Topics in Distributed Systems

An Introduction to Distributed Systems Verification

Bernard Boigelot

E-mail : bernard.boigelot@uliege.be
WWW : https://people.montefiore.uliege.be/boigelot/
https://people.montefiore.uliege.be/boigelot/courses/tds

Introduction

Motivation: Distributed systems are often employed in critical applications.

(Source: Airbus)

(Source: TechNode)

(Source: RailEngineer)

(Source: Distrelec)

In such applications, software defects can have serious consequences.

Famous (and unfortunate) examples

- Therac-25 (1986-87): 4 deaths, 2 serious injuries.

(Source: `medium.com`)

- Ariane 5 maiden flight (1996): $>$ 300 M€.

(Source: ESA)

- AT&T long-distance network collapse (9h, 1990): 60 M$ + indirect costs.

(Source: AT&T)

- AT&T mobile network outage (11h, 2024): > 500 M$ + indirect costs.

- A400M test flight crash (2015): 4 deaths.

(Source: Wikimedia Commons)

- Uber self-driving car collision with cyclist (2018): 1 death.

(Source: NTSB)

What can be done?

- Approach 1: Testing

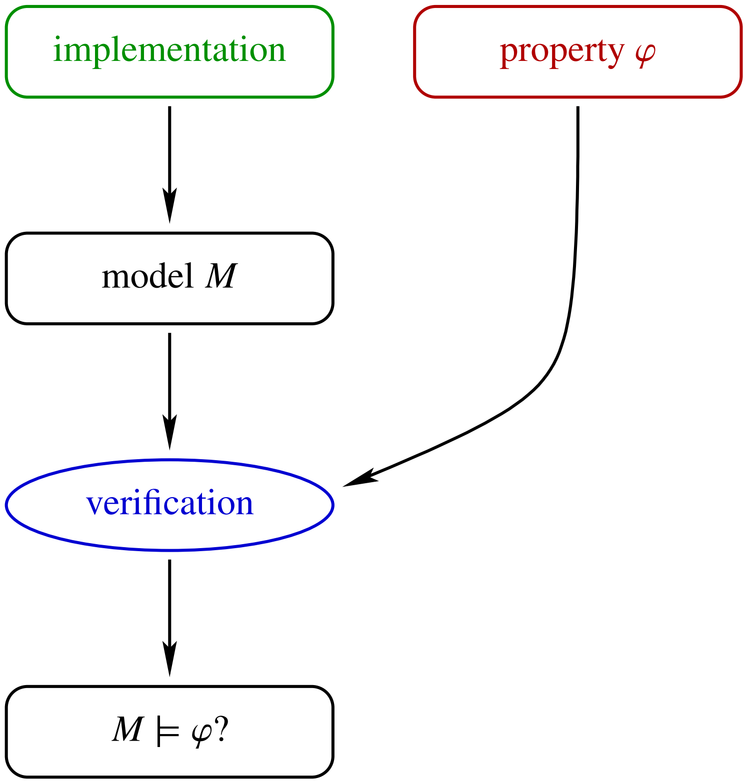
  Essential, but not sufficient for distributed systems because

  - concurrency, and
  - their unpredictable environment

  usually make them highly non-deterministic.

- Approach 2: Formal proof of correctness
  - Tedious and difficult.
  - No guarantee that the proofs hold for actual implementations.

