Social Path Planning: Generic Human-Robot Interaction Framework for Robotic Navigation Tasks

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Abstract— This paper formulates a generic framework for the social path planning problem. Previous works deal with very specific cases of socially-aware motion planning. Instead, here we propose a mathematical formulation that can be considered the preliminary steps towards a general theoretical setting, incorporating previous work as sub-cases. Social path planning is analyzed into 6 different subproblems. Furthermore, a review of the state of the art about the different aspects of the social path planning problem is included. Most importantly, an extended model for the O-space for groups of people engaged in a social interaction is proposed. Preliminary results for actual social path planning solutions under the proposed formulation are shown, proving the powerfulness of the approach and its generalization. Finally, a concrete discussion about future work is provided.

I. INTRODUCTION

The classical path planning problem has been deeply studied during the last decades. Many different, interesting versions of the problem and solutions have been proposed, turning this field into one of the most studied within artificial intelligence as applied to robotics.

For instance, 2D navigation can be considered as a solved problem in most of the scenarios. However, very often the community proposes new problems that previous approaches are not able to successfully solve. This is the case of the socially-aware path planning, which has recently become much more important as human-robot interaction becomes increasingly frequent, and robots are expected to become an increasingly important part of our everyday lives. If the common path planning approaches are applied in mixed human-robot environments, the robot will execute movements that might well distract humans, making them feeling uncomfortable. Also, if the task of the robot is to interact with humans, the robot should be able to gracefully approach humans, and enter and exit their conversational circles, without alarming or distressing them, while signaling its intentions in a human-understandable way.

As shown in the following section, a lot of work have been done towards enabling robots to properly behave in a socially-acceptable manner. However, most of these approaches seem to be very ad-hoc and there is not a clear formulation of the problem. Therefore, one of the main points of this paper is to establish the basic formulation of the social path planning problem, so future works can be addressed under the same mathematical background. This will ease the research tracking and results comparison. This formulation takes into account the previous partial formulations available in the literature but an important novelty is introduced: the social path planning problem is divided into 6 different subproblems. By correctly combining these subproblems, most of the navigation tasks for social robots can be modeled.

The paper is structured as follows: the next section provides a detailed state of the art of the social path planning problem. Section III describes the mathematical formulation which is intended to be a basis for future work in this field. Following that, in section IV social spaces models are detailed. In section V the current work in this field is outlined. Lastly, section VII outlines the conclusions and discusses future work.

II. BACKGROUND

How social space is managed by humans was first studied by E.T. Hall in 1966 [1]. He demonstrated that some factors, such as the distance between two people, are influenced by their relationship. The study of these factors and their role in the human-human relationships received the name of *proxemics* [2]. Concretely, Hall defines proxemics as "the study of humankind's perception and use of space". A modern study about the distance-relationship influences is shown in [3].

During the recent years, human-robot interaction has became one of the most important fields in the current robotic research. Robots do not have to carry out their task only efficiently, but also in a human-friendly way, engaging and behaving in a socially acceptable manner in a robot companion context [4]. Recent work can be divided into 3 categories (figure 1): 1) human-robot proxemics, 2) human-aware planning and navigation, and 3) robot-to-human approaching and behaving. While the first group focuses on the low-level and mainly static configurations, the other two groups study high-level robot behaviors.

In the first group, human-robot proxemics, we consider those works which study the human reactions to robots in different contexts. Walters et al. [5] found that the social rules applied for a robot when interacting with humans depend on whether the human is standing in an open space, against a wall, or sitting. This work was later extended by taking into account the robot's shape, height and task carried out [6]. The recent sensing technologies allow the creation of novel algorithms for labelling and measuring of

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proxemics variables. Mead et al. [7] details a set of metrics extracted from social sciences works and carry out a pilot study with these metrics, showing that real-time automated annotation is possible. This supposes a great advance towards online robot actuation scaling depending on the human reactions. Actuation scaling refers to the modification of different variables according to the parameters sensed from the environment (i.e. robot velocity modulation depending on the distance to the objective). Within this group, we would like to include the works related to automatic identification of positive/negative interaction acceptance from the human point of view, as this an important feature for a social robot towards appropiate behaviour [8]. Finally, Laga et al. [9] proposes a model for the personal space modelling based on the mixture of two Gaussian functions. This model is deeply analyzed in [10]. This model is extended in [11] using a mixture of four Gaussian functions.

