Distortion of filtered signals

MATLAB tutorial series (Part 3)

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Distortionless response system

A system has distortionless response if the input signal $x[n]$ and the output signal $y[n]$ have the same shape.

Distortionless response system

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It means:

$$y[n] = Gx[n - n_d]$$

G, n_d: **constant**

Distortionless response system

A system has distortionless response if the input signal $x[n]$ and the output signal $y[n]$ have the same shape.

It means:

$$Y\left(e^{j\omega}\right) = G e^{-j\omega n_d} X\left(e^{j\omega}\right),$$

Distortionless response system

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It means:

$$Y\left(e^{j\omega}\right) = Ge^{-j\omega n_d}X\left(e^{j\omega}\right),$$

$$H\left(e^{j\omega}\right) = \frac{Y\left(e^{j\omega}\right)}{X\left(e^{j\omega}\right)} = Ge^{-j\omega n_d}$$

Distortionless response system

A system has distortionless response if the input signal $x[n]$ and the output signal $y[n]$ have the same shape.

It means:

$$\left|H\left(e^{j\omega}\right)\right| = G,$$

$$\angle H\left(e^{j\omega}\right) = -n_d\omega.$$

Distortionless response system

A system has distortionless response if the input signal $x[n]$ and the output signal $y[n]$ have the same shape.

It means:

$$\left|H\left(e^{j\omega}\right)\right| = G,$$

$$\angle H\left(e^{j\omega}\right) = -n_d\omega.$$

Notice: phase response passes from the origin !

Example (pp. 216-218)

$$x[n] = \cos(\omega_0 n) - \frac{1}{3}\cos(3\omega_0 n) + \frac{1}{5}\cos(5\omega_0 n),$$

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$$x[n] = \cos(\omega_0 n) - \frac{1}{3}\cos(3\omega_0 n) + \frac{1}{5}\cos(5\omega_0 n),$$

$$y_i[n] = c_1 \cos(\omega_0 n + \varphi_1) + c_2 \cos(3\omega_0 n + \varphi_2)$$

$$+c_3 \cos(5\omega_0 n + \varphi_3).$$

Example (pp. 216-218)

$$x[n] = \cos(\omega_0 n) - \frac{1}{3}\cos(3\omega_0 n) + \frac{1}{5}\cos(5\omega_0 n),$$

$$y_0[n] = 1\cos(\omega_0 n + 0) - 1/3\cos(3\omega_0 n + 0)$$

$$+1/5\cos(5\omega_0 n + 0).$$

Original signal
no change !

Example (pp. 216-218)

$$x[n] = \cos(\omega_0 n) - \frac{1}{3}\cos(3\omega_0 n) + \frac{1}{5}\cos(5\omega_0 n),$$

$$y_1[n] = 1/4\cos(\omega_0 n + 0) - 1/3\cos(3\omega_0 n + 0)$$

$$+1/5\cos(5\omega_0 n + 0).$$

High pass filter

Low frequency attenuated !

Example (pp. 216-218)

$$x[n] = \cos(\omega_0 n) - \frac{1}{3}\cos(3\omega_0 n) + \frac{1}{5}\cos(5\omega_0 n),$$

$$y_2[n] = \cos(\omega_0 n + 0) - 1/6\cos(3\omega_0 n + 0)$$

$$+1/10\cos(5\omega_0 n + 0).$$

Low pass filter

High frequencies attenuated !

Example (pp. 216-218)

$$x[n] = \cos(\omega_0 n) - \frac{1}{3}\cos(3\omega_0 n) + \frac{1}{5}\cos(5\omega_0 n),$$

$$y_3[n] = \cos(\omega_0 n + \pi/6) - 1/3\cos(3\omega_0 n + \pi/6)$$

$$+1/5\cos(5\omega_0 n + \pi/6).$$

Constant phase

Example (pp. 216-218)

$$x[n] = \cos(\omega_0 n) - \frac{1}{3}\cos(3\omega_0 n) + \frac{1}{5}\cos(5\omega_0 n),$$

$$y_4[n] = \cos(\omega_0 n - \pi/4) - 1/3\cos(3\omega_0 n - 3\pi/4)$$

$$+1/5\cos(5\omega_0 n - 5\pi/4).$$

Linear phase

Example (pp. 216-218)

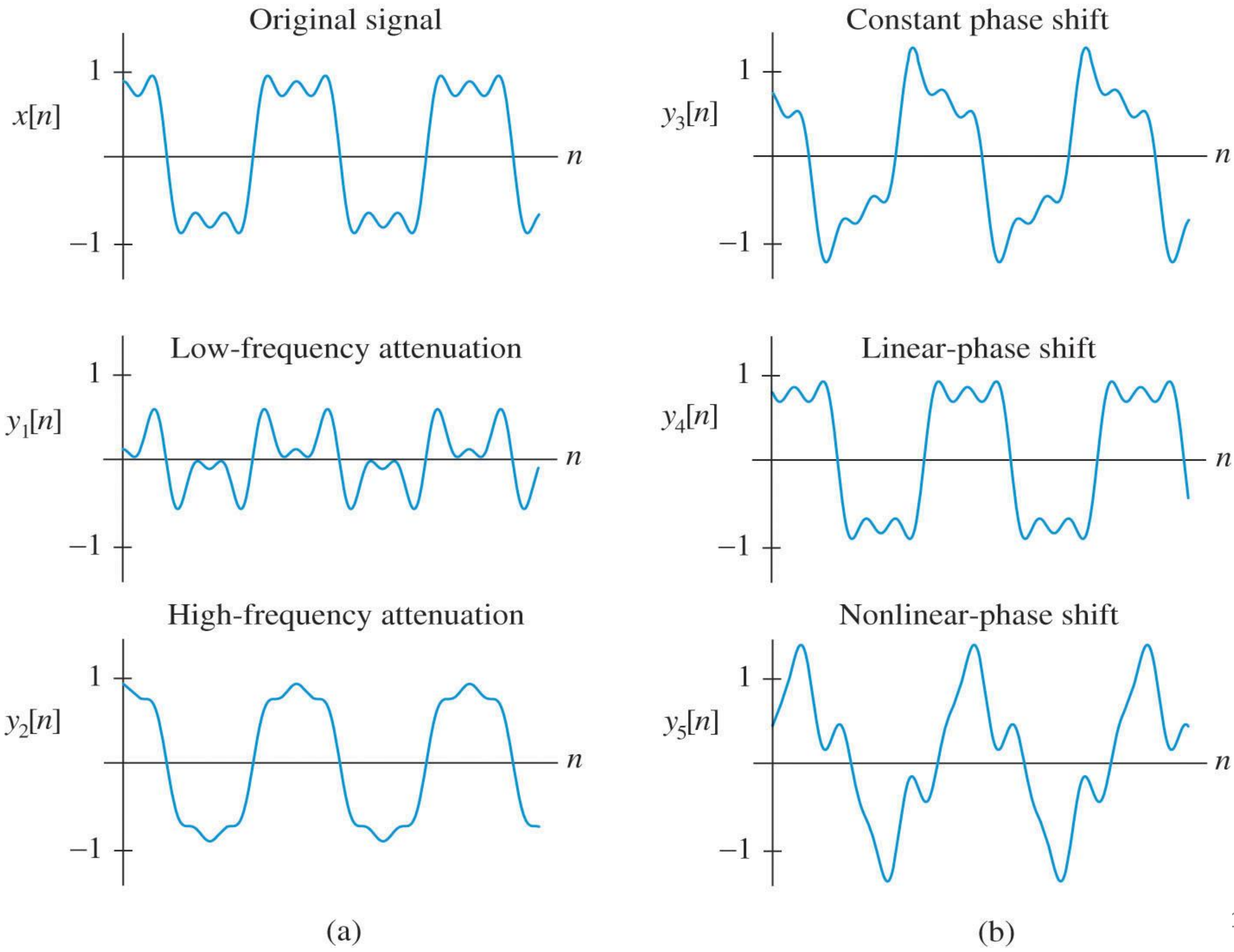
$$x[n] = \cos(\omega_0 n) - \frac{1}{3}\cos(3\omega_0 n) + \frac{1}{5}\cos(5\omega_0 n),$$

$$y_5[n] = \cos(\omega_0 n - \pi/3) - 1/3\cos(3\omega_0 n + \pi/4)$$

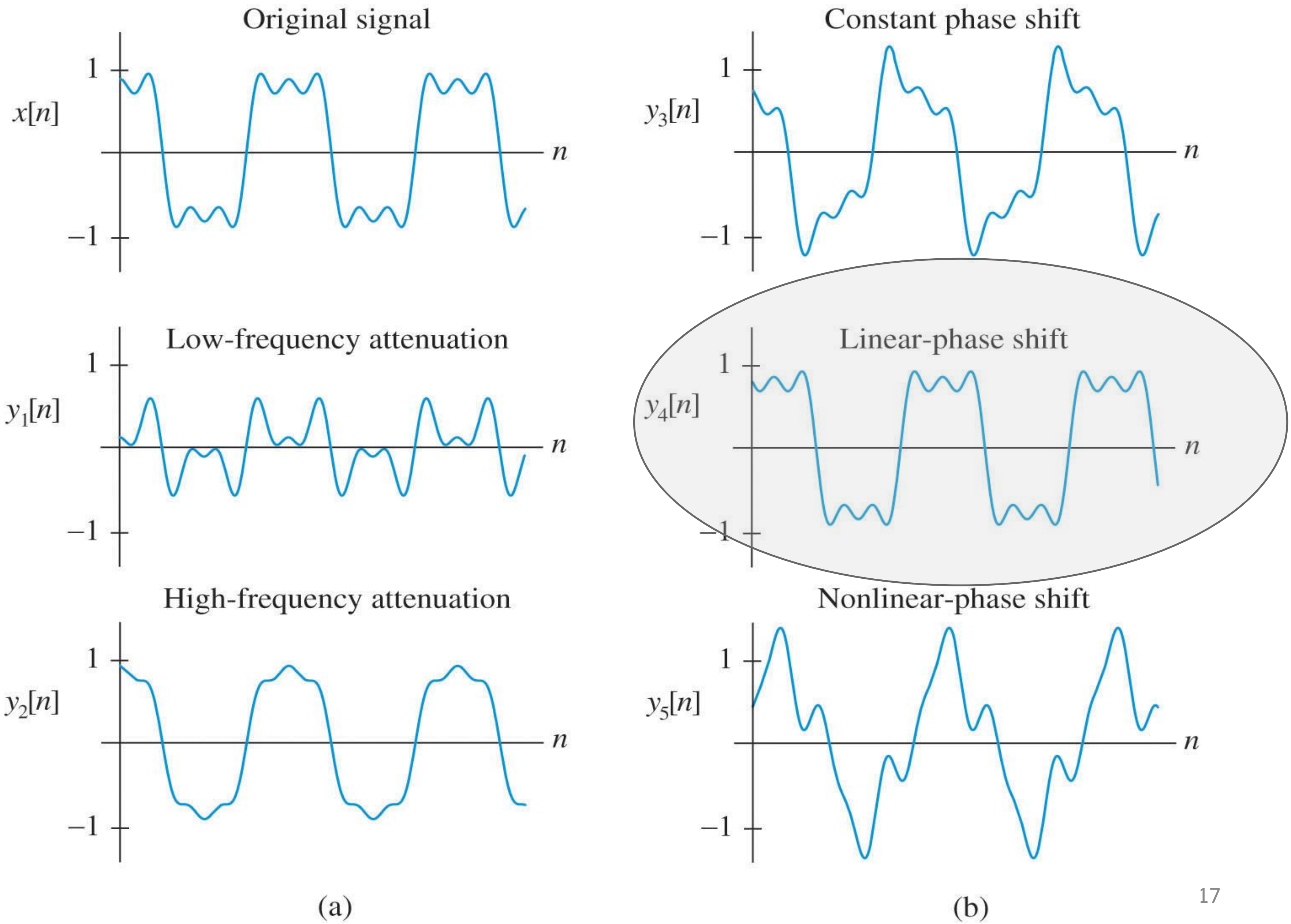
$$+1/5\cos(5\omega_0 n + \pi/7).$$

Nonlinear phase

Example (pp. 216-218)



Example (pp. 216-218)