Other approach: Automated verification



implementation

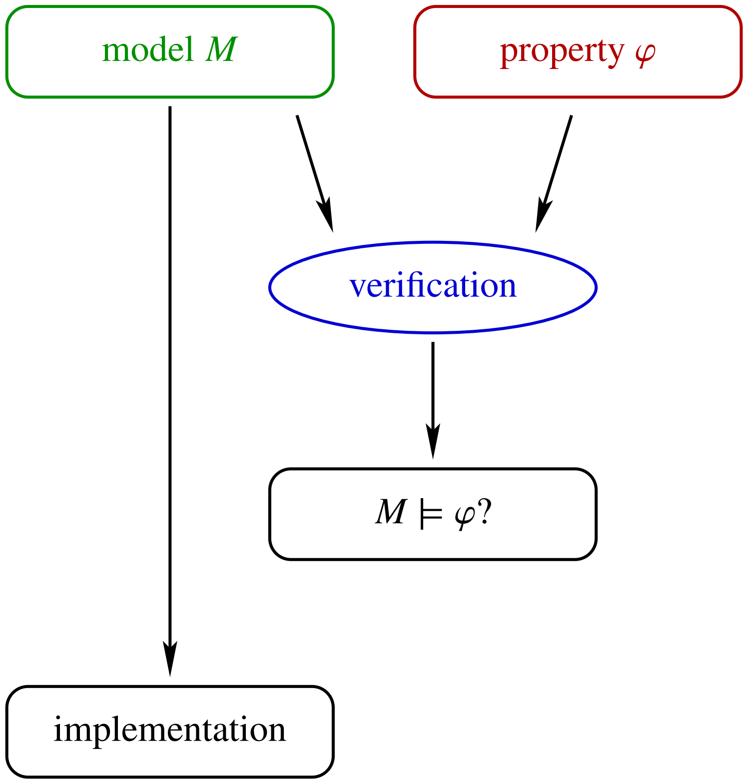
property φ

model M

verification

$M \models \varphi$?

or



model M

property φ

verification

$M \models \varphi$?

implementation

Principles

- The model M describes the system at some level of abstraction.
- The property φ is either
  - generic: absence of deadlocks, assertion violations, buffer overflows, arithmetic exceptions, . . . , or
  - specific to the application:
    - safety properties: bad things never happen,
    - liveness properties: good things eventually happen.
- The aim is to cover 100% of the model's behaviors.

Notes:

- This usually does not guarantee that actual implementations will be 100% error-free.
- The real goal is to find elusive bugs that are missed by testing.

Case study: LeLann-Chang-Roberts's leader election algorithm on a ring

(Source: Pascal Fontaine's lectures)

```
process $i$
    is_leader $\leftarrow \bot$
    max_id $\leftarrow$ id$_i$
    next!id$_i$
    while $\top$ do
        prev?$j$
        if $j$ = id$_i$ then
            is_leader $\leftarrow \top$
            next!id$_i$
            prev?$j$
            return
        if $j$ = max_id then
            next!$j$
            return
        if $j >$ max_id then
            max_id $\leftarrow j$
            next!$j$
```

Setting:

- At first, we look at a safety property: Is the elected leader always unique?

  Note: We will later check whether a leader is always elected.

- The verification strategy is exhaustive state-space exploration.
- To obtain a finite-state model, we will fix
  - the number of processes participating in the election, and
  - the capacity of the communication channels.
- The ID of each node will be assigned non-deterministically.
- The goal is to cover all possible interleavings of process actions.

States

Definition: A state of the model is the collection of all data characterizing its possible future behaviors.

For our case study, a state is composed of

- a control location (i.e., line number in the code) for each process,
- a value for the variables id, $\mathrm{max\_id}$, $\mathrm{is\_leader}$ and j of each process, and
- the content of the communication channels.