In the second group, human-aware planning and navigation, we put together all the research focused on how a robot should navigate towards a given point in a legible, human-friendly manner. One of the basic works is [12]. Here, the rules for an harmonious human-robot interaction in navigation scenarios are specified. In this work, a great effort is done towards creating a generic framework for human-aware robot navigation. However, only individuals and other robots are taken into account. There is no specification about how the robot should behave in the presence of group of humans. Sehestedt et al. [13] proposes a learning-based approach in which the nodes of a roadmap receive different weights depending on the observed human behaviors. They tested this approach in an office environment, resulting in a robot leveraging the corridors even if the path is longer, close to the most likely human paths. More recently, Guzzi et al. [14] proposed a human-inspired reactive and proactive motion planner, where the actions the robot follows are computed with a heuristic observed for humans [15], [16]. Another different problem within this group is mobile robot navigation through crowds of dynamic agents with uncertain trajectories. Most of the approaches suffer from the *freezing problem*, when the environment is complex enough, the planner considers that all forward paths are unsafe, and the robot freezes in place. Trautman et al. [17] solved this problem by applying *joint collision avoidance*: not only one agent on the crowd should care about collisions, but all of them do. This model is called *social forces* [18].
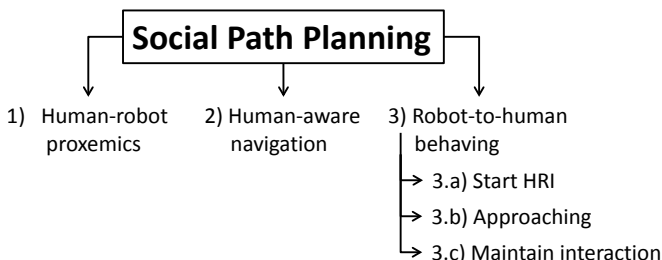

Fig. 1: State of the art classification

Other approaches focuses on how the human should be modelled when the robot wants to avoid them. A simple obstacle avoidance algorithm creates agressive movements which make the humans not comfortable with the robot motion. Kruse et al. [19] proposes a cost-based method which takes into account the human proxemics and field of view in order to navigate in a legible way. The Gaussian mixture model proposed in [9], [10] is employed in order to create a human friendly planner in [20], [21] called RiskRRT. This planner takes into account both single humans and couples of humans engaged in a social interaction.

Moving on to the last case, the main problem of the third category, i.e. robot-to-human approaching and behavior, is: how should a robot approach a human to successfully start an interaction? We can divide this problem into three different phases: a) to select the best goal pose to start the human-robot interaction, b) to perform the approaching towards that goal in a friendly, non-agressive way and c) to have a socially acceptable behaviour once the interaction has been successfully initiated. An interesting approach to static humans is the one proposed by Mead et al. [22]. They train the robot with different poses to establish the interaction depending on the objectives of the interaction so that the robot is later able to decide which pose to select depending on the purpose of the interaction. Also, Satake et al. [23] study how a robot should approach people in order to start a positive interaction in malls. For static humans, different approaches have been taken into account: Dautenhahn et al. [24] studied how the humans prefer to be approached while seating and proposed a cost-based path planning method to imitate the preferred behavior. However, Koay et al. [25] obtained results that were opposite to previous works when designing a human-friendly manipulation planner. Concretely, the difference attends to the robot approaching direction preference when the robot task is to give an object to the humans. They justified this difference by assuming that there are *cohabituation effects* in human-robot interaction that may play an important role. Avrunin et al. [26] proposes a simple approaching method depending on the shoulders and head orientation of the human to be approached. Regarding dynamic humans, Carton et al. [27] proposed a method for walking humans which estimates the future position of the human and computes a smooth path towards the predicted point.

Most of these approaches take into account the subproblems a) and b) at the same time since it is complex to separate those two problems. However, Henkel et al. [28] proposes a very interesting approach in which the robot's behavior scales according to proxemics measurements. Their experiments show that a perception-based scaling method is more comfortable for humans. This means that the robot behavior should not be fixed but should depend on what the robot senses.

