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Interior-point based algorithms for the solution of optimal power flow problems

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Abstract

Interior-point method (IPM) is a very appealing approach to the optimal power flow (OPF) problem mainly due to its speed of convergence and ease of handling inequality constraints. This paper analyzes the ability of three interior-point (IP) based algorithms, namely the pure primal-dual (PD), the predictor–corrector (PC) and the multiple centrality corrections (MCC), to solve various classical OPF problems: minimization of overall generation cost, minimization of active power losses, maximization of power system loadability and minimization of the amount of load curtailment. These OPF variants have been formulated using a rectangular model for the (complex) voltages. Numerical results on three test systems of 60, 118 and 300 buses are reported.

Keywords: Optimal power flow; Interior-point method; Nonlinear programming

1. Introduction

Since the early 60's [1] the optimal power flow (OPF) has become progressively an essential tool in power systems planning, operational planning and real-time operation, both in an integrated and a deregulated electricity industry [2].

The OPF is stated in its general form as a nonlinear, nonconvex, large-scale, static optimization problem with both continuous and discrete variables. It aims at optimizing some objective by acting on available control means while satisfying network power flow equations, physical and operational constraints.

First approaches to the complex OPF problem can be classified in: gradient methods [3], sequential quadratic programming [4] and sequential linear programming [5]. The shortcomings of these techniques concern the slow convergence especially in the neighborhood of the optimum for the first two, and a rather limited field of application, such as the optimization of the active control variables only, for the third one.

The use of the very efficient Newton method to the solution of the Karush–Kuhn Tucker (KKT) optimality conditions constitutes a breakthrough [6]. Moreover, this reference proposes sparsity techniques to considerably speed up the computations. The weakness of this approach lies in the difficulty to identify inequality constraints that are active at the optimum.

Emerged in the middle 50's [7] and largely developed in the late 60's [8], the interior-point method (IPM) becomes in the early 90's a very appealing approach to the OPF problem due to three reasons: (i) ease of handling inequality constraints by logarithmic barrier functions, (ii) speed of convergence and (iii) a strictly feasible initial point is not required [9–12].

The pure primal-dual interior-point algorithm was historically the first one used to solve OPF problems. Although enjoying the above mentioned IPM advantages, it suffers, nevertheless, from two drawbacks: (i) the heuristic to decrease the barrier parameter and (ii) the required positivity of slack variables and their corresponding dual variables at every iteration, which may drastically shorten the Newton step length(s). In order to mitigate or to remove one or both among these flaws, two classes of methods emerged: higher-order IPMs (e.g., the predictor–corrector [13], multiple predictor–corrector [14], multiple centrality corrections [15]) and non-interior point methods (e.g., the unlimited point algorithm [16], the complementarity method [17], the Jacobian smoothing method [18]). Note, incidentally, that although non-interior point methods are not

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perfectly mathematically rigorous, some of them exhibit in practice convergence performances comparable to those of the best interior-point algorithms.

In this paper we compare performances of three interior-point (IP) based algorithms: the pure primal-dual (PD), the predictor–corrector (PC) and the multiple centrality corrections (MCC), on several OPF problems, expressed in rectangular form. This paper gathers the main results of our previous research work in the OPF field [19–21].

The PC and MCC algorithms belong to the class of higher-order interior-point methods. The latter rely on the observation that the factorization of the Hessian matrix is, by far, the most expensive computational task of an interior-point algorithm iteration. Indeed, in most power system applications, the Hessian factorization takes much more time than the backward/forward solution of the already factorized linear system [9,11,12]. The aim of these methods is hence to draw maximum of profit from the factorized matrix, with little additional computational effort. Practically, they solve one or more extra linear system(s), based on the same factorized matrix, expecting to yield an improved search direction and thereby to reduce the number of iterations. Obviously, higher-order methods are of interest as long as they are able to reduce the computational time with respect to the PD algorithm.

The PC algorithm is considered to be the benchmark of IP based algorithms. It was consequently extensively used in the last decade for the solution of OPF problems [9–12,22]. The MCC algorithm, initially proposed for linear programming [15], has been less well studied so far. However, its performances have already been assessed on some OPF problems and have been shown to be comparable to those of the PC algorithm [22,23]. The MCC algorithm is motivated by the observation that the convergence of an IP algorithm is worsened by a large discrepancy between complementarity products at an iteration [15]. The MCC algorithm is based on the assumption that the closer the point to be optimized to the central path, the larger step length(s) can be afterwards. The aim of the MCC approach is twofold: (i) to increase the step length in both primal and dual spaces at the current iteration and (ii) to improve the centrality of the next iteration.

The paper is organized as follows. Section 2 introduces the general OPF problem. Section 3 offers an overview of the three IP algorithms under study: PD, PC and MCC, respectively. Section 4 provides numerical results obtained with these algorithms on several OPF variants. Finally, some conclusions and future works are presented in Section 5.

2. Statement of the optimal power flow problem

2.1. Generalities

Let us denote by: n, g, c, b, l, t, o and s the number of: buses, generators, loads, branches, lines, all transformers, transformers with controllable ratio and shunt elements, respectively.

We formulate the OPF problem with voltages expressed in rectangular coordinates, choice which will be explained in Section 2.6. In rectangular coordinates the complex voltage $\underline{V}_i$ is expressed as:

$$\underline{V}_i = e_i + jf_i, \quad i = 1, \ldots, n$$

where e_i and f_i are its real and imaginary part, respectively.

2.2. Objective functions

In this paper we deal with four classical objectives, namely minimum generation cost (1), minimum active power losses (2), maximum power system loadability (3) and minimum load shedding (4):

$$\min \sum_{i=1}^{g} (c_{0i} + c_{1i} P_{gi} + c_{2i} P_{gi}^2) \tag{1}$$

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} G_{ij} [(e_i - e_j)^2 + (f_i - f_j)^2] \tag{2}$$

$$\max S = -\min(-S) \tag{3}$$

$$\min \sum_{i=1}^{c} \phi_i P_{ci} \tag{4}$$

where for the i-th generator, c_{01}, c_{1i} and c_{2i} are cost coefficients describing its cost curve (usually assumed not to include terms higher than quadratic) and P_{gi} is its active output, G_{ij} is the conductance of the branch linking buses i and j, S is a scalar representing the loadability factor and, for the i-th load, ϕ_i is its percentage of load curtailment and P_{ci} is its active demand.

