Math 471: Assignment 7 – due Wed 11/29

A rectangular plate with isolated faces, heated to a uniform temperature u_0, is suddenly allowed to cool with three of its edges maintained at $0°$ while the last edge is kept insulated.

Mathematically, this problem can be written as follows:

$$u_t = \kappa \nabla^2 u, \qquad 0 \le x \le L, \qquad 0 \le y \le H,$$

$$u(0,y,t) = 0 = u(L,y,t), \qquad u(x,0,t) = 0, \qquad \frac{\partial u}{\partial y}(x,H,t) = 0.$$

1. Solve for $u(x,y,t)$ subject to an initial condition $u(x,y,0) = u_0 = 100$.
2. Find the smallest eigenvalue λ and the first term approximation (i.e. the term with $e^{-\lambda\kappa t}$).
3. For fixed $t = t_0 \gg 0$, sketch the level curves $u =$ constant in the xy-plane.
4. Of all rectangular plates of equal area, which will cool the slowest? Hint: for each type of plate, the smallest eigenvalue gives the rate of cooling.