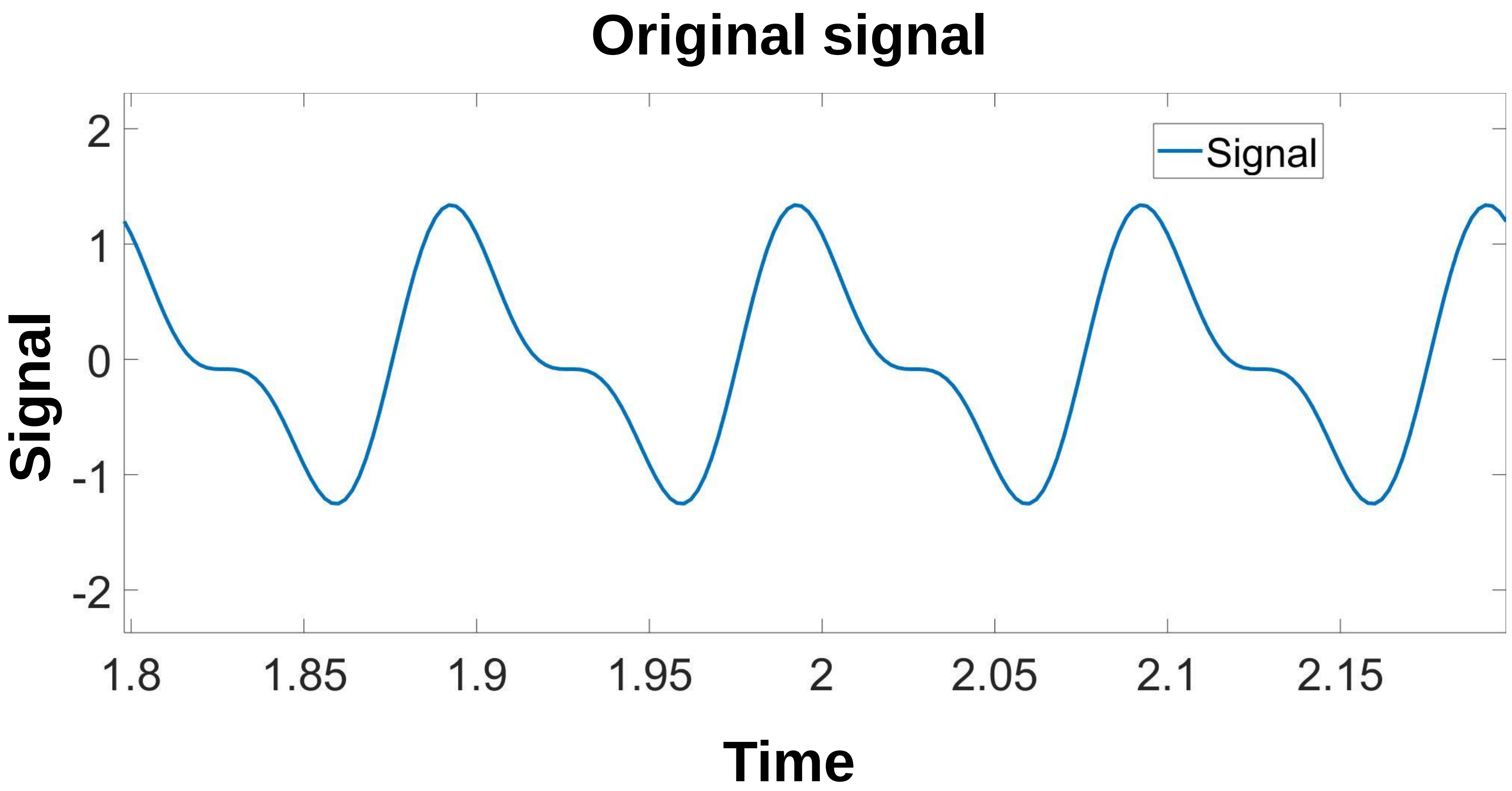


FIR has one main advantage and many disadvantages rather IIR …

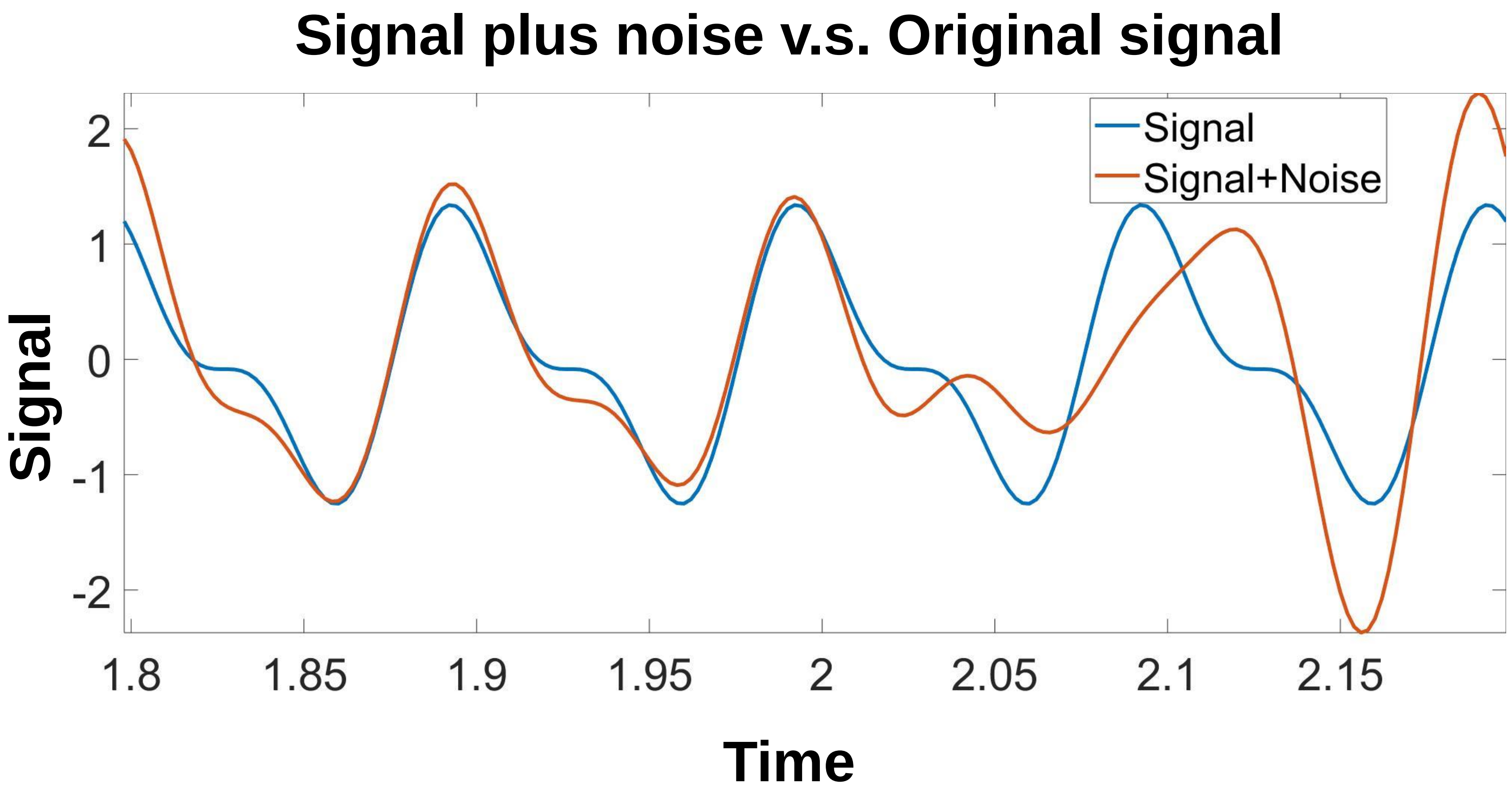
FIR has linear phase response !

FIR filters are the best choice to remove the noises from signal without distortion.

Example

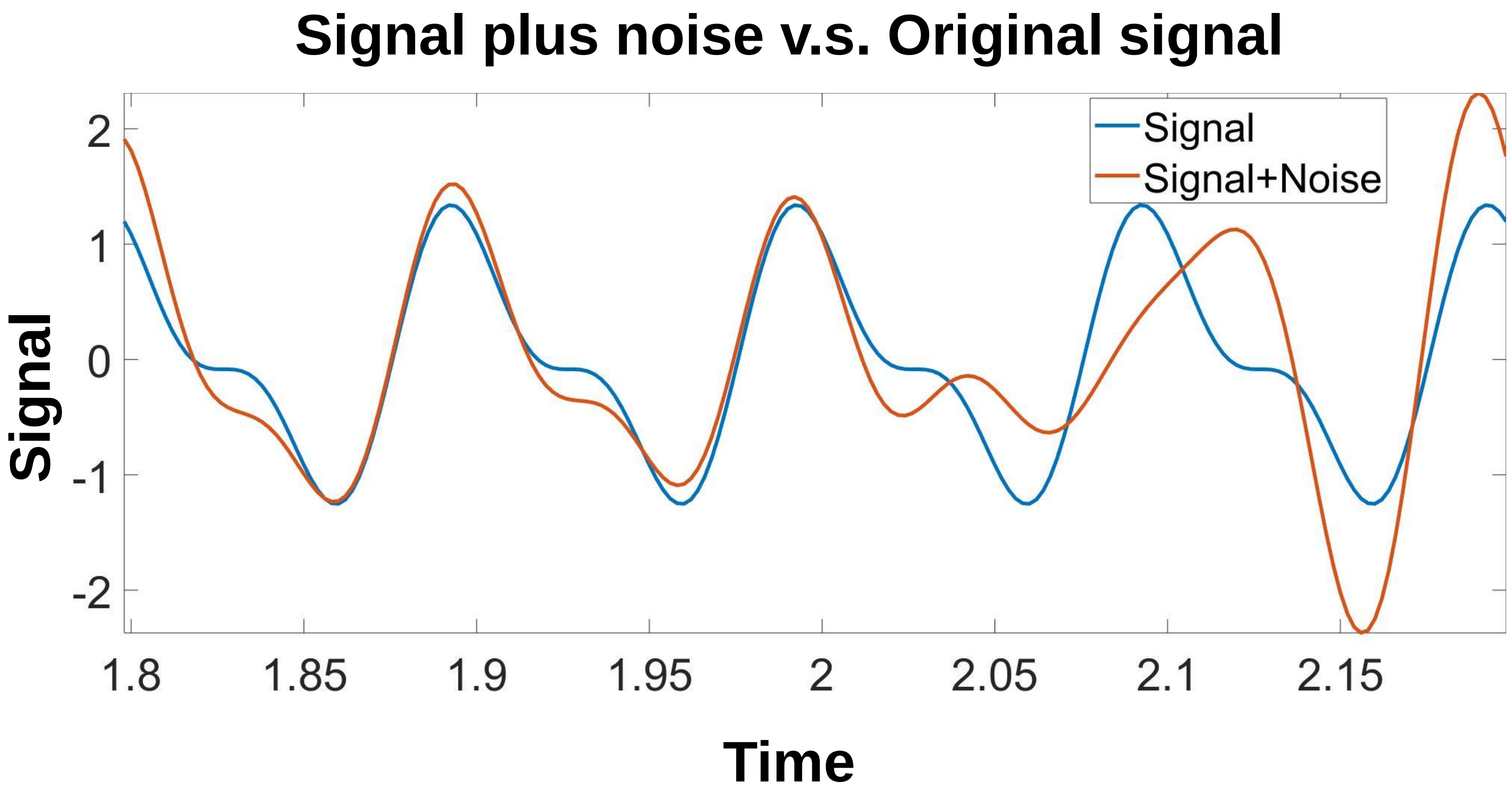


Example



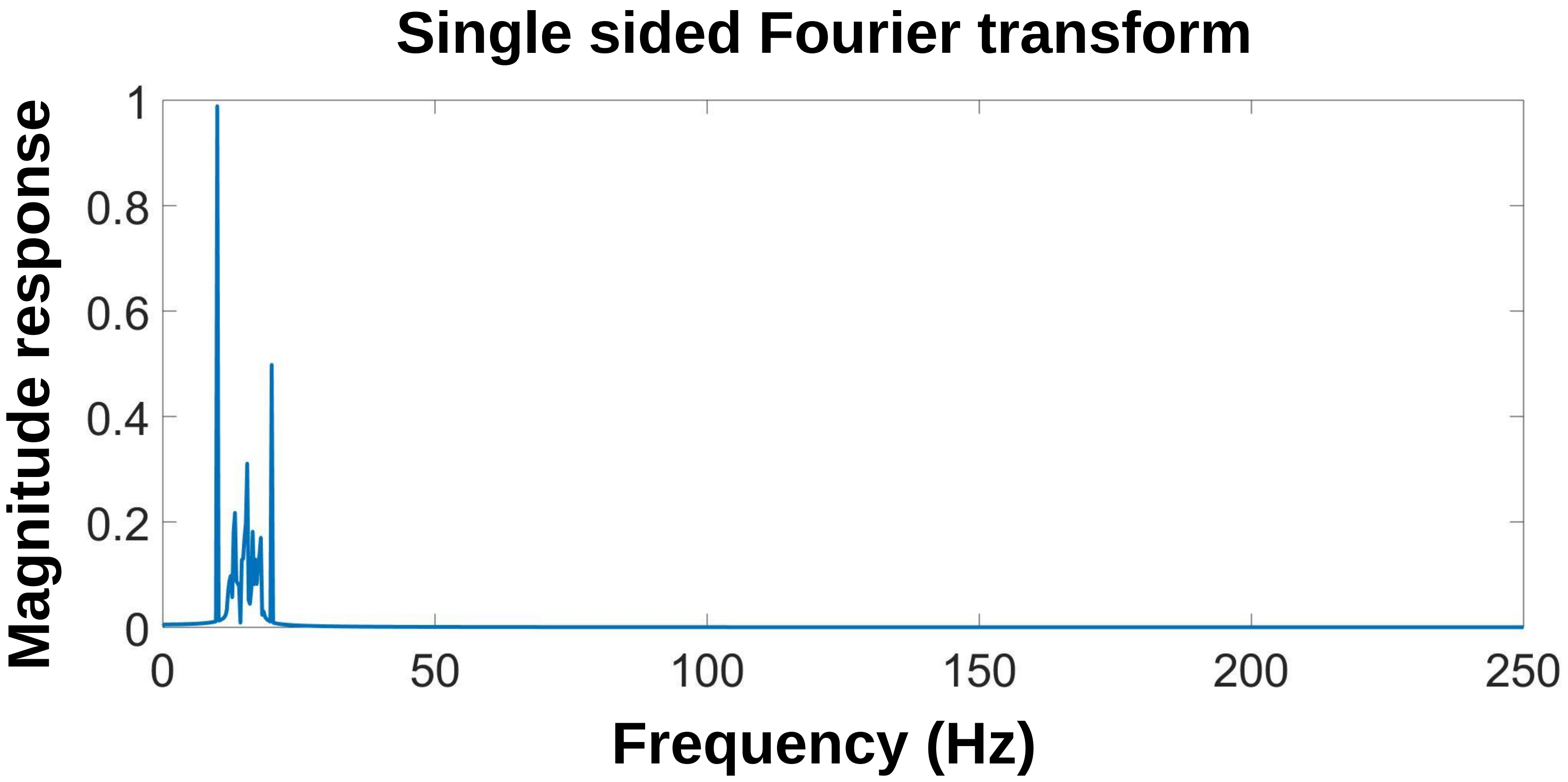
Signal plus noise v.s. Original signal
Signal
Signal+Noise
2
1
0
-1
-2
1.8
1.85
1.9
1.95
2
2.05
2.1
2.15
Time
Signal