2.3. Control variables

For the sake of presentation clarity we have restricted ourselves to the most widespread control variables, e.g., generator active power, generator reactive power (or generator terminal voltage), controllable transformer ratio, shunt reactance and percentage of load curtailment. Note, however, that our OPF can also take into account control variables such as: phase shifter transformer angle, SVC reactance, TCSC reactance, etc.

2.4. Equality constraints

Equality constraints mainly involve nodal active and reactive power balance equations, which, for the i-th bus ($i = 1, \ldots, n$), take on the form:

$$\begin{aligned} & P_{gi} - (1 + S - \phi_i) P_{ci} - V_i^2 \sum_{j \in N_i} (G_{sij} + G_{ij}) \\ & \quad + \sum_{j \in N_i} [(e_i e_j + f_i f_i) G_{ij} + (f_i e_j - e_i f_j) B_{ij}] = 0 \end{aligned} \tag{5}$$

$$\begin{aligned} & Q_{gi} - (1 + S - \phi_i) Q_{ci} + V_i^2 \left[B_{si} + \sum_{j \in N_i} (B_{sij} + B_{ij}) \right] \\ & \quad - \sum_{j \in N_i} [(e_i e_j + f_i f_j) B_{ij} + (e_i f_j - f_i e_j) G_{ij}] = 0 \end{aligned} \tag{6}$$

where P_{gi} and Q_{gi} are the active and reactive power of the generator connected at bus i, P_{ci} and Q_{ci} are the active and reactive demand of the load connected at bus i, $V_i^2 = e_i^2 + f_i^2$ is the modulus of the complex voltage at bus i, B_{si} is the shunt susceptance at bus i, G_{ij} and B_{ij} (resp. G_{sij} and B_{sij}) are the longitudinal (resp. shunt) conductance and susceptance of the branch linking buses i and j and N_i is the set of buses connected by branches to the bus i. Obviously, $S=0$, unless one deals with the objective (3) and $\phi_i=0$, $i=1, \ldots, c$ if load curtailment is not allowed as control variable.

Additional equality constraints may exist, as for example the setting of generator voltage to a specified reference:

$$e_i^2 + f_i^2 - (V_i^{\text{ref}})^2 = 0 \quad i = 1, \ldots, g. \tag{7}$$

2.5. Inequality constraints

An OPF problem encompasses two types of inequality constraints: operational (aimed to ensure a secure operation of the system) and physical limits of equipments. The former involve limits on branches current and voltages magnitude:

$$(G_{ij}^2+B_{ij}^2)[(e_i - e_j)^2+(f_i - f_j)^2] \leq (I_{ij}^{\max})^2, \quad i, j = 1, \ldots, n \tag{8}$$

$$(V_i^{\min}) \leq e_i^2 + f_i^2 \leq (V_i^{\max})^2, \quad i = 1, \ldots, n \tag{9}$$

We have chosen to express constraints on (longitudinal) branch current rather than on active power flowing through the branch because overcurrent protections and conductor heating are related to Amperes and not Mega Watts. Note, however, that active, reactive and apparent power flow constraints can be easily incorporated if needed.

Finally, physical limits of some power system devices can be expressed as:

$$P_{gi}^{\min} \leq P_{gi} \leq P_{gi}^{\max}, \quad i = 1, \ldots, g \tag{10}$$

$$Q_{gi}^{\min} \leq Q_{gi} \leq Q_{gi}^{\max}, \quad i = 1, \ldots, g \tag{11}$$

$$r_i^{\min} \leq r_i \leq r_i^{\max}, \quad i = 1, \ldots, o \tag{12}$$

$$x_i^{\min} \leq x_i \leq x_i^{\max}, \quad i = 1, \ldots, s \tag{13}$$

$$\phi_i^{\min} \leq \phi_i \leq \phi_i^{\max}, \quad i = 1, \ldots, c \tag{14}$$

where for the i-th generator $P_{gi}^{\min}$, $P_{gi}^{\max}$ (resp. $Q_{gi}^{\min}$, $Q_{gi}^{\max}$) are its active (resp. reactive) output limits, for the i-th controllable transformer $r_i^{\min}$ and $r_i^{\max}$ are bounds on its ratio, for the i-th load $\phi_i^{\min}$ and $\phi_i^{\max}$ are limits on its curtailment percentage and finally, for the i-th shunt $x_i^{\min}$ and $x_i^{\max}$ are bounds on its reactance. Note that, although not shown explicitely, the variable r_i intervenes in the OPF formulation through the terms G_{sij}, B_{sij}, G_{ij} and B_{ij}, while x_i intervenes through the term B_{si} only.

2.6. Pros and cons of using rectangular coordinates

Although most (IP-based) OPF codes rely on a voltage polar coordinates model [9–11,22,27], equally good results (in terms of number of iterations to convergence and CPU time) have been reported with the voltage rectangular model for several OPF problems as well [12,24,26]. The good results obtained with voltages expressed in rectangular coordinates are due to in most OPF applications intervenes quadratic functions only, e.g., functions (1)–(13) are at most quadratic. The major advantage of a quadratic function is that its Hessian matrix is constant, e.g., second derivatives of load flow Eqs. (5) and (6), branch current (8) and voltage bounds (9) constraints. Besides, the Hessian matrix has slightly less non-zero terms and the computation of its terms requires much less operations as with polar coordinates, as clearly shown in references [12,26]. On the other hand, the main drawback of rectangular coordinates is the handling of bus voltage magnitudes constraints (9) and imposed voltages of generators (7) as functional constraints, instead of simple bounds as in case of using polar coordinates.