Finally, the subproblem c) has been very briefly addressed in the literature. Assuming that an interaction has been already started, how should the robot behave in order to behave as the human expects? Pedica et al. [29] propose a method which provides reactive behavior to agents within

the interaction in order to simulate unconscious reactions and dynamic motions within the interaction. Feil-Seifer et al. [30] proposes a learning-based method in which a human and a robot walk together to a given point in a comfortable way for the human. A very interesting approach is given in [31]. Here, an algorithm for approaching, interacting and disengaging from a group of 3 humans is given. Although the results are poorly reported, they assure that the robot behaves as humans do.

III. MATHEMATICAL FORMULATION

In this section we detail a mathematical formulation for the socially-acceptable path planning problem. Let us consider a bi-dimensional, euclidean space $\mathcal{C}$. Those two dimensions correspond with the plane of the floor. This space is composed by the union of the obstacles space $\mathcal{C}_{obs}$ and obstacles-free space $\mathcal{C}_{free}$. Humans are expected to be in $\mathcal{C}_{free}$, so they are not treated as simple obstacles. We denote as $H = \langle H_i, \ldots, H_N \rangle$ the set composed by N humans in the environment. The state of a given human i is composed by its position, heading and velocity: $H_i = \langle x_H^i, y_H^i, \theta_H^i, v_H^i \rangle$. The set of humans can be split into two different subsets: H_{group} composed by those humans which are engaged in a social interaction, and H_{ind} the rest of individuals (walking around).

Every human within H becomes automatically part of the social space shared by all the individuals. Therefore, they create unintentional reactions on other people. Focusing on proxemics, the influence of individuals in H_{ind} is modelled with their personal spaces Φ_i. On the other hand, people belonging to H_{group} will be arranged in F-formations [32] (subgroups) which are the organization of the personal spaces shared by the humans enganged in a group. The O-space is the center space of the people within the group, the P-space surrounds the O-space and contains the people taking part in the interaction, and the R-space is the rest of the area. In this case, the social agent is the group itself and not the individual people on it. Therefore, the model to take into account should be the F-formation instead every person individually. Hence, we denote the social influence created by the group of humans H_{group}^j as Φ_j. The modelling of these social spaces are detailed in the following section.

The robot state is denoted as $R = \langle x_R, y_R, \theta_R, v_R \rangle$. For our purpose, we consider that the robot is assigned to an objective by a higher-level algorithm. Then, the robot should accomplish this objective, with a path Γ_R, in a human-friendly manner. This formulation is represented in figures 2 and 3.

Analyzing the state of the art, we have differentiated 6 different cases which can be given in a robotic social path planning framework. These cases are divided taking into account whether the humans to consider in the planning problem are engaged in social interactions or they can be considered as individuals:

1) Single human, individual (figure 2):
   a. Robot to point. Regular path planning considering humans as obstacles.

b. Full interaction: 1) approach human, 2) interact, keep interaction, 3) disengage.
   c. Follow human.
2) Group of humans (figure 3):
   a. Robot to point. Regular path planning considering group of humans as obstacles.
   b. Observe group, ask for permission to enter.
   c. Full interaction: 1) enter the group, 2) interact, keep interaction, 3) disengage.

By the correct addition of these different subproblems, it is possible to model most (if not all) the possible social path planning scenarios that could be given for a robot that is trying to navigate together with humans in its surroundings. Therefore, the free space $\mathcal{C}_{free}$ can be modelled as:

$$\mathcal{C}_{free} = \bigcup \Phi_i \quad \forall i \in H_{ind} \cup H_{group} \quad (1)$$

In the first case, single human, it is important to differentiate between full interaction (1.b) and human following (1.c). In the full interaction, we consider the problem of approaching to the human, interacting with him/her and disengaging in a human friendly manner. Following human can be considered as a way of keeping interaction. However, this implies a different kind of relative motions between human and robot so it is actually a different problem. Also, for the group of humans division, we do not consider the following group problem, as it can be reduced to following one of the humans within the group.

According to [12], a human-friendly, harmonious interaction should accomplish the next 6 rules:

1) *Collision-free:* Maintains robot safety.
2) *Interference-free:* the robot should not enter the personal space of any human unless it is its objective.
3) *Waiting:* If the robot enters the personal space of a human, it has to stop a fixed amount of time.
4) *Human priority:* Humans always have the highest priority.
5) *Robot intrusion:* If a robot enters the workspace of other robot, it should leave this space as soon as possible, while the other robots should stop their activities.
6) *Robot priority:* Robots with lower priority should yield to robots with higher priority.