Example



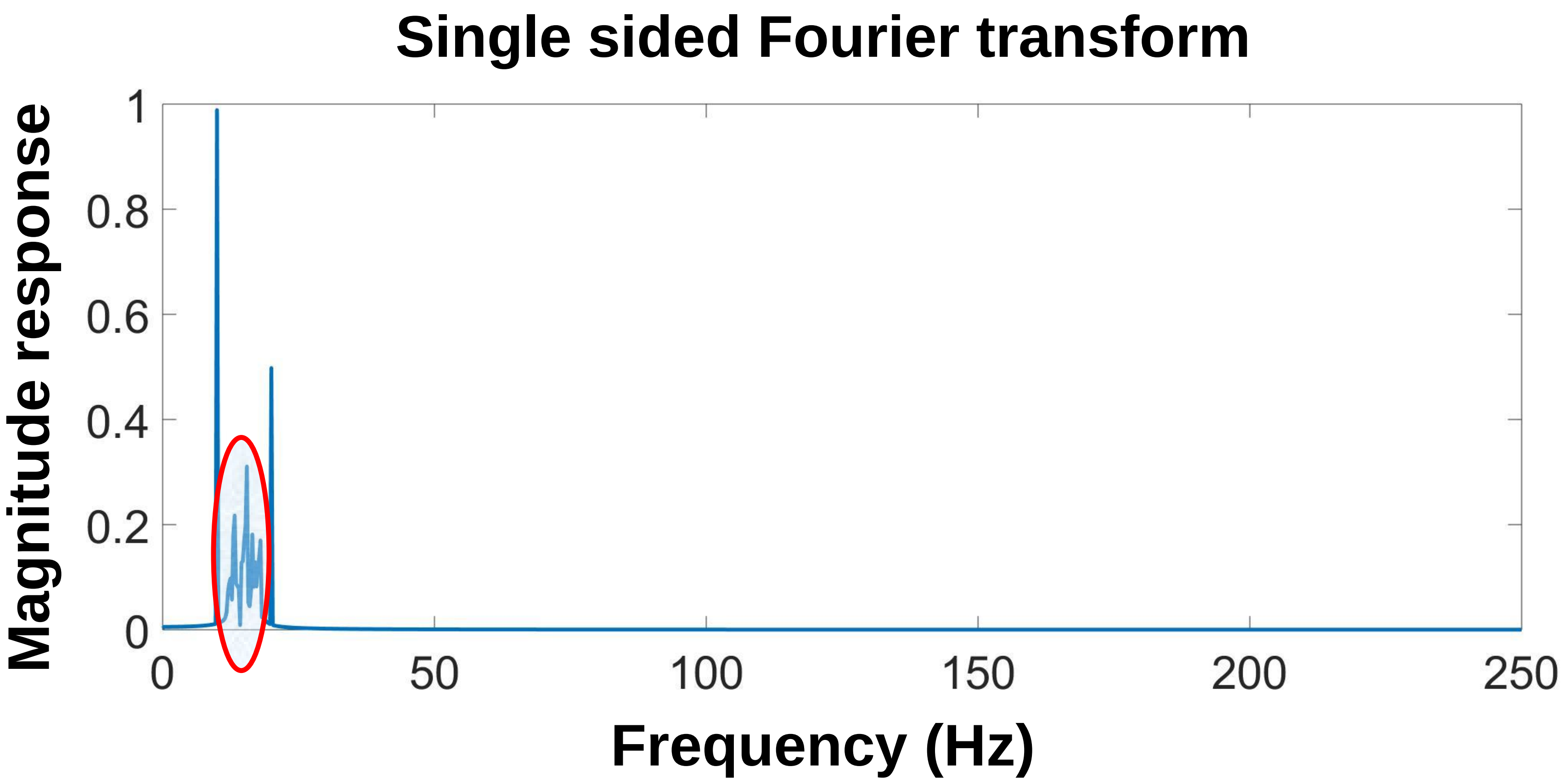
Noise source is known : 12-18 Hz

Example



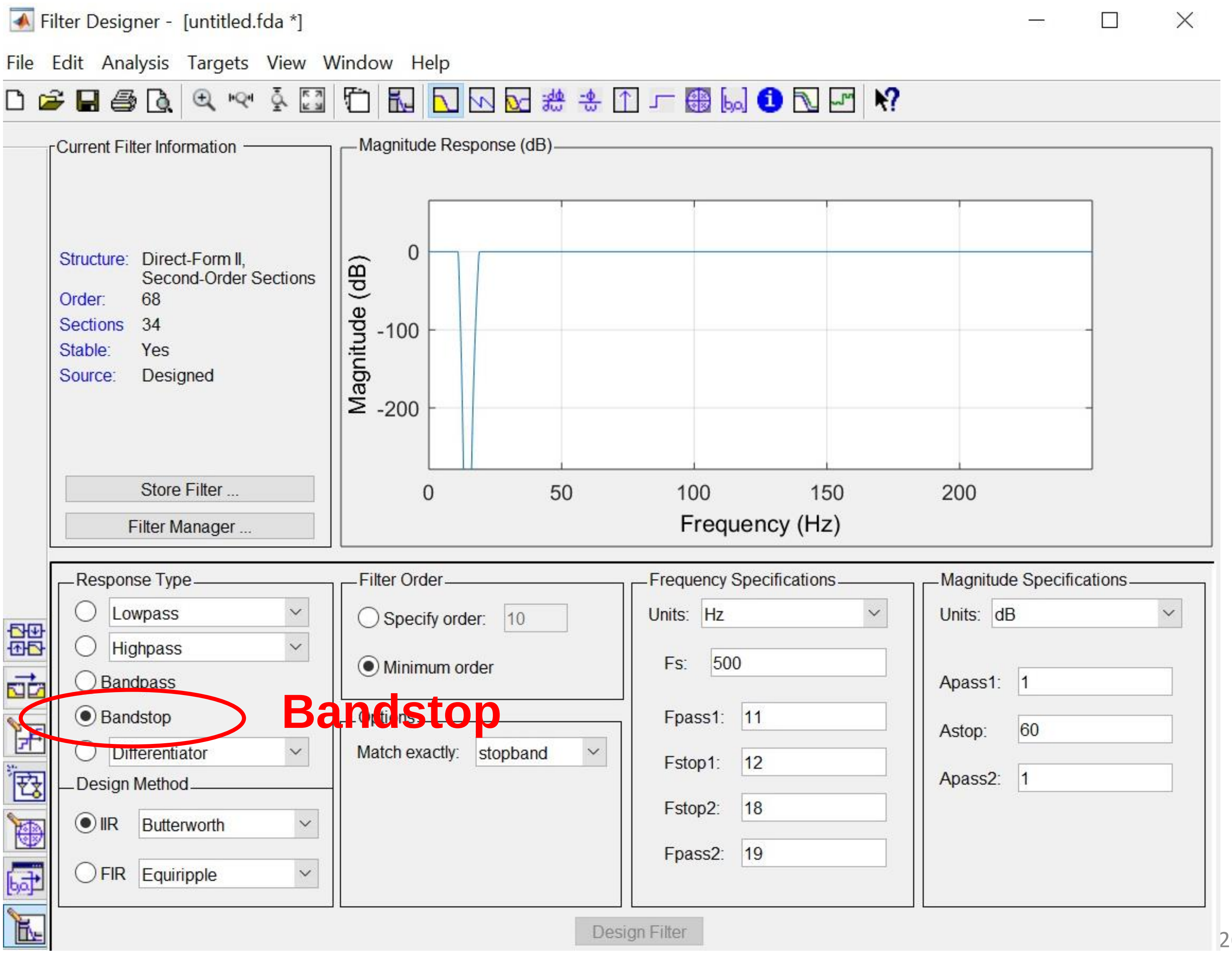
Noise source is known : 12-18 Hz

Example

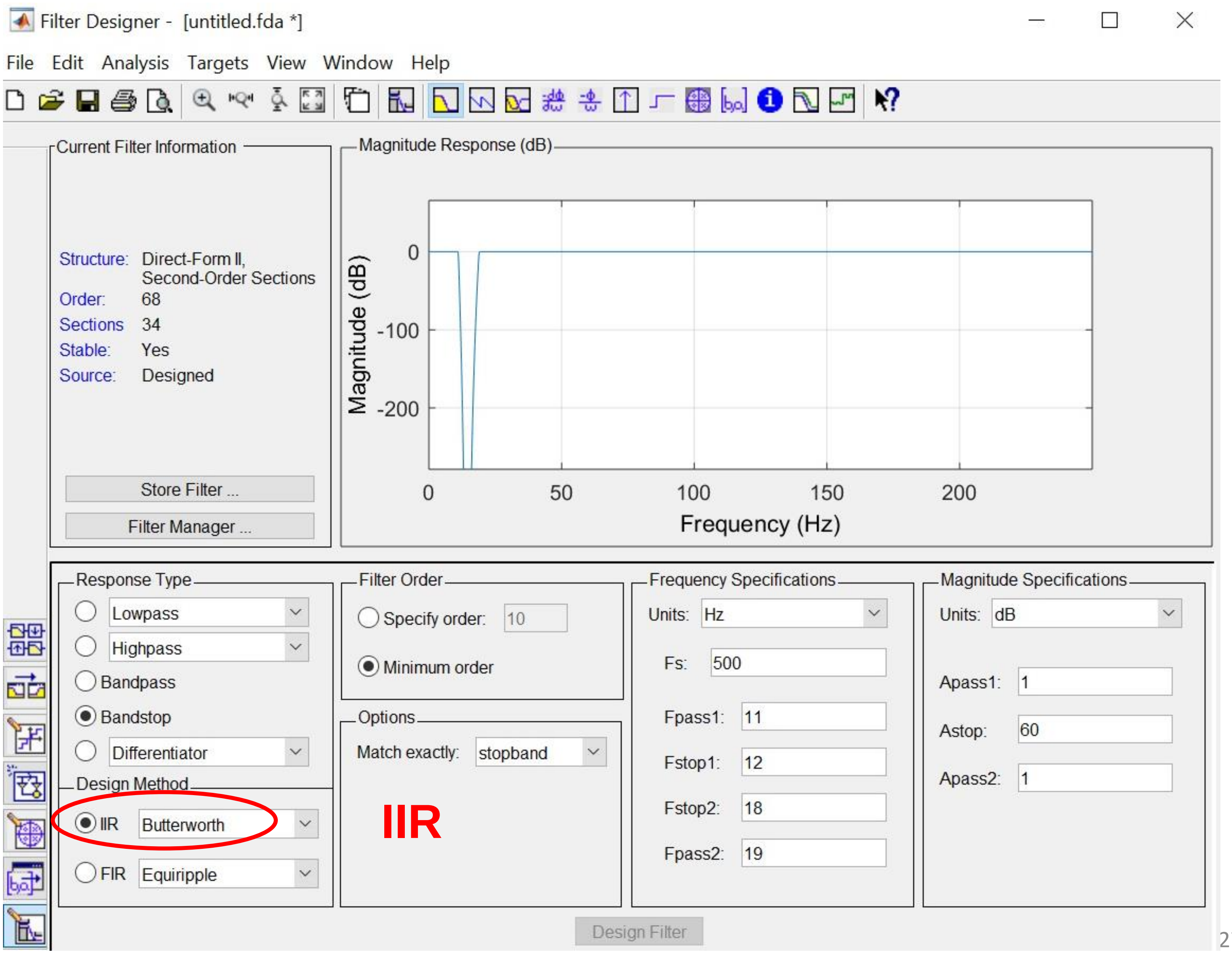


Noise source is known : 12-18 Hz

Example

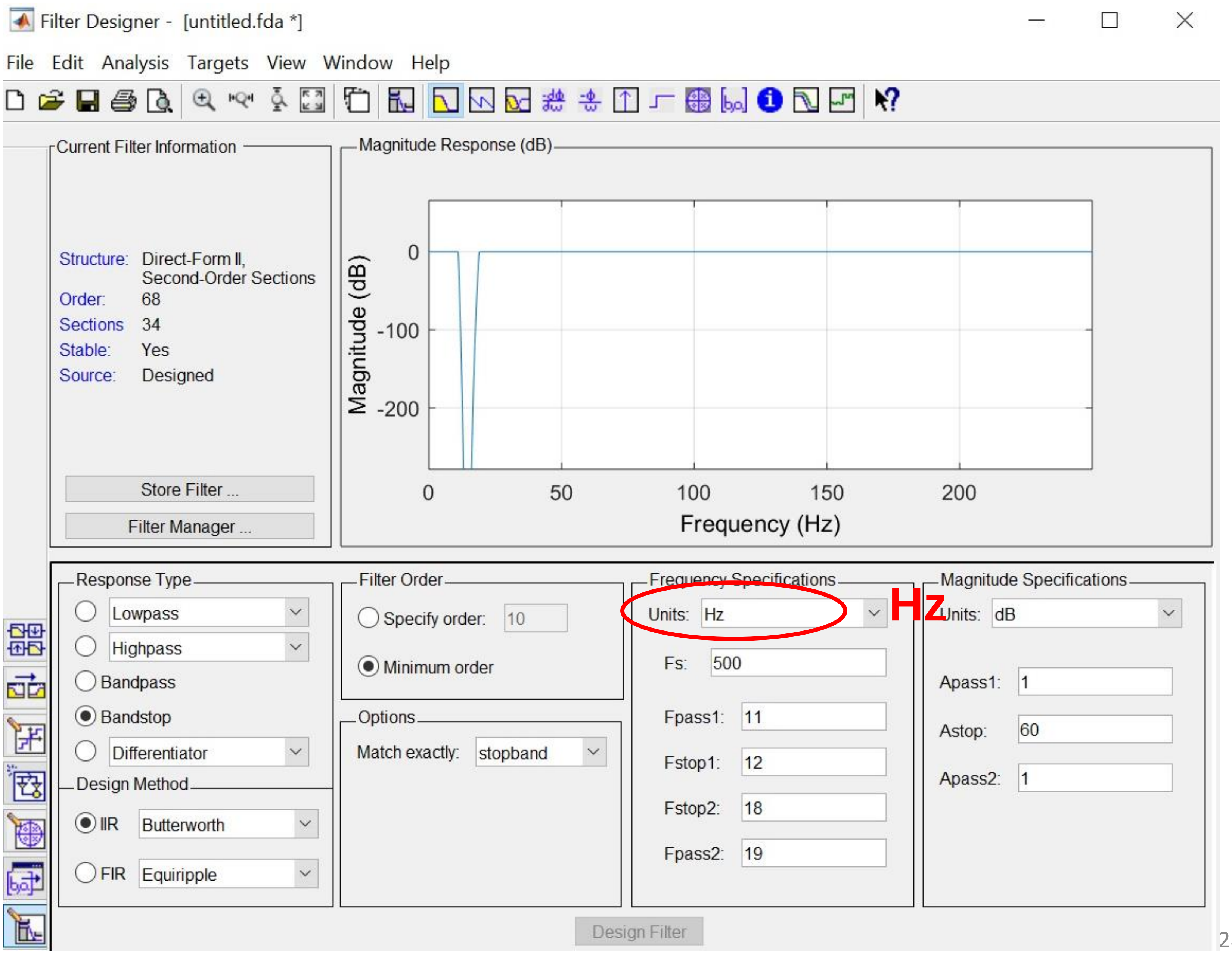

Filter Designer - [untitled.fda *]
File Edit Analysis Targets View Window Help
Current Filter Information
Structure: Direct-Form II, Second-Order Sections
Order: 68
Sections 34
Stable: Yes
Source: Designed
Store Filter ...
Filter Manager ...
Magnitude Response (dB)
Magnitude (dB)
0
-100
-200
0
50
100
150
200
Frequency (Hz)
Response Type
Lowpass
Highpass
Bandpass
Bandstop
Differentiator
Bandstop
Design Method
IIR Butterworth
FIR Equiripple
Filter Order
Specify order: 10
Minimum order
Options
Match exactly: stopband
Frequency Specifications
Units: Hz
Fs: 500
Fpass1: 11
Fstop1: 12
Fstop2: 18
Fpass2: 19
Magnitude Specifications
Units: dB
Apass1: 1
Astop: 60
Apass2: 1
Design Filter

Example



Filter Designer - [untitled.fda *]
File Edit Analysis Targets View Window Help
Current Filter Information
Structure: Direct-Form II, Second-Order Sections
Order: 68
Sections 34
Stable: Yes
Source: Designed
Store Filter ...
Filter Manager ...
Magnitude Response (dB)
0
-100
-200
Magnitude (dB)
0
50
100
150
200
Frequency (Hz)
Response Type
Lowpass
Highpass
Bandpass
Bandstop
Differentiator
Design Method
IIR Butterworth
FIR Equiripple
Filter Order
Specify order: 10
Minimum order
Options
Match exactly: stopband
IIR
Frequency Specifications
Units: Hz
Fs: 500
Fpass1: 11
Fstop1: 12
Fstop2: 18