3. Interior-point based algorithms

3.1. Obtaining the optimality conditions in the interior-point method

The above OPF formulation, optimizing one objective among (1)–(4), subject to constraints (5)–(14) can be compactly written as a general nonlinear programming problem:

$$\min f(\mathbf{x}) \tag{15}$$

subject to:

$$\mathbf{g}(\mathbf{x}) = \mathbf{0} \tag{16}$$

$$\mathbf{h}(\mathbf{x}) \geq \mathbf{0} \tag{17}$$

where $f(\mathbf{x})$, $\mathbf{g}(\mathbf{x})$ and $\mathbf{h}(\mathbf{x})$ are assumed to be twice continuously differentiable, $\mathbf{x}$ is an m-dimensional vector that encompasses both control variables and state variables (real and imaginary part of voltage at all buses), $\mathbf{g}$ is a p-dimensional vector of functions and $\mathbf{h}$ is a q-dimensional vector of functions. To simplify the presentation simple bound constraints (10)–(14) have been included into the functional inequality constraints (17).

The IPM encompasses four steps to obtain optimality conditions. One first adds slack variables to inequality constraints, transforming them into equality constraints and non-negativity conditions on slacks:

$$\min f(\mathbf{x})$$

subject to:

$$\mathbf{g}(\mathbf{x}) = \mathbf{0}, \qquad \mathbf{h}(\mathbf{x}) - \mathbf{s} = \mathbf{0}, \qquad \mathbf{s} \geq \mathbf{0}$$

where the vectors $\mathbf{x}$ and $\mathbf{s} = [s_1, \ldots, s_q]^T$ are called primal variables.

The inequality constraints are then eliminated by adding them to the objective function as logarithmic barrier terms, resulting in the following equality constrained optimization problem:

$$\min f(\mathbf{x}) - \mu \sum_{i=1}^{q} \ln s_i$$

subject to:

$$\mathbf{g}(\mathbf{x}) = \mathbf{0}, \qquad \mathbf{h}(\mathbf{x}) - \mathbf{s} = \mathbf{0}$$

where μ is a positive scalar called barrier parameter which is gradually decreased to zero as iterations progress. Let us remark that at the heart of IPM is the main theorem from [8], which proves that as μ tends to zero, the solution $\mathbf{x}(\mu)$ converges to a local optimum of the problem (15)–(17).

Next, one transforms the equality constrained optimization problem into an unconstrained one, by defining the Lagrangian:

$$L_\mu(\mathbf{y}) = f(\mathbf{x}) - \mu\sum_{i=1}^{q}\ln s_i - \boldsymbol{\lambda}^T\mathbf{g}(\mathbf{x}) - \boldsymbol{\pi}^T[\mathbf{h}(\mathbf{x}) - \mathbf{s}]$$

where the vectors of Lagrange multipliers $\boldsymbol{\lambda}$ and $\boldsymbol{\pi}$ are called dual variables and $\mathbf{y} = [\mathbf{s}, \boldsymbol{\pi}, \boldsymbol{\lambda}, \mathbf{x}]^T$.

Finally, the perturbed KKT first order necessary optimality conditions of the resulting problem are obtained by setting to zero the derivatives of the Lagrangian with respect to all unknowns [8]:

$$\begin{bmatrix} \nabla_{\mathbf{s}} L_\mu(\mathbf{y}) \\ \nabla_{\boldsymbol{\pi}} L_\mu(\mathbf{y}) \\ \nabla_{\boldsymbol{\lambda}} L_\mu(\mathbf{y}) \\ \nabla_{\mathbf{x}} L_\mu(\mathbf{y}) \end{bmatrix} = \begin{bmatrix} -\mu\mathbf{e} + \mathbf{S}\boldsymbol{\pi} \\ -\mathbf{h}(\mathbf{x}) + \mathbf{s} \\ -\mathbf{g}(\mathbf{x}) \\ \nabla f(\mathbf{x}) - \mathbf{J_g}(\mathbf{x})^T\boldsymbol{\lambda} - \mathbf{J_h}(\mathbf{x})^T\boldsymbol{\pi} \end{bmatrix} = \mathbf{0} \tag{18}$$

where $\mathbf{S}$ is a diagonal matrix of slack variables, $\mathbf{e} = [1, \ldots, 1]^T$, $\nabla f(\mathbf{x})$ is the gradient of f, $\mathbf{J_g}(\mathbf{x})$ is the Jacobian of $\mathbf{g}(\mathbf{x})$ and $\mathbf{J_h}(\mathbf{x})$ is the Jacobian of $\mathbf{h}(\mathbf{x})$.

3.2. The primal dual algorithm

We briefly outline the PD algorithm to solve KKT optimality conditions (18):

(1) Set iteration count $k = 0$. Chose $\mu^0 > 0$. Initialize $\mathbf{y}^0$, taking care that slack variables and their corresponding dual variables are strictly positive $(\mathbf{s}^0, \boldsymbol{\pi}^0) > \mathbf{0}$.

(2) Solve the linearized KKT conditions for the Newton direction $\Delta\mathbf{y}^k$:

$$\mathbf{H}(\mathbf{y}^k)\begin{bmatrix} \Delta\mathbf{s}^k \\ \Delta\boldsymbol{\pi}^k \\ \Delta\boldsymbol{\lambda}^k \\ \Delta\mathbf{x}^k \end{bmatrix} = \begin{bmatrix} \mu^k\mathbf{e} - \mathbf{S}^k\boldsymbol{\pi}^k \\ \mathbf{h}(\mathbf{x}^k) - \mathbf{s}^k \\ \mathbf{g}(\mathbf{x}^k) \\ -\nabla f(\mathbf{x}^k) + \mathbf{J_g}(\mathbf{x}^k)^T\boldsymbol{\lambda}^k + \mathbf{J_h}(\mathbf{x}^k)^T\boldsymbol{\pi}^k \end{bmatrix} \tag{19}$$

where $\mathbf{H}(\mathbf{y}^k)$ is the second derivative Hessian matrix $(\partial^2 L_\mu(\mathbf{y}^k)/\partial\mathbf{y}^2)$.