The initial states are such that

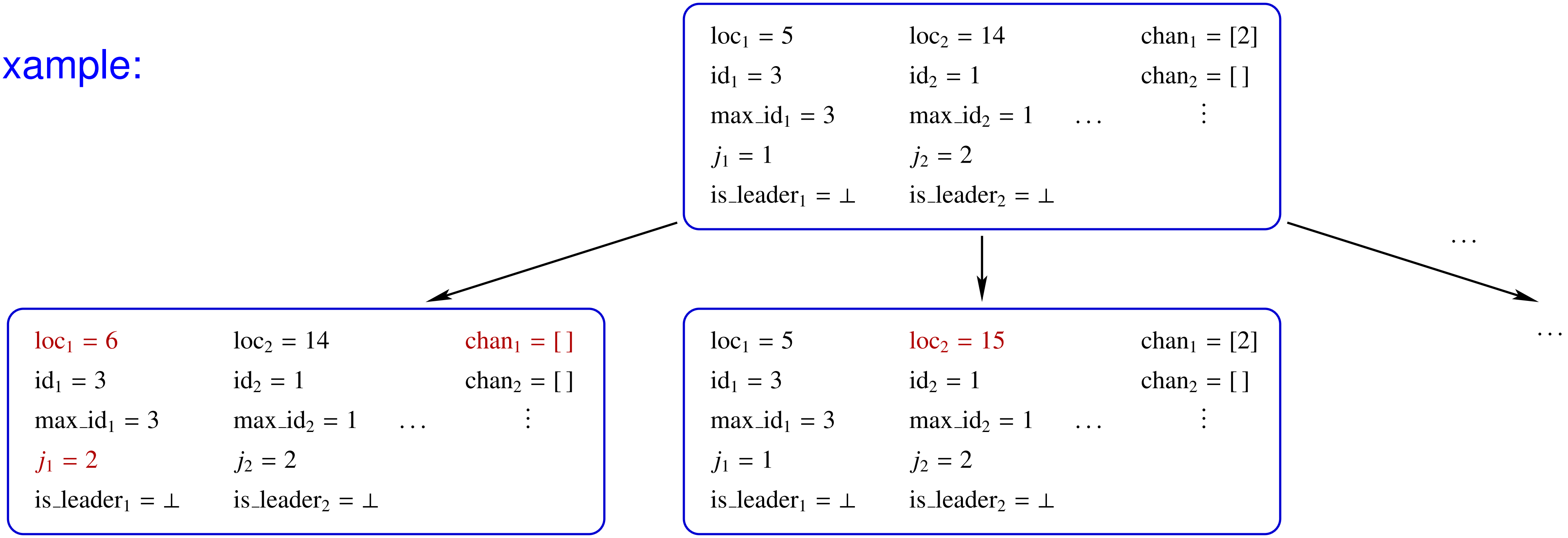
- each process is in its first control location,
- each variable has its specified initial value,
- each communication channel is empty.

Transitions

The current state of the model changes by following transitions.

- We consider the interleaving model of concurrency, in which each transition corresponds to performing one atomic operation in a single process.
- The transitions that can be followed from a state s are said to be enabled in that state. The set of such transitions is written $enabled(s)$.

Example:



$\mathrm{loc}_1 = 5 \quad \mathrm{loc}_2 = 14 \quad \mathrm{chan}_1 = [2]$

$\mathrm{id}_1 = 3 \quad \mathrm{id}_2 = 1 \quad \mathrm{chan}_2 = [\,]$

$\mathrm{max\_id}_1 = 3 \quad \mathrm{max\_id}_2 = 1 \quad \ldots \quad \vdots$

$j_1 = 1 \quad j_2 = 2$

$\mathrm{is\_leader}_1 = \bot \quad \mathrm{is\_leader}_2 = \bot$

$\mathrm{loc}_1 = 6 \quad \mathrm{loc}_2 = 14 \quad \mathrm{chan}_1 = [\,]$

$\mathrm{id}_1 = 3 \quad \mathrm{id}_2 = 1 \quad \mathrm{chan}_2 = [\,]$

$\mathrm{max\_id}_1 = 3 \quad \mathrm{max\_id}_2 = 1 \quad \ldots \quad \vdots$

$j_1 = 2 \quad j_2 = 2$

$\mathrm{is\_leader}_1 = \bot \quad \mathrm{is\_leader}_2 = \bot$

$\mathrm{loc}_1 = 5 \quad \mathrm{loc}_2 = 15 \quad \mathrm{chan}_1 = [2]$

$\mathrm{id}_1 = 3 \quad \mathrm{id}_2 = 1 \quad \mathrm{chan}_2 = [\,]$

$\mathrm{max\_id}_1 = 3 \quad \mathrm{max\_id}_2 = 1 \quad \ldots \quad \vdots$

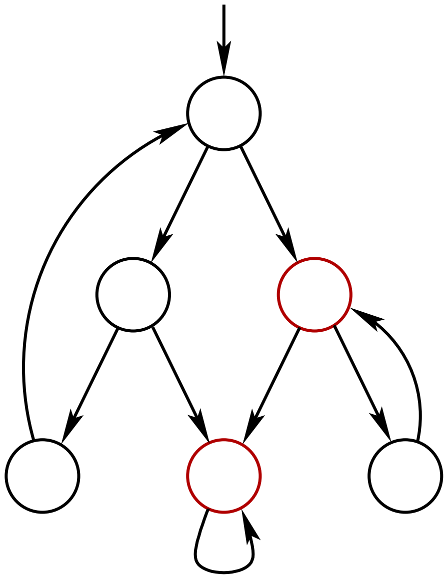
$j_1 = 1 \quad j_2 = 2$

$\mathrm{is\_leader}_1 = \bot \quad \mathrm{is\_leader}_2 = \bot$

State-space exploration

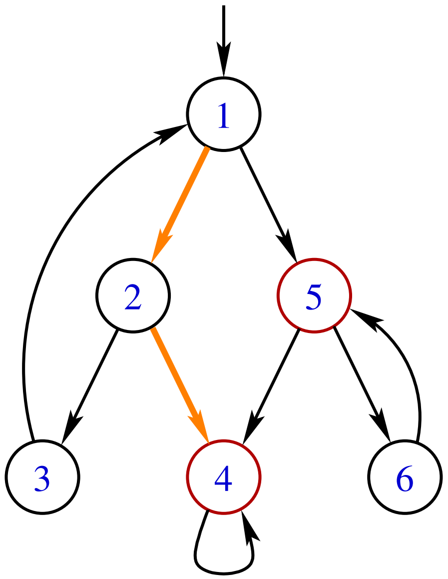
Principles:

- One explores one by one the reachable states of the model (i.e., those that can be reached by following finitely many transitions from an initial state).
- For each reachable state, one checks whether the safety properties are satisfied.
- One remembers the already visited states, in order to consider them only once.
- If a error state is explored, one extracts an execution trace that allows to reproduce the corresponding bug.



State-space exploration algorithm: Depth-First Search (DFS)

```
procedure $DFS()$:
    $St \leftarrow [\,]$
    $H \leftarrow \emptyset$
    for all initial $s_0$:
        $explore(s_0)$

procedure $explore(s)$:
    $St.push(s)$
    $H \leftarrow H \cup \{s\}$
    for all $t \in enabled(s)$:
        $s' \leftarrow succ(s,t)$
        if $s' \notin H$ then:
            $explore(s')$
    $St.pop()$
```

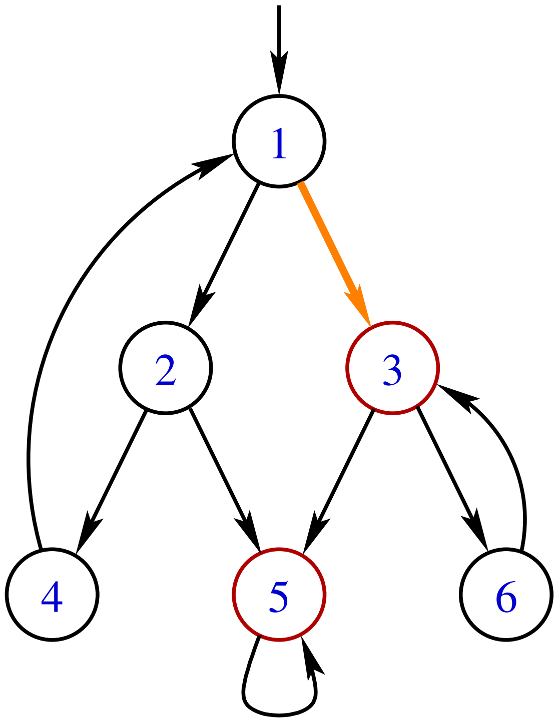


Notes:

- The function $succ(s, t)$ returns the state reached by following the transition t from s.
- The stack St does not affect the execution of the algorithm. Its purpose is to provide an execution trace that shows how an error state can be reached.
- The transition relation does not need to be stored: the state-space graph is generated on-the-fly.