Fpass2: 19
Magnitude Specifications
Units: dB
Apass1: 1
Astop: 60
Apass2: 1
Design Filter

Example



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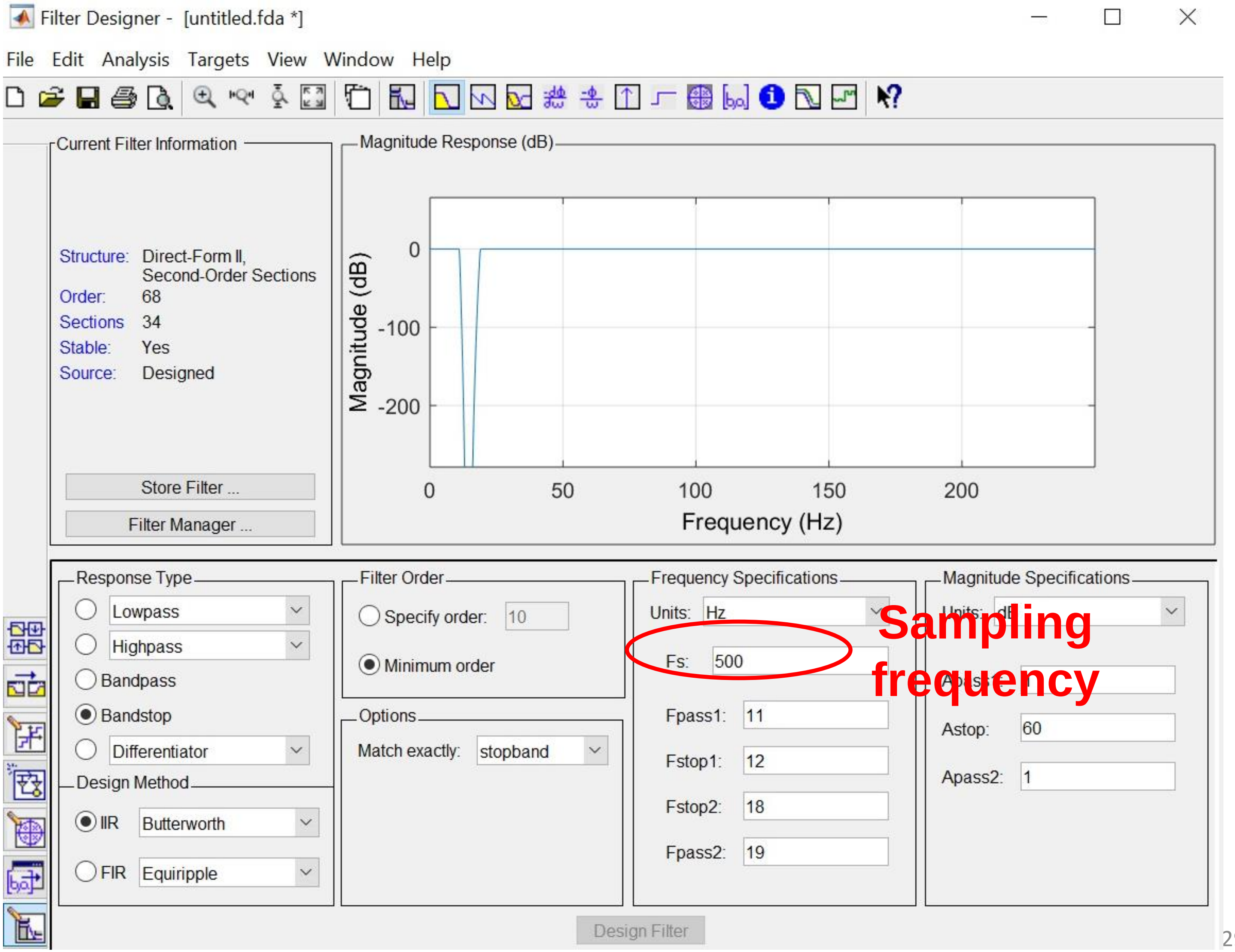
Structure: Direct-Form II, Second-Order Sections
Order: 68
Sections 34
Stable: Yes
Source: Designed
Store Filter ...
Filter Manager ...
Magnitude Response (dB)
Magnitude (dB)
0
-100
-200
0
50
100
150
200
Frequency (Hz)
Response Type
Lowpass
Highpass
Bandpass
Bandstop
Differentiator
Design Method
IIR Butterworth
FIR Equiripple
Filter Order
Specify order: 10
Minimum order
Options
Match exactly: stopband
Frequency Specifications
Units: Hz
Hz
Fs: 500
Fpass1: 11
Fstop1: 12
Fstop2: 18
Fpass2: 19
Magnitude Specifications
Units: dB
Apass1: 1
Astop: 60
Apass2: 1
Design Filter

Example



Filter Designer - [untitled.fda *]
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Current Filter Information
Structure: Direct-Form II, Second-Order Sections
Order: 68
Sections 34
Stable: Yes
Source: Designed
Store Filter ...
Filter Manager ...
Magnitude Response (dB)
Magnitude (dB)
0
-100
-200
0
50
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200
Frequency (Hz)
Response Type
Lowpass
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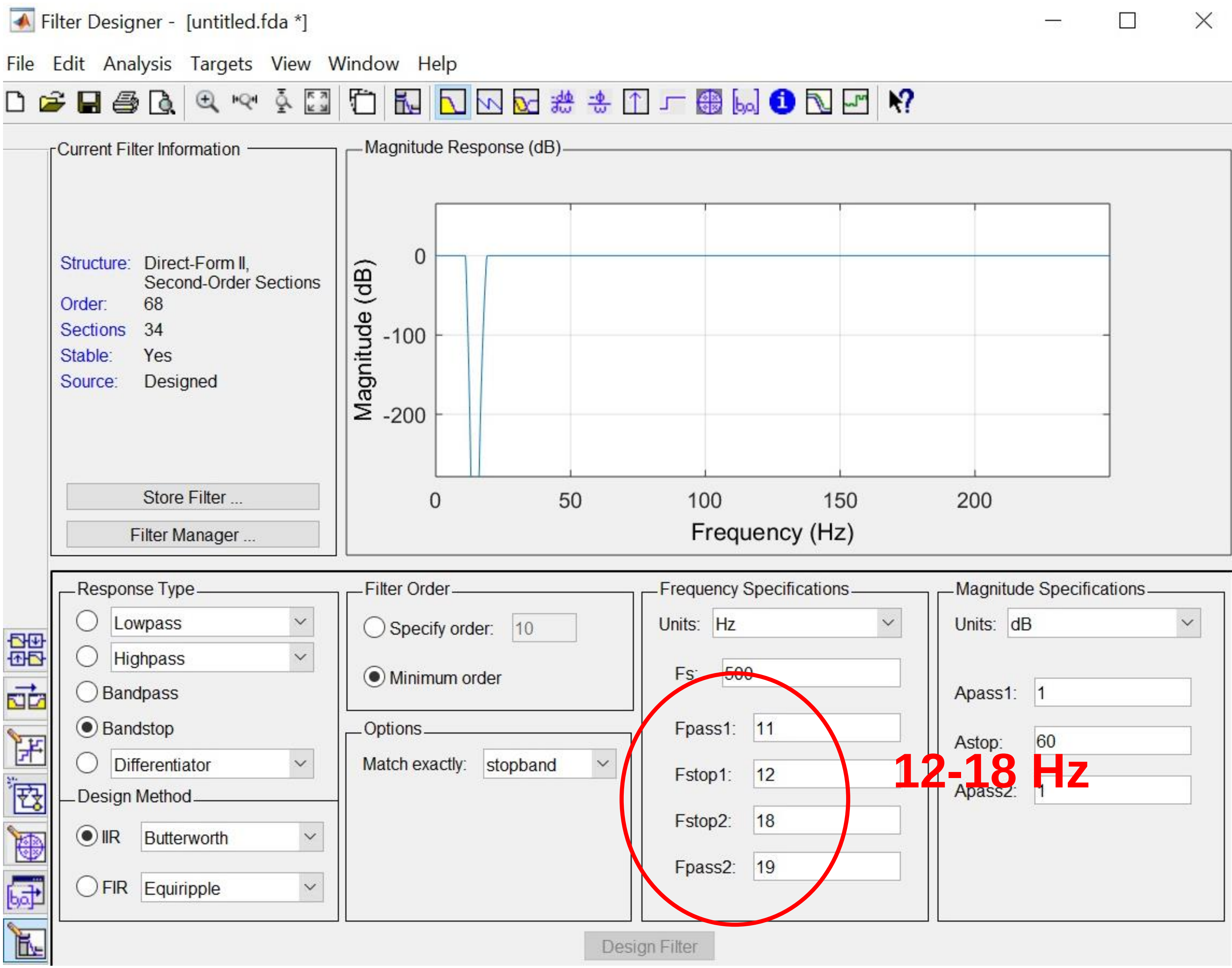
Bandstop
Differentiator
Design Method
IIR Butterworth
FIR Equiripple
Filter Order
Specify order: 10
Minimum order
Options
Match exactly: stopband
Frequency Specifications
Units: Hz
Fs: 500
Fpass1: 11
Fstop1: 12
Fstop2: 18
Fpass2: 19
Magnitude Specifications
Astop: 60
Apass2: 1
Design Filter
Sampling frequency