(3) Determine the maximum step length $\alpha^k \in (0, 1]$ along the Newton direction $\Delta\mathbf{y}^k$ such that $(\mathbf{s}^{k+1}, \boldsymbol{\pi}^{k+1}) > \mathbf{0}$:

$$\alpha^k = \min\left\{1, \gamma \min_{\Delta s_i^k < 0} \frac{-s_i^k}{\Delta s_i^k}, \gamma \min_{\Delta \pi_i^k < 0} \frac{-\pi_i^k}{\Delta \pi_i^k}\right\} \tag{20}$$

where $\gamma \in (0, 1)$ is a safety factor aiming to ensure strict positiveness of slack variables and their corresponding dual variables. A typical value of the safety factor is $\gamma = 0.99995$. Update solution:

$$\mathbf{s}^{k+1} = \mathbf{s}^k + \alpha^k\Delta\mathbf{s}^k \qquad \boldsymbol{\pi}^{k+1} = \boldsymbol{\pi}^k + \alpha^k\Delta\boldsymbol{\pi}^k$$
$$\mathbf{x}^{k+1} = \mathbf{x}^k + \alpha^k\Delta\mathbf{x}^k \qquad \boldsymbol{\lambda}^{k+1} = \boldsymbol{\lambda}^k + \alpha^k\Delta\boldsymbol{\lambda}^k$$

(4) Check convergence. A (locally) optimal solution is found and the optimization process terminates when: primal feasibility, scaled dual feasibility, scaled complementarity gap and objective function variation from an iteration to the next fall below some tolerances [10–12]:

$$\max\left\{\max_i\{-h_i(\mathbf{x}^k)\}, ||\boldsymbol{g}(\mathbf{x}^k)||_\infty\right\} \le \varepsilon_1 \tag{21}$$

$$\frac{||\nabla f(\mathbf{x}^k) - \boldsymbol{J_g}(\mathbf{x}^k)^T\boldsymbol{\lambda} - \boldsymbol{J_h}(\mathbf{x}^k)^T\boldsymbol{\pi}^k||_\infty}{1 + ||\mathbf{x}^k||_2 + ||\boldsymbol{\lambda}^k||_2 + ||\boldsymbol{\pi}^k||_2} \le \varepsilon_1 \tag{22}$$

$$\frac{\rho^k}{1 + ||\mathbf{x}^k||_2} \le \varepsilon_2 \tag{23}$$

$$\frac{|f(\mathbf{x}^k) - f(\mathbf{x}^{k-1})|}{1 + |f(\mathbf{x}^k)|} \le \varepsilon_2 \tag{24}$$

where $\rho^k = (\mathbf{s}^k)^T\boldsymbol{\pi}^k$ is called complementarity gap.

(5) If convergence was not achieved, update the barrier parameter:

$$\mu^{k+1} = \sigma\frac{\rho^k}{q}$$

where usually $\sigma = 0.2$. Set $k = k + 1$ and go to step 2.

3.3. The predictor–corrector algorithm

Here, instead of updating iteratively the unknown vector $\mathbf{y}$ as in the Newton method, we introduce the new point $\mathbf{y}^{k+1} = \mathbf{y}^k + \Delta\mathbf{y}$ directly into the Newton system (19), obtaining[1]:

$$\mathbf{H}\begin{bmatrix} \Delta\mathbf{s} \\ \Delta\boldsymbol{\pi} \\ \Delta\boldsymbol{\lambda} \\ \Delta\mathbf{x} \end{bmatrix} = \begin{bmatrix} \mu\mathbf{e} - \mathbf{S}\boldsymbol{\pi} - \Delta\mathbf{S}\Delta\boldsymbol{\pi} \\ \mathbf{h}(\mathbf{x}) + \mathbf{h}(\Delta\mathbf{x}) - \mathbf{s} \\ \mathbf{g}(\mathbf{x}) + \mathbf{g}(\Delta\mathbf{x}) \\ -\nabla f(\mathbf{x}) + \mathbf{J_g}(\mathbf{x})^T\boldsymbol{\lambda} + \mathbf{J_h}(\mathbf{x})^T\boldsymbol{\pi} \end{bmatrix} \tag{25}$$

What differs with respect to the Newton system (19) are the Δ terms from the right-hand side. Observe that this system cannot be solved directly because the higher-order terms in (25) are not known in advance. Mehrotra [13] proposed a two step procedure involving a predictor and a corrector step, which we describe below.

3.3.1. The predictor step

The predictor step objective is twofold: to approximate higher-order terms in (25) and to dynamically estimate the barrier parameter μ. To this purpose one solves the system (25) for the affine-scaling direction, obtained by neglecting in its right-

[1] In the remaining of the paper k is dropped for the sake of readability.

hand side the higher-order terms and μ, that is:

$$\mathbf{H}\begin{bmatrix}\Delta\mathbf{s}_{af}\\ \Delta\boldsymbol{\pi}_{af}\\ \Delta\boldsymbol{\lambda}_{af}\\ \Delta\mathbf{x}_{af}\end{bmatrix}=\begin{bmatrix}-\mathbf{S}\boldsymbol{\pi}\\ \mathbf{h}(\mathbf{x})-\mathbf{s}\\ \mathbf{g}(\mathbf{x})\\ -\nabla f(\mathbf{x})+\mathbf{J}_{\mathbf{g}}(\mathbf{x})^T\boldsymbol{\lambda}+\mathbf{J}_{\mathbf{h}}(\mathbf{x})^T\boldsymbol{\pi}\end{bmatrix}$$

Next the affine complementarity gap ρ_{af} is computed:

$$\rho_{af}=(\mathbf{s}+\alpha_{af}\Delta\mathbf{s}_{af})^T(\boldsymbol{\pi}+\alpha_{af}\Delta\boldsymbol{\pi}_{af})$$

where $\alpha_{af}\in(0,1]$ is the step length which would be taken along the affine scaling direction if the latter was used (20).