This rules are accepted in the literature as high-level requirements for successful human-robot interaction. However, this set of rules can create new problems if they are taken into account strictly. In section VI some of these problems are outlined.

IV. SOCIAL SPACE MODELING

A correct model of the social spaces Φ_i (personal space in case of individuals and O-space in case of groups) is mandatory if a socially acceptable robot navigation is desired. As pointed out earlier, the model completely changes depending if the humans in the environment are individuals (not engaged in any social interaction) or if they belong to an F-formation (engaged in an interaction with other humans).
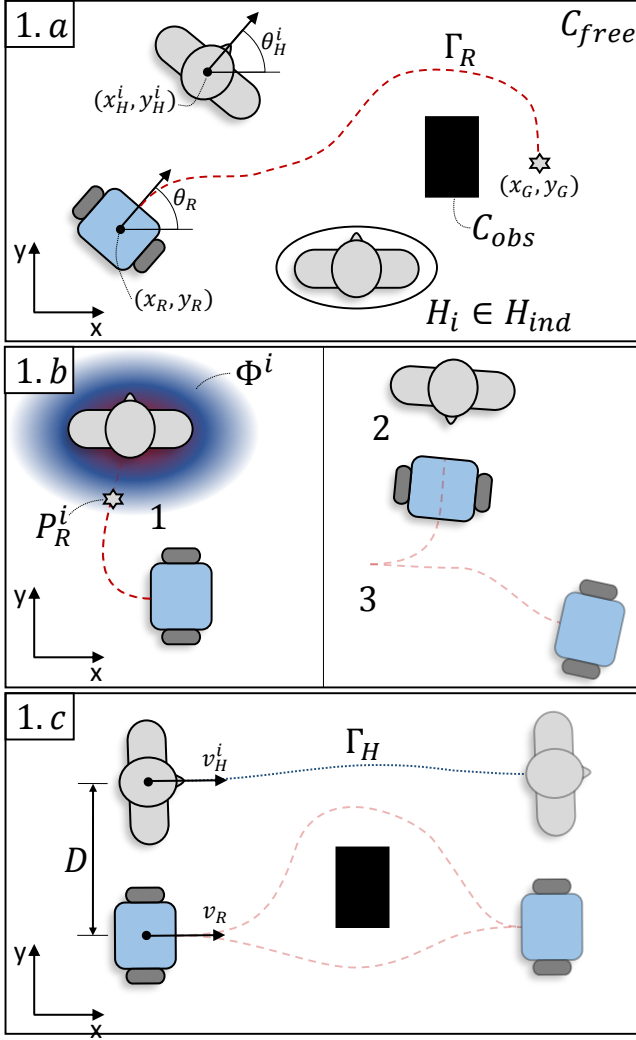
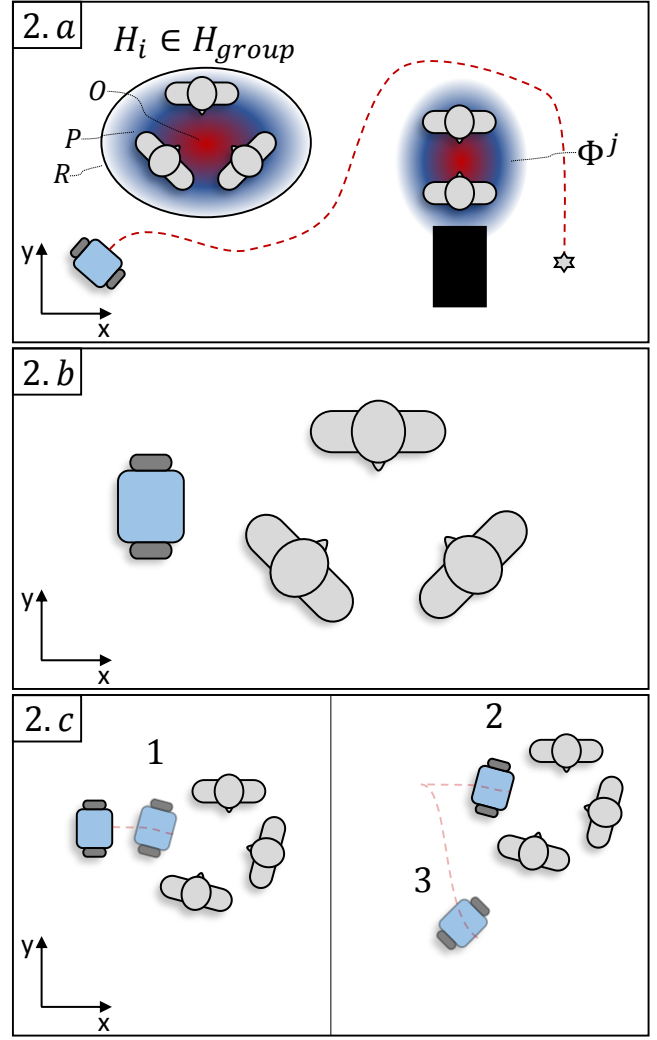Fig. 2: Subproblems for a single human.