Example



Filter Designer - [untitled.fda *]
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Structure: Direct-Form II, Second-Order Sections
Order: 68
Sections 34
Stable: Yes
Source: Designed
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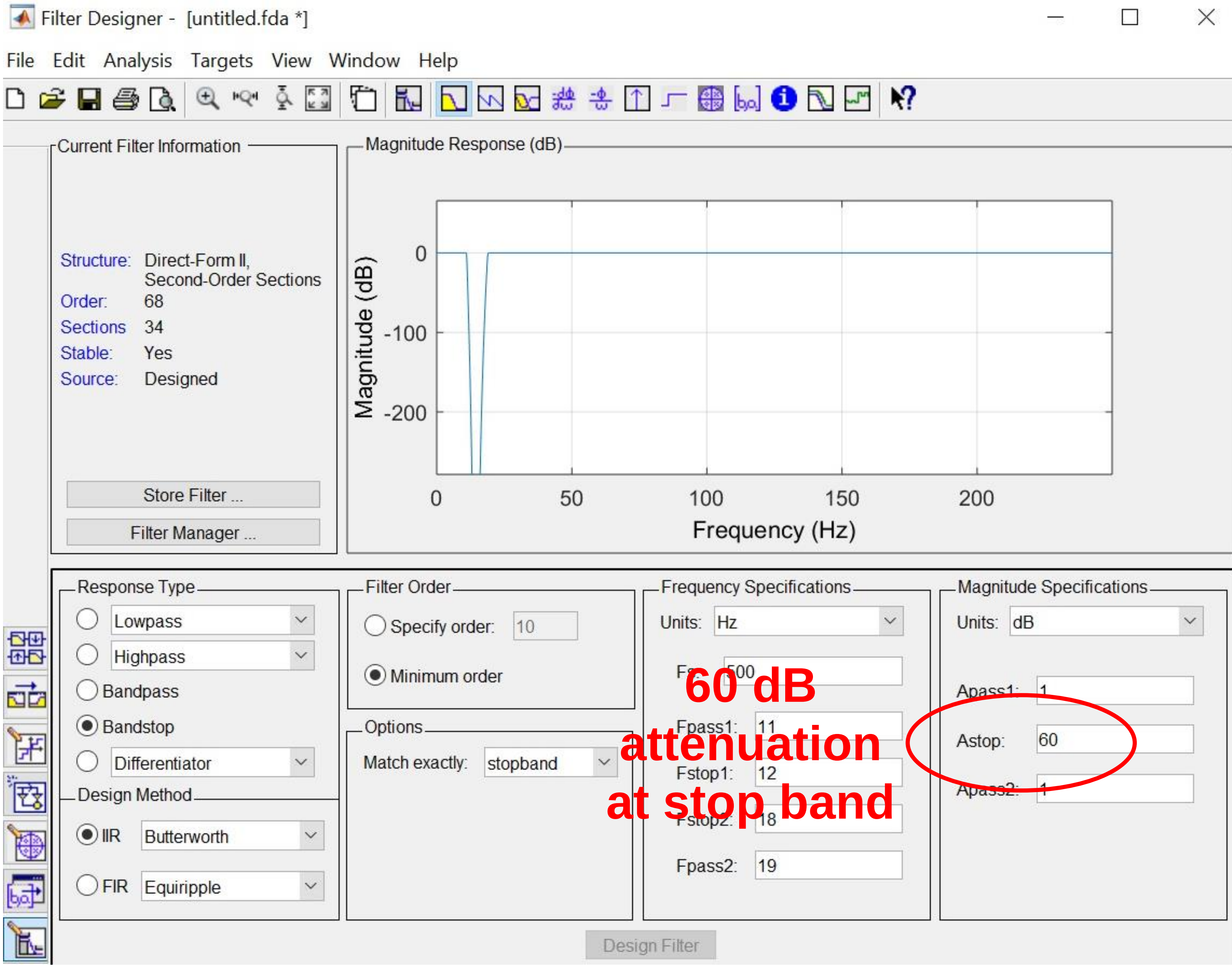
Filter Manager ...
Magnitude Response (dB)
Magnitude (dB)
0
-100
-200
0
50
100
150
200
Frequency (Hz)
Response Type
Lowpass
Highpass
Bandpass
Bandstop
Differentiator
Design Method
IIR
Butterworth
FIR
Equiripple
Filter Order
Specify order: 10
Minimum order
Options
Match exactly: stopband
Frequency Specifications
Units: Hz
Fs: 500
Fpass1: 11
Fstop1: 12
Fstop2: 18
Fpass2: 19
Magnitude Specifications
Units: dB
Apass1: 1
Astop: 60
Apass2: 1
12-18 Hz
Design Filter

Example



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Stable: Yes
Source: Designed
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Magnitude Response (dB)
Magnitude (dB)
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-100
-200
0
50
100
150
200
Frequency (Hz)
Response Type
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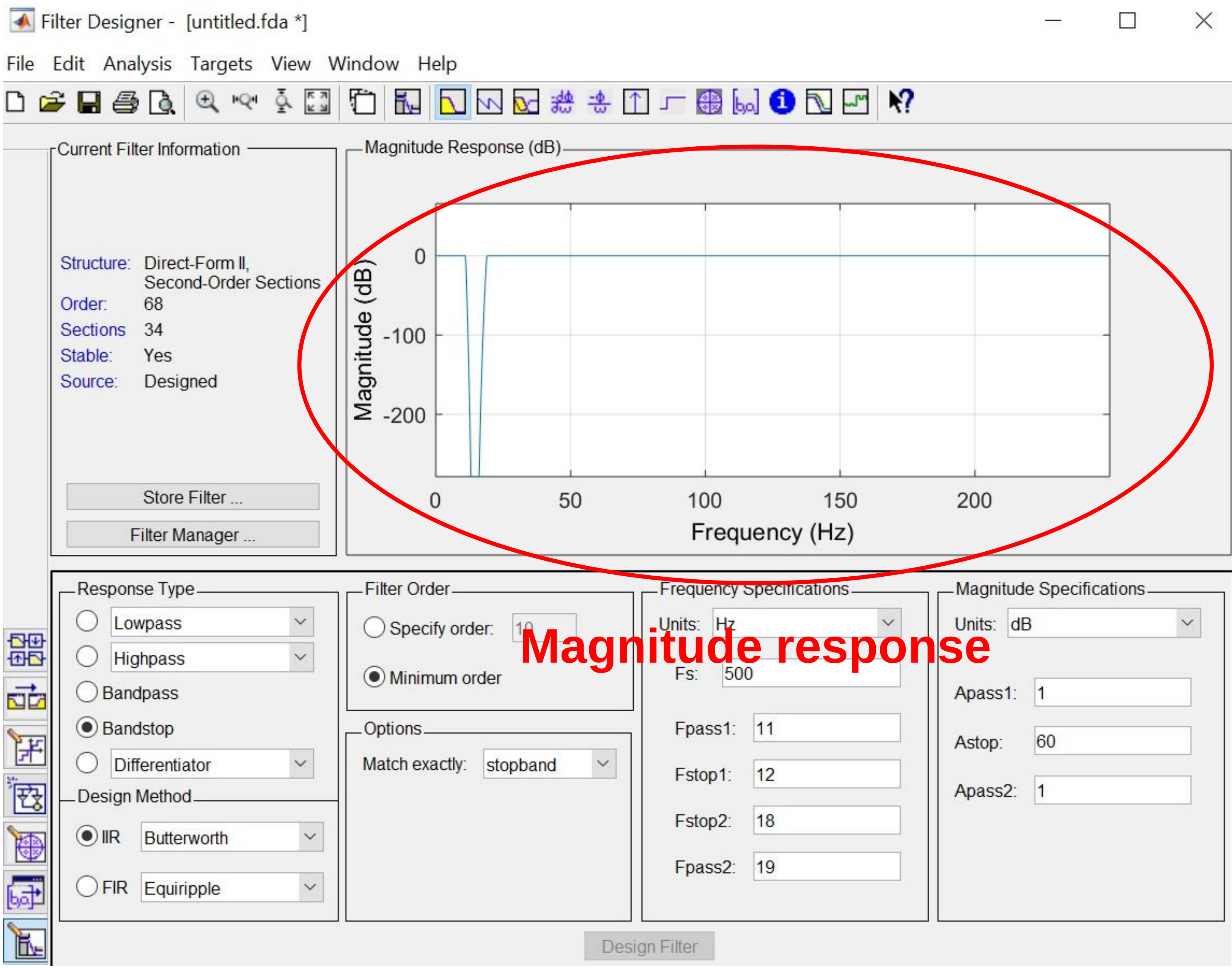
Highpass
Bandpass
Bandstop
Differentiator
Design Method
IIR
Butterworth
FIR
Equiripple
Filter Order
Specify order: 10
Minimum order
Options
Match exactly: stopband
Frequency Specifications
Units: Hz
Fs: 500
Fpass1: 11
Fstop1: 12
Fstop2: 18
Fpass2: 19
Magnitude Specifications
Units: dB
Apass1: 1
Astop: 60
Apass2: 1
Design Filter
60 dB attenuation at stop band

Example



Filter Designer - [untitled.fda *]
File Edit Analysis Targets View Window Help
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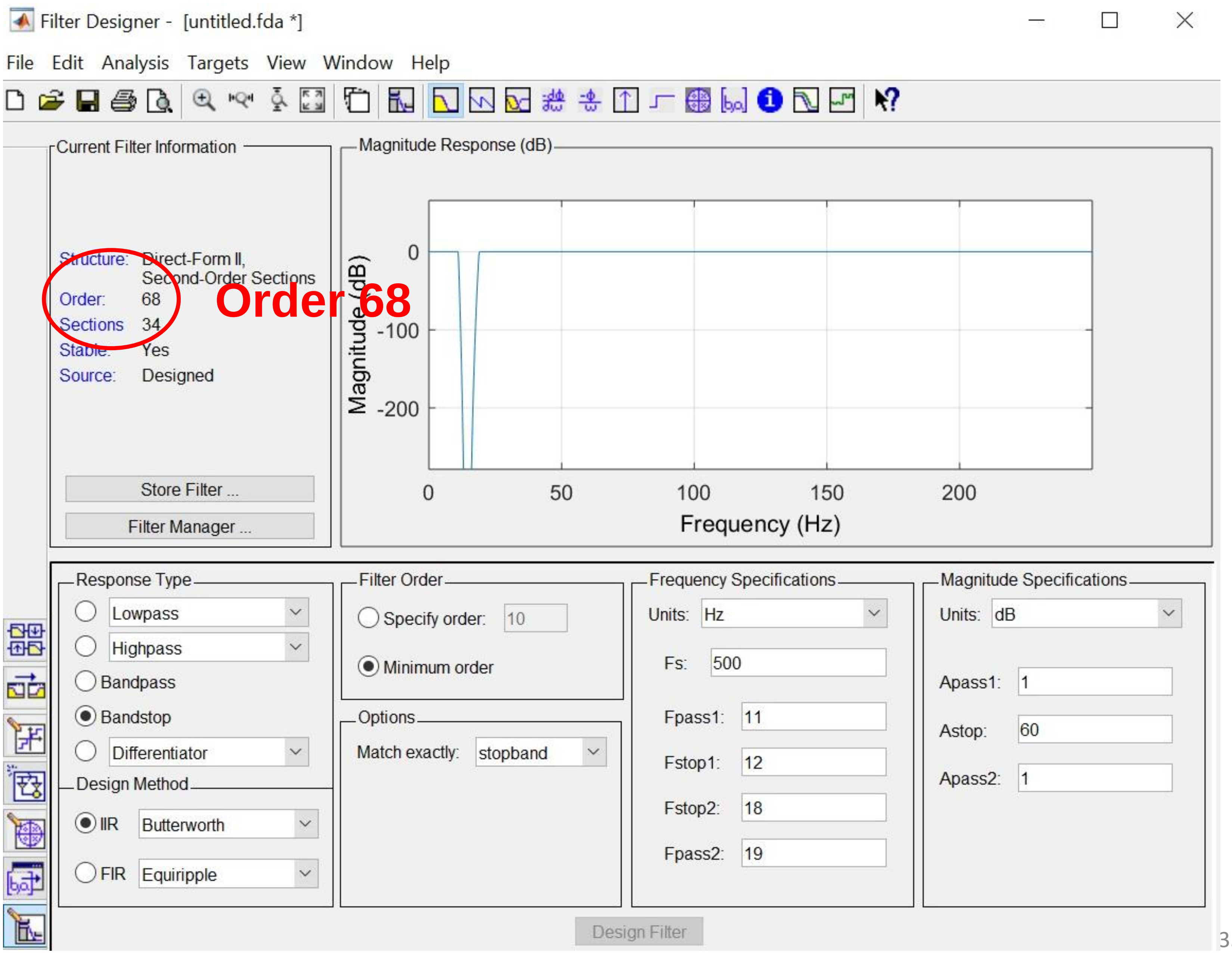
Structure: Direct-Form II, Second-Order Sections
Order: 68
Sections 34
Stable: Yes
Source: Designed
Store Filter ...
Filter Manager ...
Magnitude Response (dB)
Magnitude (dB)
0
-100
-200
0
50
100
150
200
Frequency (Hz)
Response Type
Lowpass
Highpass
Bandpass
Bandstop
Differentiator
Design Method
IIR
Butterworth
FIR
Equiripple
Filter Order
Specify order:
Minimum order
Options
Match exactly:
stopband
Frequency Specifications
Units:
Hz
Fs:
500
Fpass1:
11
Fstop1:
12
Fstop2:
18
Fpass2:
19
Magnitude Specifications
Units:
dB
Apass1:
1
Astop:
60
Apass2:
1
Design Filter
Magnitude response