Finally, the barrier parameter for the next iteration is estimated from:

$$\mu_{af}=\min\left\{\left(\frac{\rho_{af}}{\rho}\right)^2, 0.2\right\}\frac{\rho_{af}}{q}$$

where $\rho=\mathbf{s}^T\boldsymbol{\pi}$ denotes the complementarity gap at the current iterate.

The goal of this adaptive scheme is to significantly reduce the barrier parameter when a large decrease in complementarity gap from affine direction is obtained ($\rho_{af}\ll\rho$) and to slightly reduce it otherwise.

3.3.2. The corrector step

The actual Newton direction is finally computed from:

$$\mathbf{H}\begin{bmatrix}\Delta\mathbf{s}\\ \Delta\boldsymbol{\pi}\\ \Delta\boldsymbol{\lambda}\\ \Delta\mathbf{x}\end{bmatrix}=\begin{bmatrix}\mu_{af}\mathbf{e}-\mathbf{S}\boldsymbol{\pi}-\Delta\mathbf{S}_{af}\Delta\boldsymbol{\pi}_{af}\\ \mathbf{h}(\mathbf{x})+\mathbf{h}(\alpha_{af}\Delta\mathbf{x}_{af})-\mathbf{s}\\ \mathbf{g}(\mathbf{x})+\mathbf{g}(\alpha_{af}\Delta\mathbf{x}_{af})\\ -\nabla f(\mathbf{x})+\mathbf{J}_{\mathbf{g}}(\mathbf{x})^T\boldsymbol{\lambda}+\mathbf{J}_{\mathbf{h}}(\mathbf{x})^T\boldsymbol{\pi}\end{bmatrix}$$

It is useful to note that the predictor–corrector procedure involves at every iteration the solution of two linear systems of equations with different right-hand sides while relying on the same matrix factorization (done on the predictor step). The extra computational burden with respect to the PD algorithm is only one solution of the corrector system of equations with the matrix already factorized and the additional test to compute μ_{af}. Generally, even if there is an increase of computing time per iteration, the overall computing time is less, thanks to a reduction of the number of iterations.

3.4. The multiple centrality corrections algorithm

MCC algorithm consists also of a predictor and a corrector step, where the predictor step is exactly as in the Mehrotra's predictor–corrector procedure.

3.4.1. The corrector step

The aim of this step is to compute a corrector direction $\Delta\mathbf{y}_{co}$ such that: (i) a larger step size can be taken for the composite direction $\Delta\mathbf{y}=\Delta\mathbf{y}_{af}+\Delta\mathbf{y}_{co}$ and (ii) the point $\mathbf{y}$ at the current iterate is driven in a vicinity of the central path.[2]

We describe in the sequel the procedure to accomplish these objectives. Let us assume that we propose to increase the step length α_{af} to:

$$\tilde{\alpha}=\min(\alpha_{af}+\delta_\alpha, 1)$$

where δ_α is the desired improvement of the step length. Incidentally, the empirical observation that the two goals: improving centrality and increasing the step size might be contradictory, especially for large value of δ_α, constrains to use small values for δ_α (very often $\delta_\alpha\in[0.1, 0.2]$) [15].

Let $\mathbf{y}$ be the solution at the current iteration. At the trial point:

$$\tilde{\mathbf{y}}=\mathbf{y}+\tilde{\alpha}\Delta\mathbf{y}_{af}$$

some slack variables and/or their corresponding dual variables may violate strictly positivity conditions $(\mathbf{s}, \boldsymbol{\pi})>\mathbf{0}$.[3] The corrector term $\Delta\mathbf{y}_{co}$ has thus to offset for these negative terms as well as to drive the trial point in the neighbourhood of the central path. To this end a target close to the central path must be defined in the space of complementarity products. Because the most natural such target, the analytic center $\mu_{af}\mathbf{e}$ is usually unreachable, for practical purposes, we require instead that all complementarity products belong to the interval $[\beta_{\min}\mu_{af}, \beta_{\max}\mu_{af}]$. Typical values for $\beta_{\min}$ and $\beta_{\max}$ are: $\beta_{\min}=0.1$ and $\beta_{\max}=10$ [15,22,23].

To determine a value of $\Delta\mathbf{y}_{co}$ which leads to complementary products satisfying the above mentioned condition, we proceed as follows. First, we compute the complementarity products at the trial point $\tilde{\mathbf{v}}=\tilde{\mathbf{S}}\,\tilde{\boldsymbol{\pi}}$. Then we identify components of $\tilde{\mathbf{v}}$ that do not belong to the interval $[\beta_{\min}\mu_{af}, \beta_{\max}\mu_{af}]$, called outlier complementarity products. The corrector step effort focuses on correcting the outliers only in order to improve the centrality of the next iterate. To achieve this, some target complementary products $\tilde{\mathbf{v}}^t$ are defined by projecting the components of $\tilde{\mathbf{v}}$ to a hypercube $\boldsymbol{H}=[\beta_{\min}\mu_{af}, \beta_{\max}\mu_{af}]^q$:

$$v_i^t=\begin{cases}\beta_{\min}\mu_{af}, & \text{if}\quad \tilde{v}_i<\beta_{\min}\mu_{af}\\ \beta_{\max}\mu_{af}, & \text{if}\quad \tilde{v}_i>\beta_{\max}\mu_{af}\\ \tilde{v}_i, & \text{otherwise}\end{cases}$$

Finally, the corrector direction $\Delta\mathbf{y}_{co}$ is obtained as the solution of the following linear system:

$$\mathbf{H}\begin{bmatrix}\Delta\mathbf{s}_{co}\\ \Delta\boldsymbol{\pi}_{co}\\ \Delta\boldsymbol{\lambda}_{co}\\ \Delta\mathbf{x}_{co}\end{bmatrix}=\begin{bmatrix}\mathbf{v}^t-\tilde{\mathbf{v}}\\ \mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}$$

where the non-zero components of the right-hand-side correspond to the outlier complementarity products only.