Fig. 3: Subproblems for a group of humans.

The following subsections detail the models proposed in the literature for both cases.

A. Single human cases

The model for single humans have been already proposed in previous work. Actually, there are two different models, a two-Gaussian mixture model [10] and an extended, four-Gaussian mixture model [11]. Here, we focus on the initial two-Gaussian mixture model since it is simpler and already accepted by other authors.

The personal space around the human i can be defined as the mixture of two Gaussian functions, one for the front of the individual Φ_i^F and another one for its rear part Φ_i^R. A Gaussian function Φ is defined by its center p and its covariance matrix Σ as follows:

$$\Phi(q) = e^{(-\frac{1}{2}(q-p)^T\Sigma^{-1}(q-p))} \qquad (2)$$

Therefore, the personal space Φ_i can be evaluated at every point q in the human surroundings, taking p as the human position $p = (x_H^i, y_H^i)$, as:

$$\Phi_i(q) = \delta(y_q)\Phi_i^F(q) + (1 - \delta(y_q))\Phi_i^R(q) \qquad (3)$$

where $q = (x_q, y_q)^T$ and $\delta(y_q) = 1$ if $y_q \geq 0$ (q is in front of the human), and 0 otherwise. This allows to orientate the model according the human heading. The covariances matrices for each Gaussian are as follows:

$$\Sigma_i^F = \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & 4\sigma_x^2 \end{pmatrix} \qquad \Sigma_i^R = \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_x^2 \end{pmatrix} \qquad (4)$$

with $\sigma_x^2 = 0.45/2 = 0.255m$ when modeling the personal space. The original formulation includes some modifications to the Gaussian function attending to the age and gender.

Figure 4 shows the results of this model in humans with different headings.

B. Group of Humans Cases

Now, humans are not treated individually but every group is modeled as a unique agent. An O-space model for F-formations is proposed in [20]. However, it is mandatory to expand this model since it is valid only for groups of 2

people. Therefore, we create a more generic version of the O-space model in which as many humans as needed can be taken into account.

First, the original model is based on the pose (position and orientation) of two humans engaged in a interaction. Given the positions of two humans, $H_1 = (x_1, y_1)$ and $H_2 = (x_2, y_2)$, and their orientations with respect the global frame, ϕ_1 and ϕ_2, a point V_i is computed as the intersection of the vectors beginning in H_1 and H_2 with directions ϕ_1 and ϕ_2 respectively. Also, the point H_{12} is defined as the mean point between the two humans. The point C is defined as the mean point between V_i and H_{12}. Also, the distance D_H is defined as the Euclidean distance between H_1 and H_2. Finally, the distance D_i is defined as the Euclidean distance between H_{12} and V_i

Therefore, the O-space is modelled as a 2-dimensional Gaussian Φ_i as follows:

$$\Phi_i(Q) = e^{(-\frac{1}{2}(Q-C)^T \Sigma^{-1}(Q-C))} \tag{5}$$

where C is the center of the Gaussian function and Q is the point which is being evaluated. Σ is the covariance matrix defined as:

$$\Sigma = \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{pmatrix} \tag{6}$$

with $\sigma_x = D_H/4$ and $\sigma_y = D_i/2$ except for the case where $\phi_1 = -\phi_2$, where $\sigma_y = 0.6$. The Gaussian function has to be rotated in order to have the direction of $\overrightarrow{H_{12}C}$. This model is shown in figure 5.a).