Example



Filter Designer - [untitled.fda *]
File Edit Analysis Targets View Window Help
Current Filter Information
Structure: Direct-Form II, Second-Order Sections
Order: 68
Sections 34
Stable: Yes
Source: Designed
Order 68
Store Filter ...
Filter Manager ...
Magnitude Response (dB)
Magnitude (dB)
0
-100
-200
0
50
100
150
200
Frequency (Hz)
Response Type
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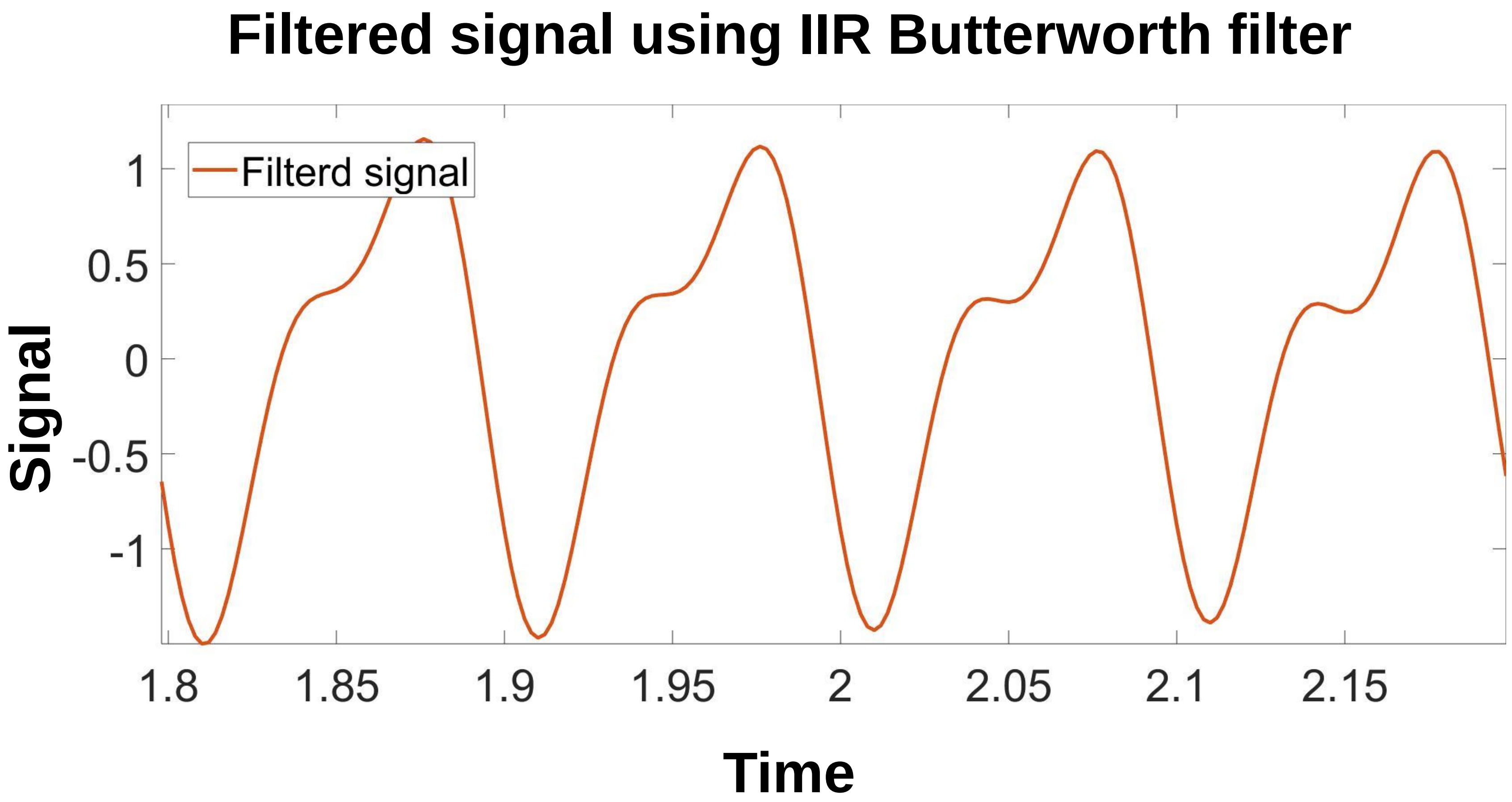
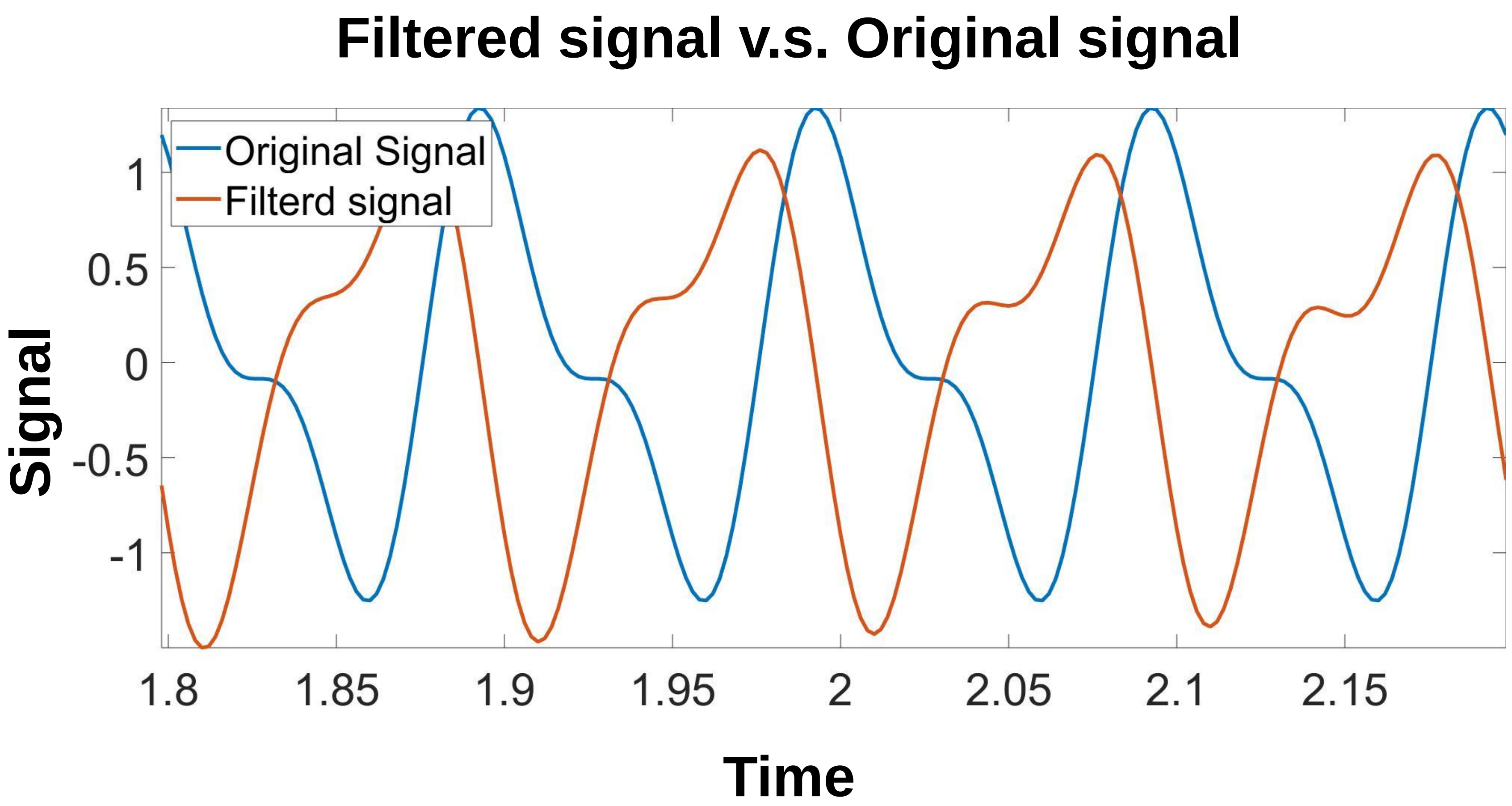
Highpass
Bandpass
Bandstop
Differentiator
Design Method
IIR
Butterworth
FIR
Equiripple
Filter Order
Specify order: 10
Minimum order
Options
Match exactly: stopband
Frequency Specifications
Units: Hz
Fs: 500
Fpass1: 11
Fstop1: 12
Fstop2: 18
Fpass2: 19
Magnitude Specifications
Units: dB
Apass1: 1
Astop: 60
Apass2: 1
Design Filter

Example



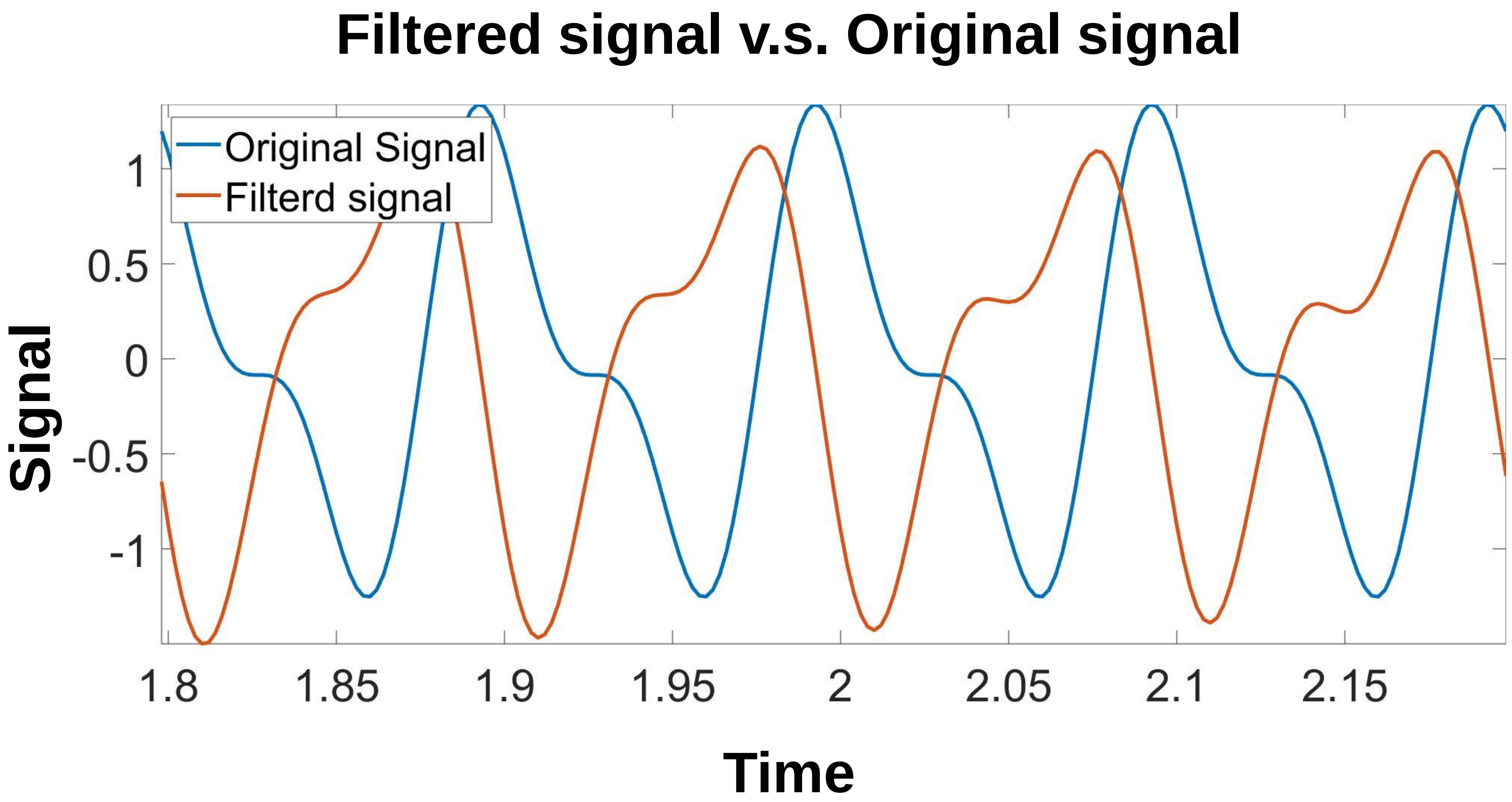
Filtered signal using IIR Butterworth filter
Filterd signal
1
0.5
0
-0.5
-1
Signal
1.8
1.85
1.9
1.95
2
2.05
2.1
2.15
Time

Example



Filtered signal v.s. Original signal
Original Signal
Filterd signal
1
0.5
0
-0.5
-1
Signal
1.8
1.85
1.9
1.95
2
2.05
2.1
2.15
Time

Example



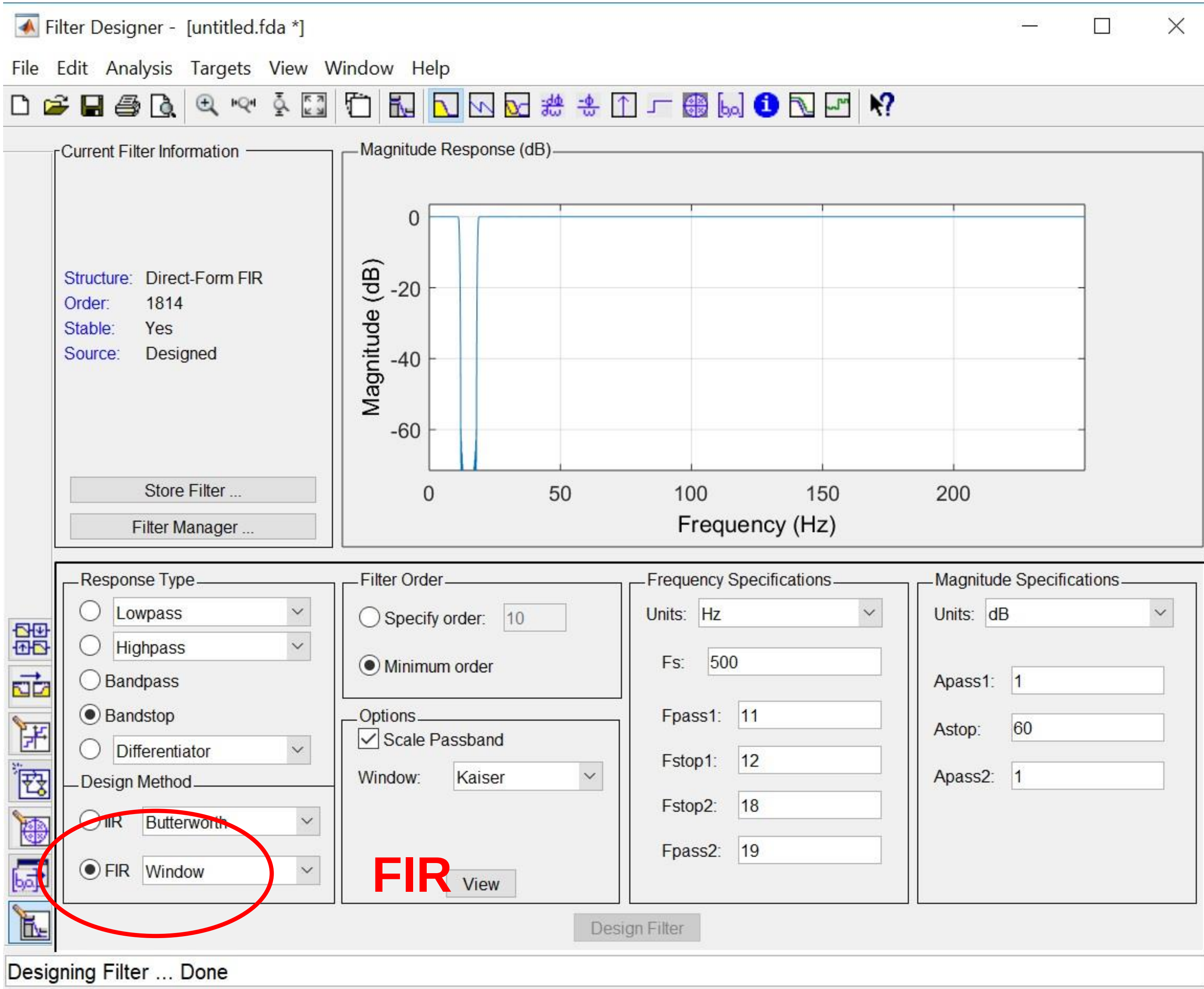
IIR filters have nonlinear phase response => Distortion