The new search direction can now be obtained:

$$\Delta\mathbf{y}=\Delta\mathbf{y}_{af}+\Delta\mathbf{y}_{co}$$

[2] The central path can be defined as the set of points $\mathbf{y}(\mu)$, with $0<\mu<\infty$, which satisfy the KKT Eq. (18).

[3] This is especially true when $\alpha_{af}<1$, since it indicates that adding $\alpha_{af}\,\Delta\mathbf{y}_{af}$ to $\mathbf{y}$ will already drive some slack or dual variables close to 0 (see Eq. (20)).

Then, the actual step length α is computed so as to preserve non-negativity conditions and variables are updated:

$$\mathbf{y} \leftarrow \mathbf{y} + \alpha \Delta \mathbf{y}$$

The corrector step can be applied several times. In such a case, the current direction $\Delta\mathbf{y}$ becomes the predictor for a new corrector, that is:

$$\Delta \mathbf{y}_{af} \leftarrow \Delta \mathbf{y}_{af} + \Delta \mathbf{y}_{co} \quad \text{and} \quad \alpha_{af} \leftarrow \alpha$$

The computation of a new centrality correction terminates as soon as one of the following conditions is achieved: (i) the maximal number of corrections allowed K is reached, (ii) the gain in step length $(\alpha - \alpha_{af})$ is insignificant and (iii) the step length becomes $\alpha = 1$.

3.4.2. Some implementation issues of the MCC algorithm

To lighten the presentation, we have considered a common step length α for updating both primal and dual variables. Our experience showed, however, that the MCC algorithm behaves better when applying separate step lengths in primal and dual spaces.

As regards the choice of the desired improvement of the step size δ_α, two solutions are proposed: either the use of a constant value $\delta_\alpha = 0.1$ [15,23], or the use of an adaptive value according to the formula $\delta_\alpha = (1 - \alpha_{af})/K$ while imposing additionally that δ_α ought not to be smaller than 0.1 or greater than 0.2 [23,22]. In order to reduce the number of centrality corrections computed at an iteration while aiming to obtain the largest increase in the step size we suggest to set $\delta_\alpha = 0.2$. We have found that this setting offers better convergence performances in terms of CPU time and number of iterations than the above mentioned proposals. We have not encountered any convergence trouble caused by this “optimistic” setting.

In our implementation we compute a new centrality correction only if it improves considerably the step lengths. To this end, we allow the computation of a new centrality correction only if the gain in step length $(\alpha - \alpha_{af})$ is superior to $\varepsilon_\alpha = 0.03$. Some other authors have suggested that a value for ε_α as small as 0.01 may be accepted [15,23].

The most challenging problem of the MCC algorithm is the choice of the maximal number of corrections allowed K, as the objective is not only to reduce the number of iterations comparatively to the PC algorithm but also to save some CPU time. To this respect some heuristics were proposed in the literature [15,23]. We instead determine K as follows. Assume a given power system and OPF variant. We first solve this OPF variant by progressively increasing the maximal number of corrections allowed K (going from 1 to 10, for instance). Then we randomly perturb (e.g. modifying generators active power, load consumptions, topology, etc.) the initial operating point of the power system to be optimized and repeat the previous step a predefined number of times. Finally, results are gathered together and the value of K that led to the overall smallest CPU time is determined. This value is automatically used for all subsequent solutions of the same type of OPF variant and the same power system. This procedure for choosing K proved to be quite robust.

Note, finally, that the algorithms performance in terms of CPU time may also depend somewhat on to the solver used for the solution of the linear system of equations. In this work we rely on the sparse object-oriented library SPOOLES [25]. This library, to the best of our knowledge, is not widely used but exhibits excellent performance of its linear algebra kernel (see also Netlib web-site: www.netlib.org).

4. Numerical results

In this section we present numerical results of four OPF applications while comparing PD, PC and MCC algorithms performance. The OPF has been coded in C++ and runs under Cygwin or Linux environments. It has been tested on three test systems, namely a 60 bus system which is a modified variant of the Nordic32 system, and the popular IEEE118 and IEEE300 systems. A summary of these test systems is given in Table 1.

All tests have been performed on a PC Pentium IV of 1.7 GHz and 512 MB RAM. The MCC algorithm runs with the following parameters: $\beta_{\min} = 0.1$, $\beta_{\max} = 10$, $\delta_\alpha = 0.2$ and $\varepsilon_\alpha = 0.03$.

The convergence tolerances are set to $\varepsilon_1 = 10^{-4}$ for the primal feasibility (21) and the scaled dual feasibility (22), and $\varepsilon_2 = 10^{-6}$ for the scaled complementarity gap (23) and the scaled objective function variation from an iteration to the next (24).

4.1. Minimizing active power losses

We first deal with the minimization of transmission active power losses (2), counted as the sum of active losses over all branches of the system. We consider the following control variables: slack generator active power, generators reactive power, controllable transformers ratio and shunt reactance. The equality constraints involve the buses power balance (5) and (6). The inequality constraints concern bounds on: generators reactive power (11), voltage magnitudes (9), transformer with controllable ratio (12) and shunt reactance (13). Voltage magnitudes are allowed to vary between 0.95 pu and 1.05 pu in all buses.