In order to generalise this model, let us assume N humans engadged in an iteration. We assume that the people involved in the group are keeping a formation close to a circle, which is a typical assumption taking into account the shape of the F-formations. Also, we assume an ordered numbering of the humans within the group (clockwise or counter-clockwise). For every couple of adjacent humans, i and j, the original model is computed. Therefore, there will be N centers C_{ij}. The center for the new Gaussian is computed as the centroid for all the computed centers C_{ij}. D_H is therefore redefined as the average distances between all the humans in the group:

$$D_H = \frac{1}{N}\left(\sum_{i=1}^{N-1}(D_{H,i\ i+1}) + D_{H,N1}\right) \tag{7}$$

and D_i is computed as the average of twice the distance of the average point for every couple H_{ij} and their corresponding center C_{ij}:

$$D_i = \frac{2}{N}\left(\sum_{j=1}^{N-1}(D_{i,j\ j+1}) + D_{i,N1}\right) \tag{8}$$

An schema of the proposed model is depicted in figure 5.b). Its results are shown figure 6.

V. CURRENT WORK

In order to prove the validity of the proposed formulation, we include preliminary results of the current research in this field. We have integrated the proposed formulation in the Fast Marching Square (FM^2) path planning algorithm [33] which has proved to be a very reliable and versatile algorithm [34]. The detail of this approach is a matter of future work. However, the results included in this paper delight the usefulness of the novel approach. For instance, in figure 7 depicts the results of this method applied to the subproblem 1.a: humans as obstacles. Concretely, figure 7 b) shows the FM^2 velocities map (an artificial velocities potential which is an intermediary step of the algorithm) and how the humans are modeled with the personal space model detailed previously. This velocities map can generate a velocity profile for the computed path that also includes desirable characteristics for a human-friendly path planner (such slowing down when approaching humans).

Also, figure 8 includes the preliminary results for the subproblem 2.a: groups of humans as obstacles. In this case, the FM^2 velocities map shows the O-space model included in the approach. The result is a path that smoothly avoids the group.

VI. DISCUSSION

Although it is not the focus of this paper, we consider that the set of rules proposed in [12] are very strict. In fact, rules 2, 3 and 4 can turn the robot into a clumsy agent in a social environment, freezing it for a while or always

Fig. 4: Two-Gaussian mixture personal space model for individuals.

(a) Previous model.
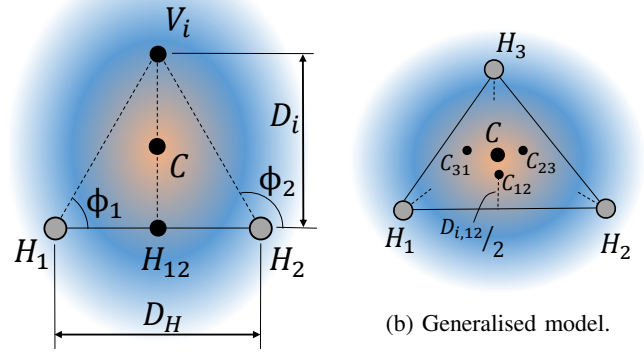
(b) Generalised model.

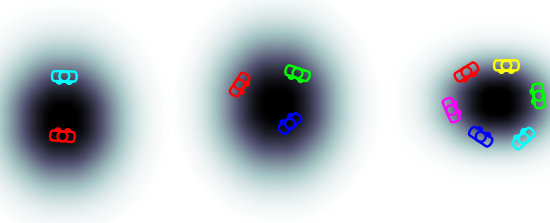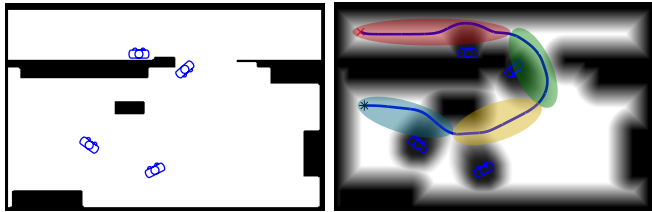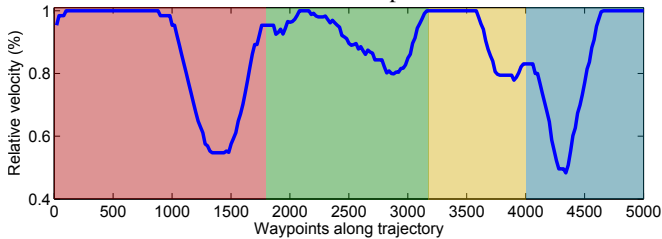Fig. 5: Generalisation of the O-space model.