Alternative exploration strategy: Breadth-First Search (BFS)

```
procedure $BFS()$:
    $H \leftarrow \emptyset$
    $S \leftarrow \emptyset$
    for all initial $s_0$:
        $S \leftarrow S \cup \{s_0\}$
    while $S \neq \emptyset$:
        $H \leftarrow H \cup S$
        $S' \leftarrow \emptyset$
        for all $s \in S$:
            for all $t \in enabled(s)$:
                $s' \leftarrow succ(s,t)$
                if $s' \notin H$ then:
                    $S' \leftarrow S' \cup \{s'\}$
        $S \leftarrow S'$
```



Notes:

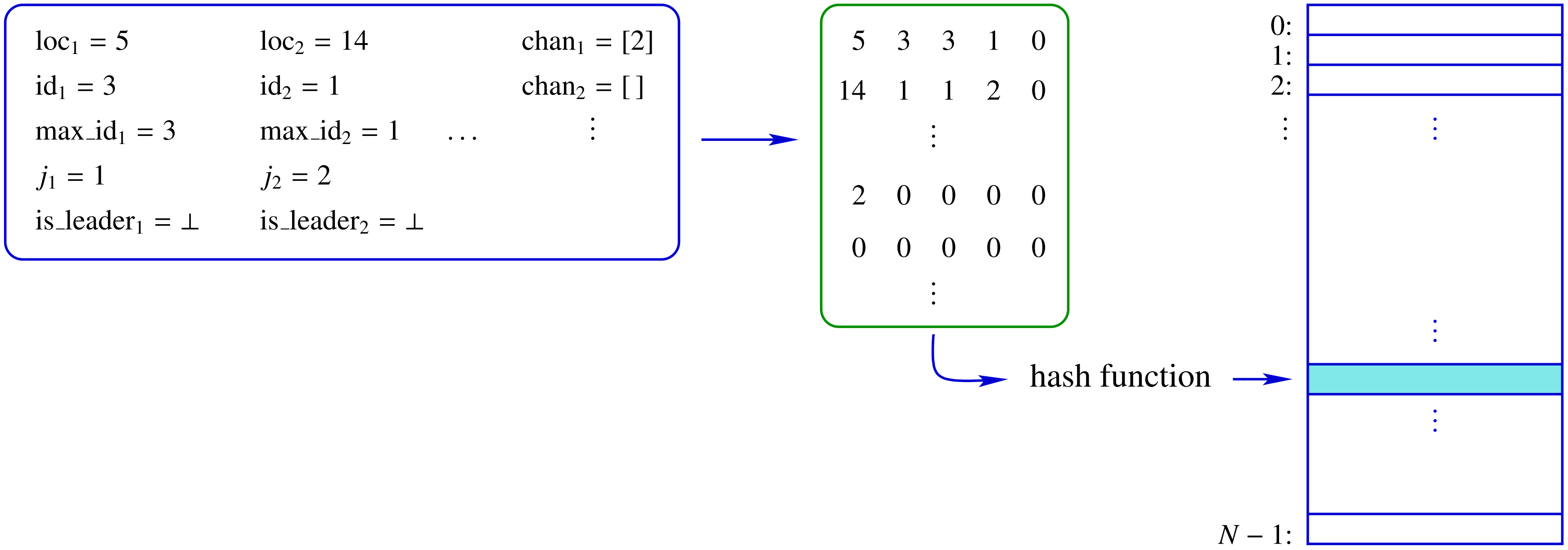
- This version is more memory-consuming, since one has to store the current state set S in addition to the set H of already visited states.
- It is however able to produce the shortest trace leading to an error state.

Storing the visited states

The main bottleneck is the size of the data structure representing the set H of already visited states. It should be such that

- adding a new state s to H, or checking whether $s \in H$, are both performed in average $O(1)$ time.
- this data structure requires as little memory as possible.

Option 1: Explicitly store the state vector in a hash table.