Table 1
Test systems summary

System	n	g	c	b	l	t	o	s
Nordic32	60	23	22	81	57	31	4	12
IEEE118	118	54	91	186	175	11	9	14
IEEE300	300	69	198	411	282	129	50	14

Table 2
Number of iterations to convergence and CPU times

Interior-point algorithm	Nordic32		IEEE118		IEEE300	
	Iters.	Time	Iters.	Time	Iters.	Time
PD	11	0.31	13	0.62	16	1.63
PC	8	0.26	10	0.58	12	1.44
MCC	6	0.24	7	0.52	10	1.48

Table 3
Number and type of active constraints

System	Active constraints				
	V	Q_g	r	x	Total
Nordic32	21	0	0	4	25
IEEE118	15	12	0	4	31
IEEE300	60	23	1	4	88

Table 4
Number of iterations to convergence and CPU times

Interior-point algorithm	Nordic32		IEEE118		IEEE300	
	Iters.	Time	Iters.	Time	Iters.	Time
PD	21	0.82	18	1.10	23	3.02
PC	13	0.61	11	0.80	15	2.33
MCC	12	0.71	10	0.90	11	2.16

Table 2 provides the number of iterations to convergence and the CPU times[4] (in seconds) for the PD, PC and MCC algorithms, while Table 3 shows the number and the type of binding constraints at optimum.

The active losses at the optimum are: 151.74 MW for the Nordic32 system, 116.52 MW for the IEEE118 system and 386.6 MW for the IEEE300 system, respectively. This corresponds, respectively, to 7.90%, 12.31%, and 5.37% of less active losses than in the base case.

As bus voltages are free to vary between bounds, in order to reduce active losses, all algorithms tend to increase voltages to their upper bound, which explains the quite high number of active voltage constraints. On the other hand, limits on: generators reactive power, transformers ratio and shunts reactance prevent some voltages to be further increased. For the IEEE118 and the IEEE300 systems, since many generators have a (very) narrow reactive capability, most of them hit one of their reactive power production limits.

The Nordic32 system was the easiest to optimize because at the initial point all voltages belong to their allowable variation interval and most generators have rather large reactive capability. In contrast, the IEEE300 system was the hardest to optimize due to the (very) tight reactive capability of many generators as well as the very broad voltage profile at the initial point, voltages being in the range of 0.92–1.08 pu.

The three algorithms under study behave well for this OPF variant. In this case, PC and MCC algorithms perform slightly better than the PD one. For all the test systems, the MCC algorithm needs a smaller number of iterations to convergence compared to the PC algorithm, while leading to a slight reduction of the CPU time only for the first two systems. The MCC algorithm has been run for this optimization problem with $K = 4$ for the Nordic32 system and $K = 5$ for both the IEEE118 and the IEEE300 systems.

4.2. *Minimizing overall generation costs*

We now focus on minimizing the overall generation costs (1) by playing on the following control variables: generator active and reactive powers, controllable transformer ratio and shunt reactance. The equality constraints are the bus active and reactive power flow Eqs. (5) and (6). The inequality constraints include bounds on all above mentioned control variables (10)–(13) as well as limits on voltage magnitudes (9) and branch currents (8).

[4] CPU time concerns the optimization process only.

Note that, the active power losses are implicitly taken into account by both reactive control variables (generator reactive power, controllable transformer ratio and shunt reactance) and active control variables (generator active power) and assigned to the cheapest generator(s).

Table 4 displays the number of iterations to convergence as well as the CPU times for the three algorithms studied. Table 5 shows the number and the type of binding constraints at optimum, where P_g, Q_g, I, V, r and x refer to constraints relative to generator active power, generator reactive power, branch current, bus voltage magnitude, controllable transformer ratio and shunt reactance, respectively.

For the IEEE118 and the IEEE300 systems we have considered a quadratic cost curve for all generators participating in the optimization, while for the Nordic32 system we have chosen a linear cost curve ($c_{2i} = 0$, $i = 1, \ldots, g$ in (1)), which explains the high number of active power generator constraints binding at optimum.

Once again, all algorithms behave well also for this OPF variant, with higher-order algorithms (PC and MCC) clearly outperforming the pure PD algorithm in terms of both number of iterations and CPU time. The MCC algorithm leads to a reduction of number of iterations with respect to the PC algorithm for all three-test systems. Nevertheless, except for the IEEE300 system, this reduction of iterations count does not translate in a CPU time saving.

As regards the MCC algorithm, it has been run with $K = 5$ for the Nordic32 system, $K = 6$ for the IEEE118 system and $K = 7$ for the IEEE300 system.

Note that, the dual variables at the optimal solution yield very precious information. They are equal to the sensitivity of the objective to a small constraint shift. In particular, the dual variable (Lagrange multiplier) associated with each active power flow equation represents the variation of the overall generation cost for an increment of the active load at that bus. They are called nodal prices and used as a method of pricing in some deregulated electricity markets [2].

Table 5
Number and type of active constraints

System	Active constraints						
	P_g	Q_g	I	V	r	x	Total
Nordic32	15	1	5	26	0	4	51
IEEE118	3	8	3	25	0	5	44
IEEE300	3	26	4	59	3	2	97

Table 6
Number of iterations to convergence and CPU times

Interior-point algorithm	Nordic32		IEEE118		IEEE300	
	Iters.	Time	Iters.	Time	Iters.	Time
PD	19	0.48	21	0.99	47	4.65
PC	11	0.37	15	0.83	19	2.60
MCC	9	0.33	11	0.78	14	2.46

4.3. Maximizing power system loadability

We now focus on the determination of the maximum loadability of a power system (3). We assume that all loads are increased proportionally to their base case consumptions, increase covered by a single "slack" generator. The control variables considered are: generators reactive power, variable ratio of controllable transformers, shunts reactance and slack generator active output. The equality constraints concern buses active and reactive power balance (5) and (6). The inequality constraints include limits on: generator reactive power (11), transformer variable ratio (12), shunt reactance (13) and bus voltage magnitudes (9) only. The latter are allowed to vary between 0.95 pu and 1.05 pu.

Table 6 gives the number of iterations to convergence and CPU times for the PD, PC and MCC algorithms, while Table 7 shows the number and the type of binding constraints at optimum.

The maximum loadability margin is: 537.8 MW for the Nordic32 system, 1119.1 MW for the IEEE118 system and 445.9 MW for the IEEE300 system. These margins are limited mostly by some voltages reaching their minimal bound (0.95 pu).