Fig. 6: Gaussian model of the O-space for groups of N people.

(a) Environment map with individuals on it.

(b) FM^2 velocities map and the final path.

(c) Velocities profile along the trajectory.

Fig. 7: Preliminary results. The start point is at the top of the map and the path provided avoids personal spaces except when it means to get very close to obstacles.

Fig. 8: Results for the solution of a group of humans as obstacle. $\mathbf{W}$ with the O-space model, saturation at 0.5m and the final path.

try to maximize their individual comfort [9]. However, there is no evidence that the human reactions are optimal to achieve this comfort. In other words, if robots are designed to imitate humans it could happen that this is not the best approach to maximize the comfort of the humans the robot is interacting with.

VII. CONCLUSIONS

Along this paper a set of contributions have been detailed, with the aim of establishing the basis for the future work in the field of human-robot interaction, specially in robot navigation tasks.

First of all, a detailed review of the state of the art was included. The papers discussed are classified attending the specific problem they try to solve. The special classification framework that we have introduced (three categories of papers, and six specific subproblems) was also useful towards exposing the partiality off existing works. While some of these papers have no mathematical formulation, others propose very different formulations depending on the different subproblems.

Therefore, in this paper we have detailed a novel classification as well a mathematical formalization of all the different cases that can be given while navigating close to humans. All the elements to take into account in each subproblem are detailed. Therefore, a generic formulation is given, that can act as a concrete basis for future work. This will allow to easily compare approaches across their structural characteristics, but also in terms of performance, efficiency, etc.

Also, within the context of a generic problem formulation, a novel O-space model was introduced. The contribution is the generalization of this model to groups of N people. Groups of people have been very briefly studied in the human-robot literature, and thus we are addressing this omission through our model.

Finally, preliminary results of a real-world implementation, and several interesting points were discussed which we

giving priority for humans. Also, there are some cases in which it is impossible to avoid to interfere in the personal space of the humans, such as crowded places, small rooms or tight corridors. Also, an strict application of these rules can decrease optimality in robot the robot behavior. Although we are trying the robot to have a friendly behavior, we also have to keep the paths and motions as optimal as possible in terms of path length, smoothness, safety, energy consumption and execution time. Therefore, it could happen that a small interference with a human could save a lot of energy or time to the robot, which is worthy to take into account from an engineering point of view.

Despite all the developed work, there is a very important issue that has not been deeply taken into account. For a given situation it is probable that robots should not behave as humans [35]. To assume that robot-human interaction should be similar than human-human interaction should always be verified. In fact, if robots are going to interact with humans they should have a socially acceptable behavior. But this does not mean that the acceptable behavior for a robot have to be the same than for a human. Also, human social behavior is, most of the times, unintentional. When interacting, humans
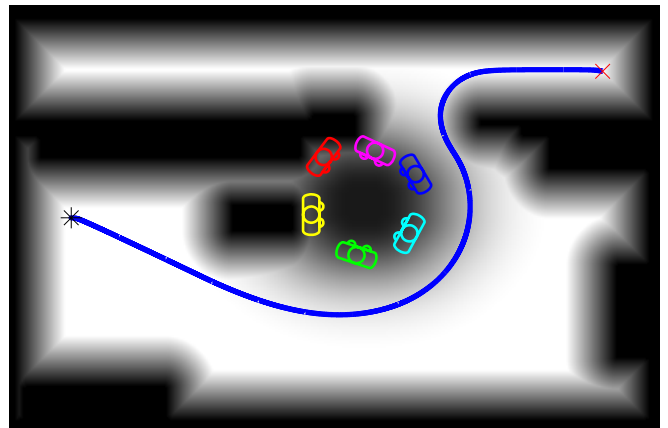
consider as some of the key points of this field during the next years.

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