$\mathrm{loc}_1 = 5$ $\quad$ $\mathrm{loc}_2 = 14$ $\quad$ $\mathrm{chan}_1 = [2]$

$\mathrm{id}_1 = 3$ $\quad$ $\mathrm{id}_2 = 1$ $\quad$ $\mathrm{chan}_2 = [\,]$

$\mathrm{max\_id}_1 = 3$ $\quad$ $\mathrm{max\_id}_2 = 1$ $\quad \dots \quad \vdots$

$j_1 = 1$ $\quad$ $j_2 = 2$

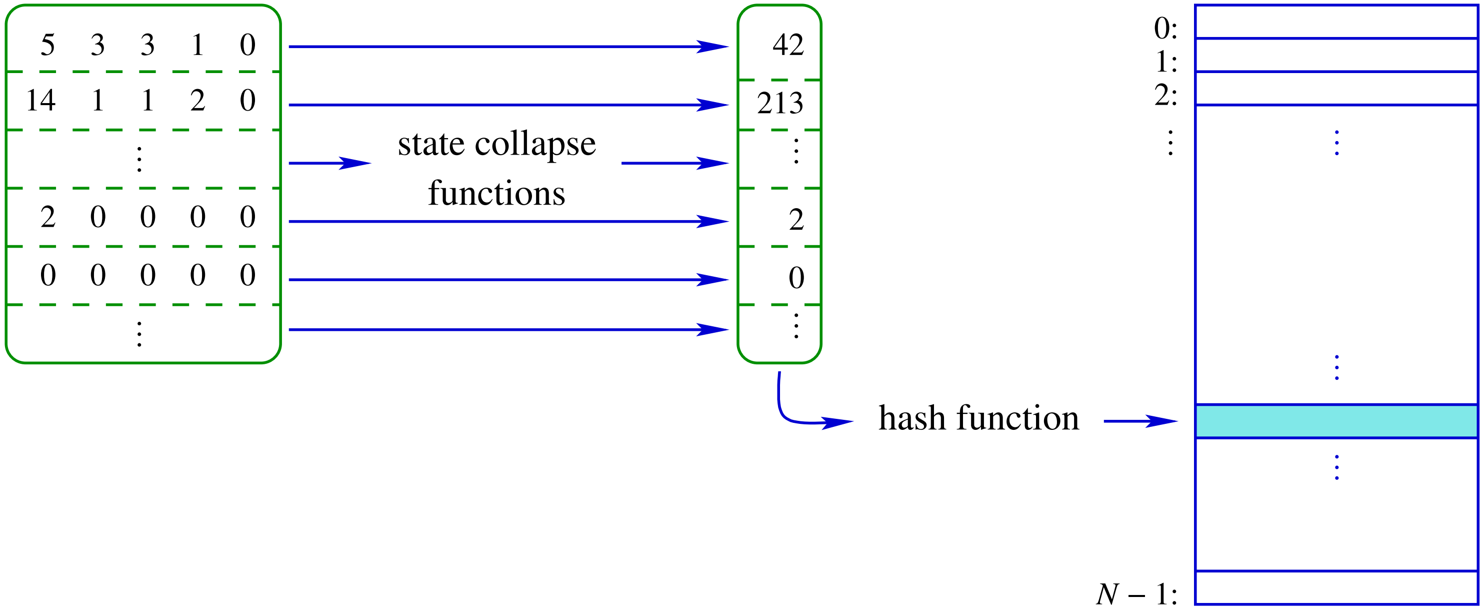
$\mathrm{is\_leader}_1 = \bot$ $\quad$ $\mathrm{is\_leader}_2 = \bot$

$$\begin{array}{ccccc} 5 & 3 & 3 & 1 & 0 \\ 14 & 1 & 1 & 2 & 0 \\ & & \vdots & & \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ & & \vdots & & \end{array}$$

hash function

0:
1:
2:
$\vdots$
$N-1$:

Option 2: Use an additional structure encoding parts of the state vector (for instance, the state of each process and each channel) with a reduced amount of information.



(Motivation: a process with a n-bit state vector very often has much less than 2^n reachable configurations.)

Option 3: Use a probabilistic approach.

