pi}{\lambda^2}\right) \boldsymbol{e} = 0$$

Non-dimensional numbers $\begin{cases} g_1 = \left(\frac{\lambda}{L}\right)^2 \\ g_2 = \left(\frac{\delta}{L}\right)^2 \\ g_3 = \left(\frac{\lambda}{\delta}\right)^2 \end{cases}$

$g_1 \gg 1$ uncoupled electric or magnetic problems

- $g_2 \gg 1$ Magnetostatics
- $g_2 \lesssim 1$ magnetodynamics
- $g_3 \gg 1$ electrokinetics
- $g_3 \approx 1$ electrodynamics
- $g_3 \ll 1$ electrostatics

$g_1 \lesssim 1$ full wave ($g_1 \to 0$ high-frequency asymptotics)