Persevering the shape of the signals not important in most of the applications

…

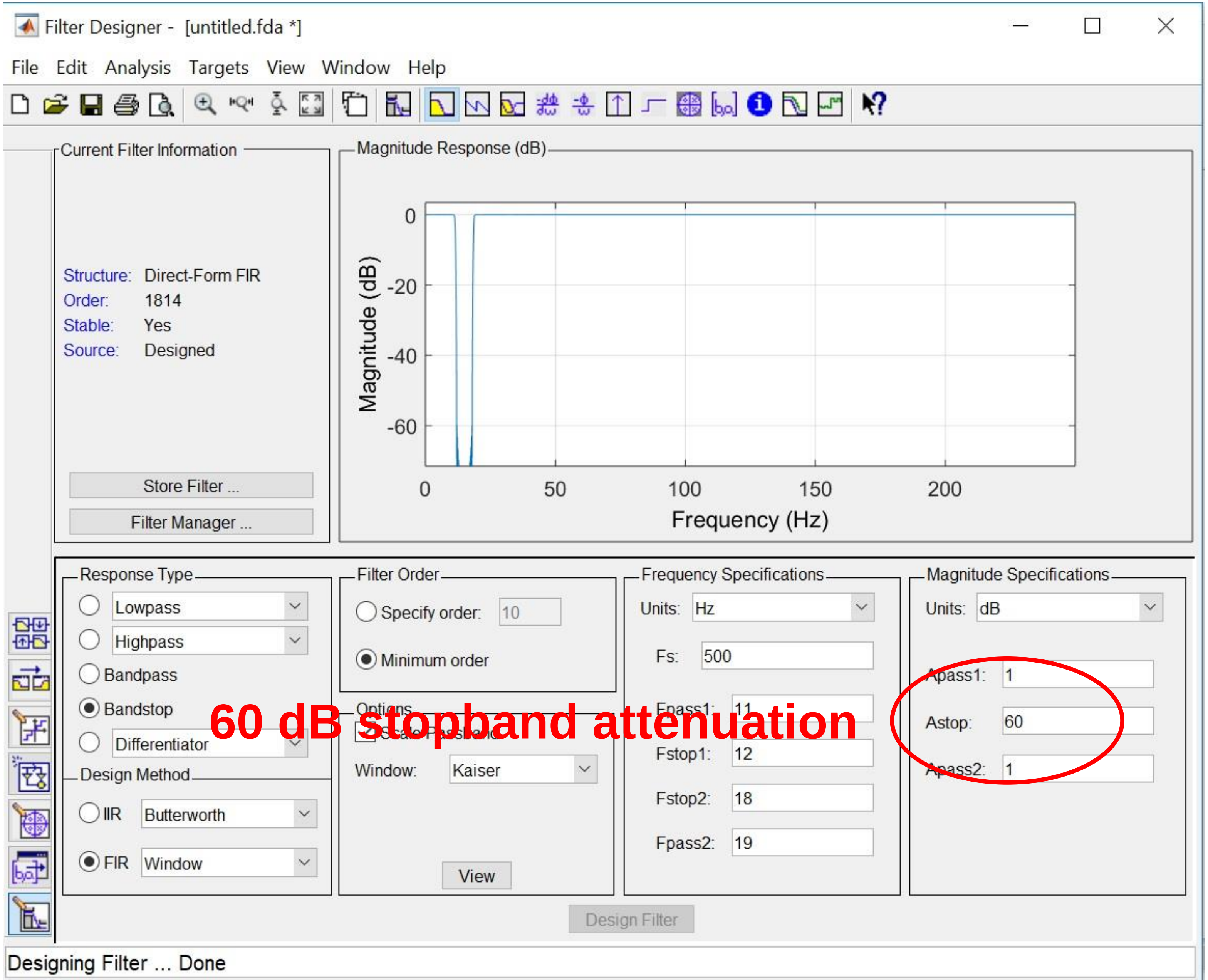
For example in audio applications, because human hearing system is not sensitive to distortion.

Example



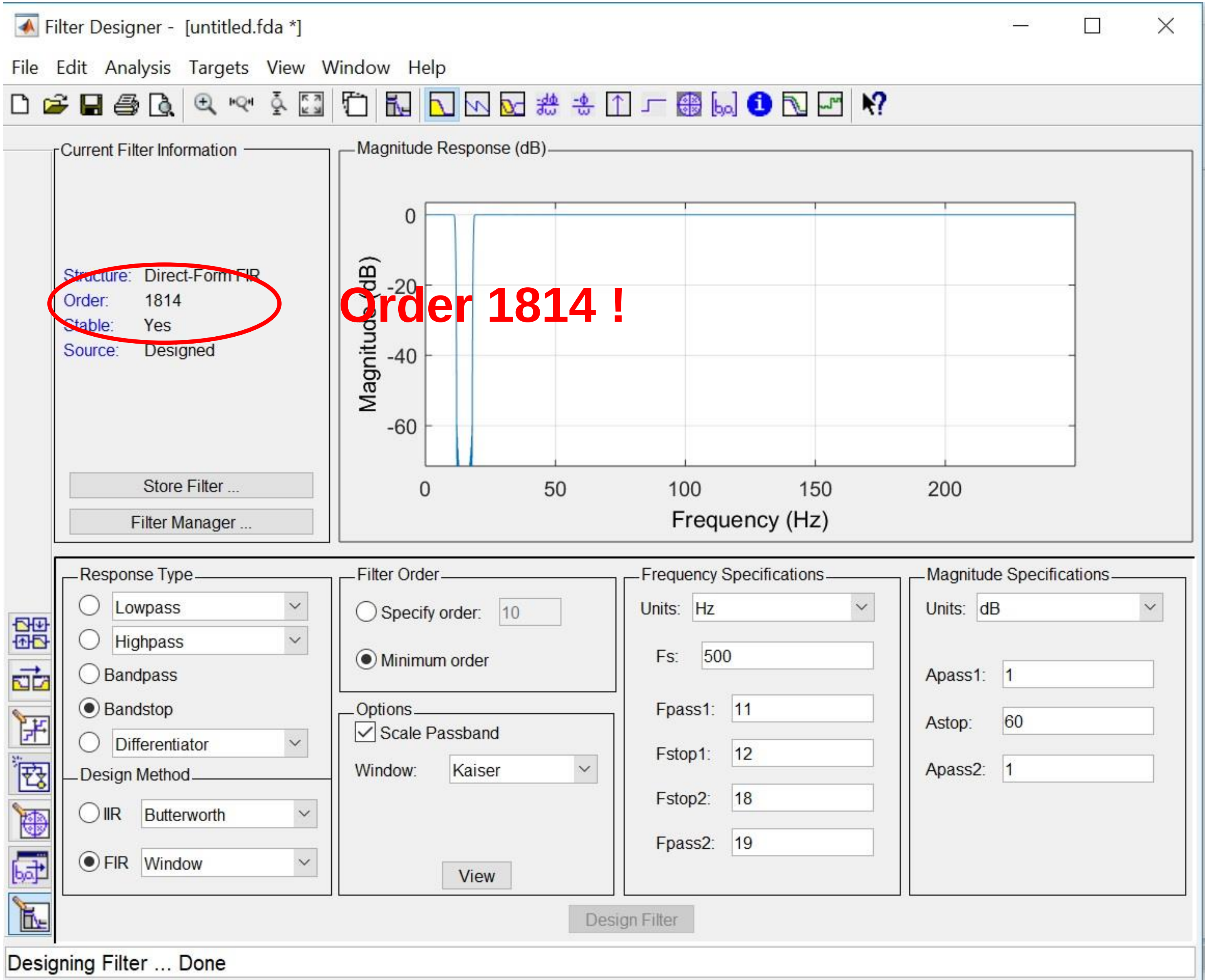
Filter Designer - [untitled.fda *]
File Edit Analysis Targets View Window Help
Current Filter Information
Structure: Direct-Form FIR
Order: 1814
Stable: Yes
Source: Designed
Store Filter ...
Filter Manager ...
Magnitude Response (dB)
0
-20
-40
-60
Magnitude (dB)
0 50 100 150 200
Frequency (Hz)
Response Type
Lowpass
Highpass
Bandpass
Bandstop
Differentiator
Design Method
IIR Butterworth
FIR Window
Filter Order
Specify order: 10
Minimum order
Options
Scale Passband
Window: Kaiser
FIR
View
Frequency Specifications
Units: Hz
Fs: 500
Fpass1: 11
Fstop1: 12
Fstop2: 18
Fpass2: 19
Magnitude Specifications
Units: dB
Apass1: 1
Astop: 60
Apass2: 1
Design Filter
Designing Filter ... Done

Example



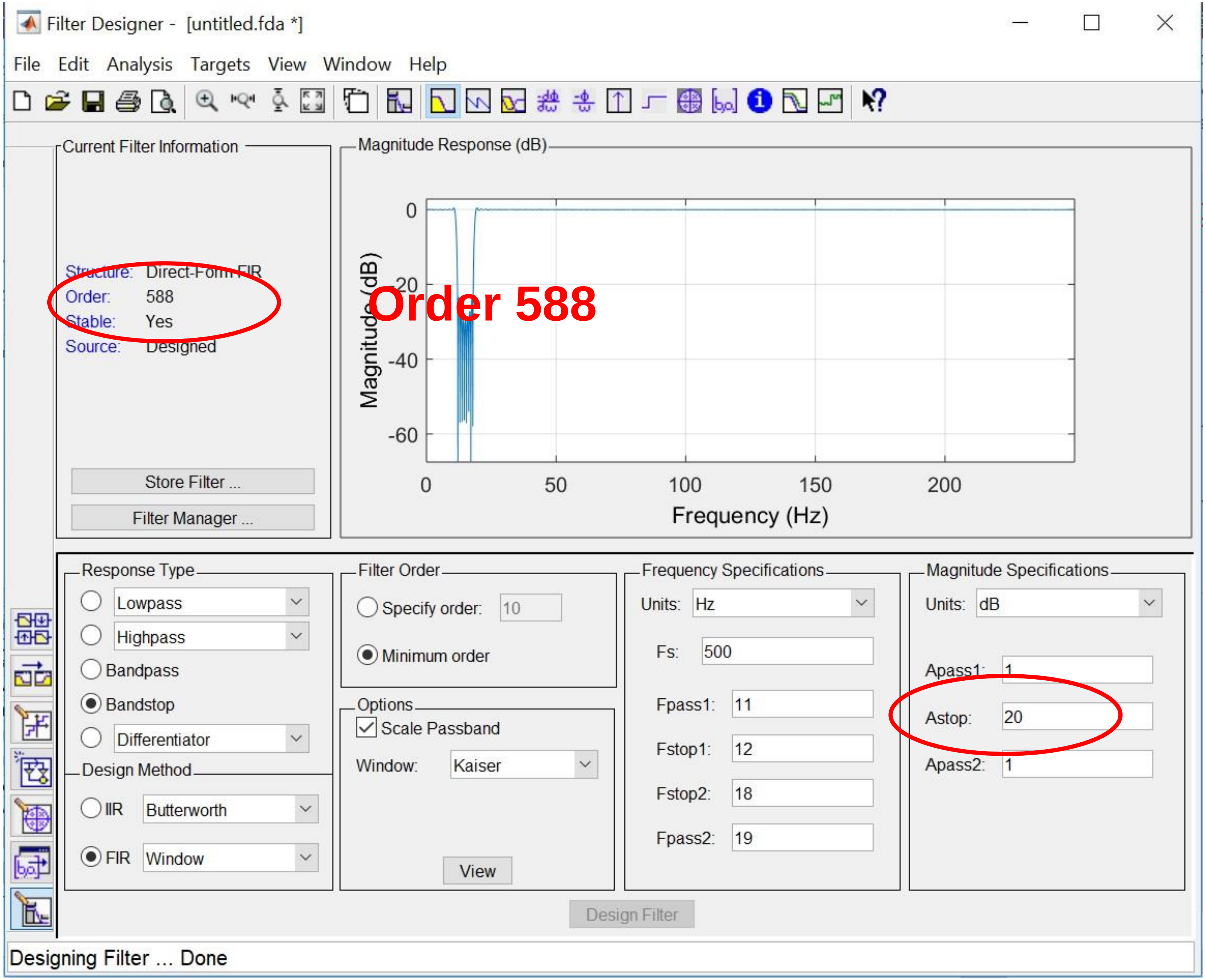
Filter Designer - [untitled.fda *]
File Edit Analysis Targets View Window Help
Current Filter Information
Structure: Direct-Form FIR
Order: 1814
Stable: Yes
Source: Designed
Store Filter ...
Filter Manager ...
Magnitude Response (dB)
Magnitude (dB)
0
-20
-40
-60
0
50
100
150
200
Frequency (Hz)
Response Type
Lowpass
Highpass
Bandpass
Bandstop
Differentiator
Design Method
IIR
Butterworth
FIR
Window
Filter Order
Specify order:
10
Minimum order
Options
Window:
Kaiser
View
Frequency Specifications
Units:
Hz
Fs:
500
Fpass1:
11
Fstop1:
12
Fstop2:
18
Fpass2:
19
Magnitude Specifications
Units:
dB
Apass1:
1
Astop:
60
Apass2:
1
Design Filter
Designing Filter ... Done
60 dB stopband attenuation

