Math 471: Assignment 3 – due Wed 09/27

1. Let
$$f(x) = \frac{1-x}{2}, \quad 0 < x < 1.$$

   (a) Show that the Fourier sine series cannot be differentiated term-by-term.

   (b) Show that the Fourier cosine series converges uniformly.

2. Consider a radioactive rod lying along the x-axis, $0 \leq x \leq \ell$. Conservation of neutrons leads to the following PDE in terms of the neutron density $n(x, t)$ at position x and time t:

$$\frac{\partial n(x, t)}{\partial t} = D\frac{\partial^2 n(x, t)}{\partial x^2} + kn(x, t),$$

where $D > 0$ is a diffusion coefficient and $k > 0$ is a fission constant. When the neutron density is zero at both ends $(n(0, t) = n(\ell, t) = 0)$ show that the rod will explode $(n \to \infty)$ if and only if $k > \pi^2 D/\ell^2$.