Math 471: Assignment 4 – due Wed 10/11

Consider the following problem:

$$\begin{aligned} &u_{tt}(x,t) = u_{xx}(x,t), && 0 < x < \pi, \quad t > 0 \\ &u(0,t) = 0, \quad u_x(\pi,t) = 0, && t > 0 \\ &u(x,0) = \sin(x), \quad u_t(x,0) = e^x, && 0 < x < \pi. \end{aligned}$$

1. What is the associated Sturm-Liouville Problem that arises when the separation of variables technique is applied?

2. The Rayleigh quotient for the general Stourm Liouville boundary value problem

$$\frac{d}{dx}\left[p(x)\frac{d\phi(x)}{dx}\right] + q(x)\phi(x) + \lambda\sigma(x)\phi(x) = 0, \quad a < x < b$$

   is

$$\lambda = \frac{-p(x)\phi(x)\frac{d\phi(x)}{dx}\Big|_a^b + \int_a^b \left[p(x)\left(\frac{d\phi(x)}{dx}\right)^2 - q(x)\phi(x)^2\right]dx}{\int_a^b \phi(x)^2\sigma(x)dx}.$$

   What is the expression of the Rayleigh quotient for the Sturm-Liouville problem you obtained in point 1 above?

3. Using this Rayleight quotient, what can you say about the eigenvalues of the Sturm-Liouville problem?

4. What are the eigenvalues of the Sturm-Liouville problem?

5. Solve the original problem by separation of variables.