Example



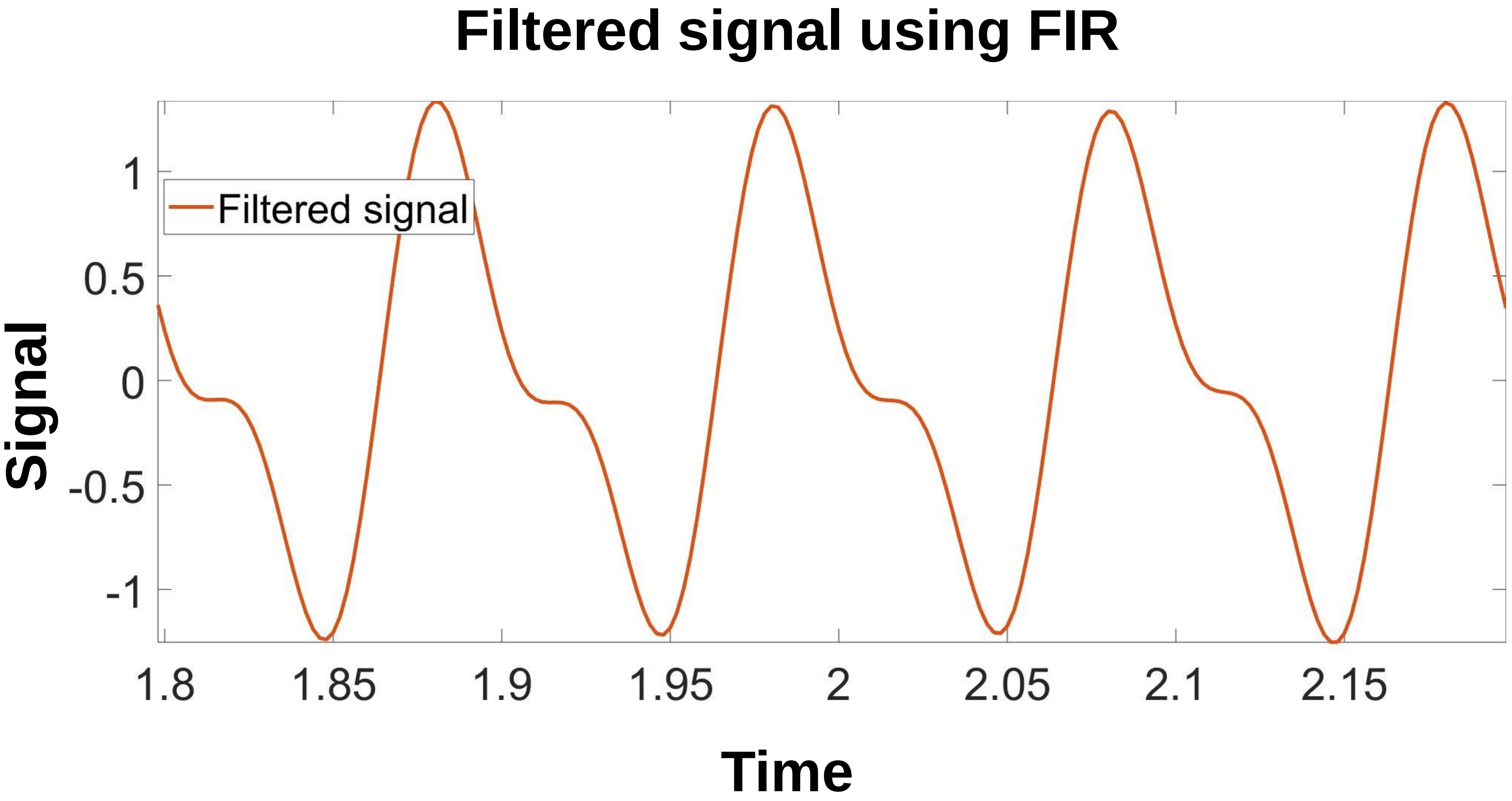
Filter Designer - [untitled.fda *]
File Edit Analysis Targets View Window Help
Current Filter Information
Structure: Direct-Form FIR
Order: 1814
Stable: Yes
Source: Designed
Store Filter ...
Filter Manager ...
Magnitude Response (dB)
Order 1814 !
Magnitude (dB)
0
-20
-40
-60
0
50
100
150
200
Frequency (Hz)
Response Type
Lowpass
Highpass
Bandpass
Bandstop
Differentiator
Design Method
IIR
Butterworth
FIR
Window
Filter Order
Specify order: 10
Minimum order
Options
Scale Passband
Window: Kaiser
View
Frequency Specifications
Units: Hz
Fs: 500
Fpass1: 11
Fstop1: 12
Fstop2: 18
Fpass2: 19
Magnitude Specifications
Units: dB
Apass1: 1
Astop: 60
Apass2: 1
Design Filter
Designing Filter ... Done

Example



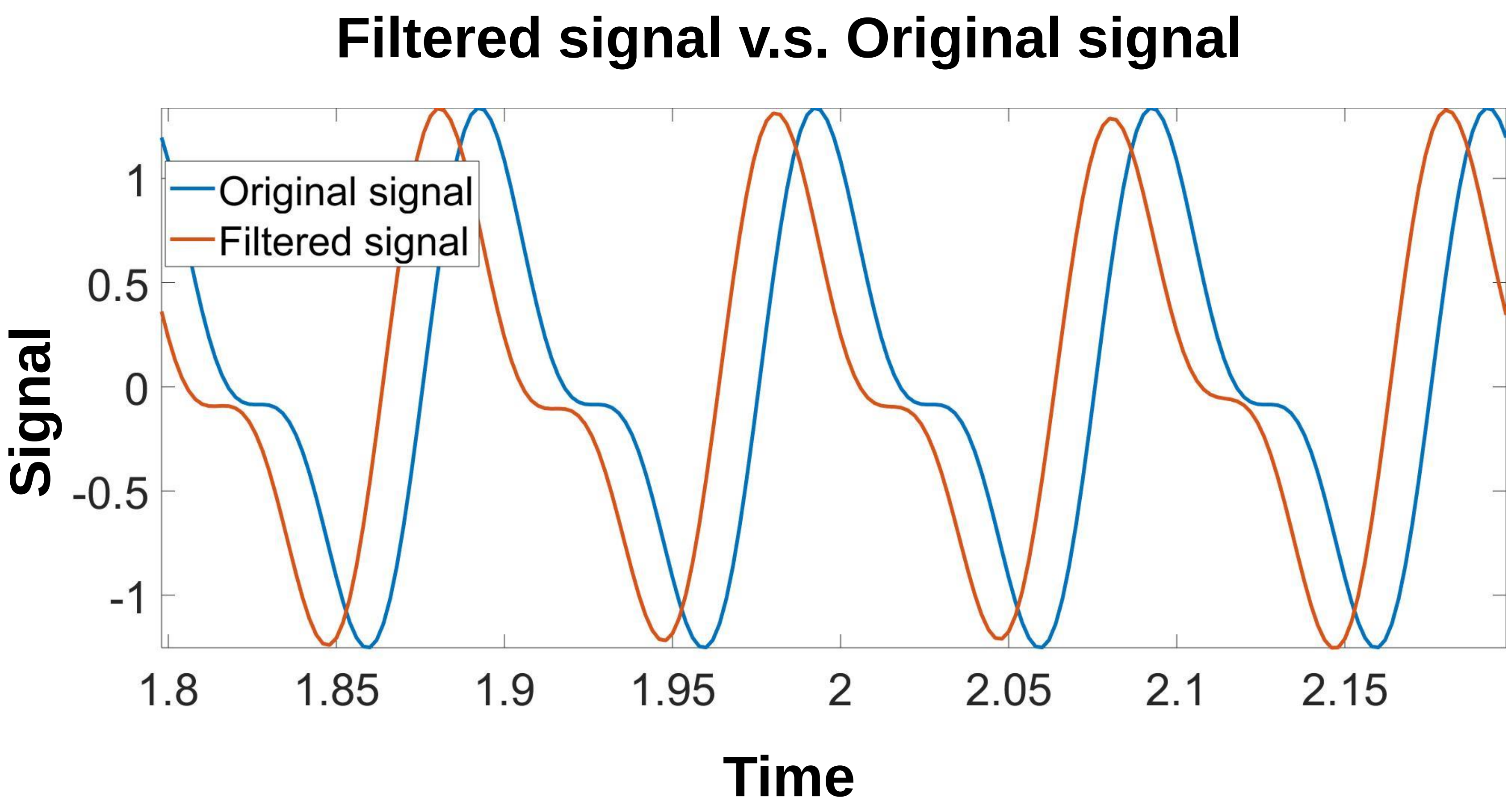
Filter Designer - [untitled.fda *]
File Edit Analysis Targets View Window Help
Current Filter Information
Structure: Direct-Form FIR
Order: 588
Stable: Yes
Source: Designed
Store Filter ...
Filter Manager ...
Magnitude Response (dB)
Order 588
Magnitude (dB)
0
-20
-40
-60
0 50 100 150 200
Frequency (Hz)
Response Type
Lowpass
Highpass
Bandpass
Bandstop
Differentiator
Design Method
IIR Butterworth
FIR Window
Filter Order
Specify order: 10
Minimum order
Options
Scale Passband
Window: Kaiser
View
Frequency Specifications
Units: Hz
Fs: 500
Fpass1: 11
Fstop1: 12
Fstop2: 18
Fpass2: 19
Magnitude Specifications
Units: dB
Apass1: 1
Astop: 20
Apass2: 1
Design Filter
Designing Filter ... Done

Example



Filtered signal using FIR
Filtered signal
Signal
1
0.5
0
-0.5
-1
1.8
1.85
1.9
1.95
2
2.05
2.1
2.15
Time

Example



FIR filters have linear phase response !

Persevering the shape of the signals is important in bio-signals applications

Example

```
%% Producing the oreginal signal

% Sampling period
Fs = 500;
% Sampling interval
Ts=1/Fs;
% Length of the signal
N=2000;
% Maximum time
Tmax=(N-1)*Ts;
% Time vector
t=0:Ts:Tmax;
```

Example

```
% Main frequencies & phase of the oreginal signal
F1=10;
F2=20;
phi1=1.4;
% Oreginal signal
x=cos(2*pi*F1*t)+0.5*cos(2*pi*F2*t+phi1);
% Plot range
plot_range =(N/2-100:N/2+100);
% Plot signal in the range
figure(1)
plot(t(plot_range),x(plot_range),'LineWidth',2.5);
axis tight
```

Example

```
%% Generate noise in a specific frequency band (12-18 Hz)

% Generate white Gaussian noise
ns = randn(1,length(x))*3;
% Design and load pass band filter: 12 to 18 Hz
load PB_12_18;
fvtool(PB_12_18)
% Construct in-band noise
ns_filtered=filter(PB_12_18,ns);
% Signal + Noise
x_ns=x+ns_filtered;
```

Example

```
% Plot oreginal signal and signal plus noise
figure(3)
plot(t(plot_range),x(plot_range),'LineWidth',2.5);
hold on
plot(t(plot_range),x_ns(plot_range),'LineWidth',2.5);
axis tight
```

Example

```
%% single-sided frequency spectrum of the signal plus noise

% Compute fft
X=fft(x_ns);
% Take abs and scale it
X2=abs(X/N);
% Pick the first half
X1=X2(1:N/2+1);
% Multiply by 2 (except the DC part), to compenseate
% the removed side from the spectrum.
X1(2:end-1) = 2*X1(2:end-1);
```

Example

```
% Frequency range
F = Fs*(0:(N/2))/N;
% Plot single-sided spectrum
figure(4)
plot(F,X1,'LineWidth',2.5)
title('Single-Sided Amplitude Spectrum')
xlabel('f (Hz)');
```

Example

```
%% Remove noise usin band-stop IIR filter

% Design and load IIR band stop filter: 12 to 18 Hz
load SB_12_18
fvtool(SB_12_18)
% Filter the noise out
x_clean_IIR=filter(SB_12_18,x_ns);
```

Example

```
% Single sided spectrum of cleaned signal
% Compute fft
X=fft(x_clean_IIR);
% Take abs and scale it
X2=abs(X/N);
% Pick the first half
X1=X2(1:N/2+1);
% Multiply by 2 (except the DC part), to compenseate
% the removed side from the spectrum.
X1(2:end-1) = 2*X1(2:end-1);
```

Example

```
% Plot single-sided spectrum
figure(6)
plot(F,X1,'LineWidth',2.5)
title('Single-Sided Amplitude Spectrum')
xlabel('f (Hz)');
figure(7)
plot(t(plot_range),x(plot_range),'LineWidth',2.5);
hold on
plot(t(plot_range),x_clean_IIR(plot_range),'LineWidth',
2.5);
axis tight
```

Example

```
%% Remove noise usin band-stop FIR filter
% Design and load FIR band stop filter: 12 to 18 Hz
load SB_12_18_FIR
fvtool(SB_12_18_FIR)
% Filter the noise out
x_clean_FIR=filter(SB_12_18_FIR,x_ns);
% Single sided spectrum of cleaned signal
% Compute fft
X=fft(x_clean_FIR);
% Take abs and scale it
X2=abs(X/N);
% Pick the first half
X1=X2(1:N/2+1);
```

Example

```
% Multiply by 2 (except the DC part), to compenseate
% the removed side from the spectrum.
X1(2:end-1) = 2*X1(2:end-1);
% Frequency range
F = Fs*(0:(N/2))/N;
% Plot single-sided spectrum
figure(9)
plot(F,X1,'LineWidth',2.5)
title('Single-Sided Amplitude Spectrum')
xlabel('f (Hz)');
```

Example

```
figure(10)
plot(t(plot_range),x(plot_range),'LineWidth',2.5);
hold on
plot(t(plot_range),x_clean_FIR(plot_range),'LineWidth'
,2.5);
axis tight
```

Useful links

- http://www.montefiore.ulg.ac.be/~ebrahimbabaie/applieddigtial.htm