The MCC algorithm behaves better than the PD and PC ones for all three-test systems. The MCC algorithm runs, for this optimization problem, with $K = 4$ for the Nordic32, $K = 5$ for the IEEE118 system and $K = 6$ for the IEEE300 system. Note the very slow convergence of the PD algorithm for the IEEE300 system, behaviour which sometimes happens especially for very nonlinear OPF problems.

4.4. Minimizing the amount of load shedding

We finally tackle the problem of minimizing the amount of load shedding (4) for an infeasible power system situation such that an equilibrium point is restored. We assume that at each load bus the load shedding is done under constant power factor and $0 \leq \phi_i \leq 0.1$, $i = 1, \ldots, c$ in (14). The slack generator only compensates for the active power imbalance. The control variables used are: load curtailment percentage (ϕ), controllable ratio of transformers, shunts reactance and slack generator active output. The equality constraints involve the buses power balance (5) and (6) and imposed voltage of generators (7). The inequality constraints include limits on: the amount of bus load shedding (14), controllable transformer ratio (12) and shunt reactance (13).

Table 7
Number and type of active constraints

System	Active constraints				
	V	Q_g	r	x	Total
Nordic32	17	5	0	6	28
IEEE118	27	21	0	10	58
IEEE300	57	29	4	5	95

Table 8
Number of iterations to convergence and CPU times

Interior-point algorithm	Nordic32		IEEE118		IEEE300	
	Iters.	Time	Iters.	Time	Iters.	Time
PD	25	0.44	47	1.98	20	1.47
PC	11	0.23	10	0.55	13	1.14
MCC	10	0.21	6	0.39	9	1.02

Table 9
Number and type of active constraints

System	Active constraints			
	ϕ	r	x	Total
Nordic32	20	0	12	32
IEEE118	91	0	0	91
IEEE300	177	14	5	196

Table 8 displays the number of iterations to convergence and CPU times for the PD, PC and MCC algorithms, while Table 9 provides the number and the type of binding constraints at optimum.

The overall load curtailed is 349.3 MW for the Nordic32 system, 1078.8 MW for IEEE300 system while no load is shed for the IEEE118 system. The curtailment effort is shared by 9 and 88 loads for the Nordic32 system and IEEE300 system, respectively.

The MCC algorithm compares again favorably with the PC one in terms of both iterations number and CPU time. Once again, these two algorithms behave much better than the PD one. As regards the MCC algorithm, it has been run with $K = 3$ for the Nordic32 system, $K = 5$ for the IEEE118 system and $K = 6$ for the IEEE300 system.

As a general remark, we have found performances of the MCC algorithm less sensitive to the initial value of the barrier parameter μ^0 than those of the PC and PD algorithms.

Note finally that there is room to improve the CPU times of the optimization process presented along this paper because, up to now, we focused more on the flexibility of the software code than on its computational speed.

5. Conclusions and future works

5.1. Conclusions

This paper has presented and compared the performances of three IP based algorithms: PD, PC and MCC. They have been able to solve successfully several OPF variants on test systems ranging from 60 till 300 buses.

Our experience with these algorithms confirms other results from the literature, that is, the PC generally outperforms the

PD algorithm in terms of CPU time, and thereby number of iterations, on large-scale optimization problems. The opposite situation sometimes happens, especially for rather simple problems.

The results obtained suggest also that the MCC algorithm is a highly viable alternative to the successful PC algorithm. The good performances of the MCC algorithm emphasize once more the importance of keeping the current iterate along the central path.

5.2. Future works

As regards the MCC algorithm, future work aims to find a better heuristic scheme to automatically choose the maximum number of corrections allowed K for a given problem, in as much as the best value of K shifts from an OPF variant to another and from a test system to another.

A natural extension of this work is the development of a hybrid algorithm, in which the corrector step combines both MCC and PC features, in order to take advantage of their respective qualities, as reported in [22].

A challenging future work is the development of a security constrained OPF (SCOPF), which handles together base case constraints and steady-state security constraints relative to credible (mainly "$N-1$") contingencies [28]. The benchmark SCOPF is stated as an extension of the base case OPF model (1)–(14) that includes a set of equality and inequality constraints of type (5)–(14) for each post-contingency state. One may distinguish between two SCOPF formulations: "preventive" and "corrective". The difference between these variants is that, in the preventive SCOPF, the re-scheduling of control variables in post-contingency states is not allowed. The major drawback of the brute force approach to the SCOPF benchmark is the high dimensionality of the problem, especially for large power systems and many contingent cases.

Since not all contingencies constrain the optimum, clearly, there is a need to filter out such "harmless" contingencies and to perform the SCOPF with "harmful" contingencies only. The simplest technique for contingency filtering is post-contingency constraint relaxation. Thus, one first runs the SCOPF with the base case constraints only. Then, at the resulted optimal solution, contingencies are simulated one by one. If no contingency violates any security limit the solution is found, otherwise the equality and inequality constraints relative to those contingencies which violate some limits or are very close to them are added to the SCOPF.

A widely used common approach to both SCOPF variants consists in adding to the base case constraints only relevant post-contingency inequalities, linearized around the base case optimized operating point, while dropping all post-contingency equality constraints. The latter constraints are however checked at the optimal solution. This approach requires iterating between the solution of the SCOPF and the linearization of post-contingency inequality constraints until some convergence criteria are met.

Alternatively, Benders decomposition is a very promising technique to deal with a corrective SCOPF [29]. In this approach the original SCOPF problem is decomposed into a master problem and some slave problems, each corresponding to a harmful contingency case. Thus, a slave problem encompasses mainly the post-contingency constraints relative to a contingency and provides a linear constraint (Benders cut) to the master problem. The latter contains base case constraints and Benders cuts stemming from slave problems. At each iteration the slave problems fed the master problem with improved Benders cuts until convergence is reached. Clearly, the simultaneous solution of slave problems, distributed among several processors, is possible; it can significantly speed-up computations.

Besides, finding a trade-off between both preventive and corrective SCOPF variants still remains an open question [30].

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