Idea (naive):

- Store in the hash table only one bit per visited state.
- Make the table big enough for the probability of a collision to be negligible.
- The guarantee of correctness is lost, but the method is still useful for finding bugs.

Problem: The birthday paradox makes this solution unfeasible:

If m values are stored in a hash table of size N, the probability of a collision is

$$p_c(m, N) = 1 - \left(1 - \frac{1}{N}\right)\left(1 - \frac{2}{N}\right) \cdots \left(1 - \frac{m-1}{N}\right) = 1 - \frac{N!}{N^m\,(N-m)!}.$$

For large N, we have $1 - \frac{1}{N} \approx e^{-\frac{1}{N}}$, which yields

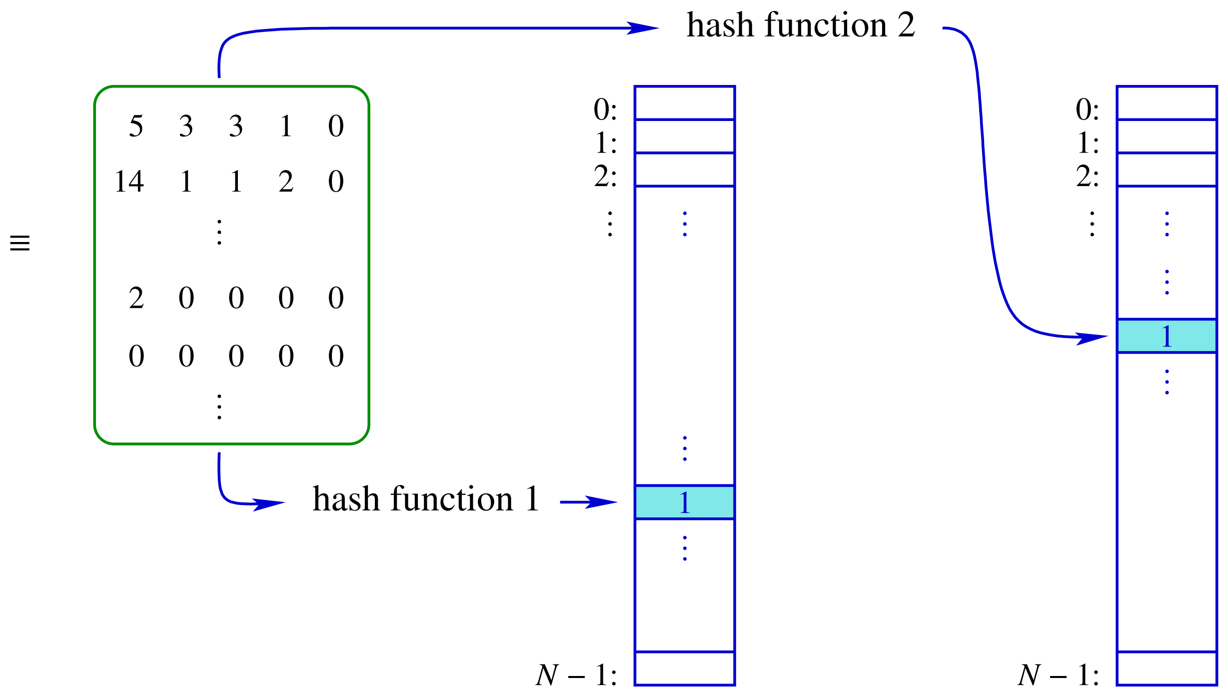
$$p_c(m, N) \approx 1 - e^{-\frac{m^2}{2N}}.$$

Examples:

- $N = 365$, $m = 40$: $p_c(m, N) \approx 89\%$.
- $N = 10^9$, $m = 10^6$: $p_c(m, N) \approx 99{,}999999\ldots\%$ (with 217 nines).
- For $m = 10^6$, the smallest value of N that gives $p_c(m, N) < 10^{-3}$ is $\approx 5.10^{14}$, corresponding to a hash table of ≈ 57 TB.

Note: It is important to avoid collisions as much as possible, since the successors of states wrongly marked as being already visited are not explored.

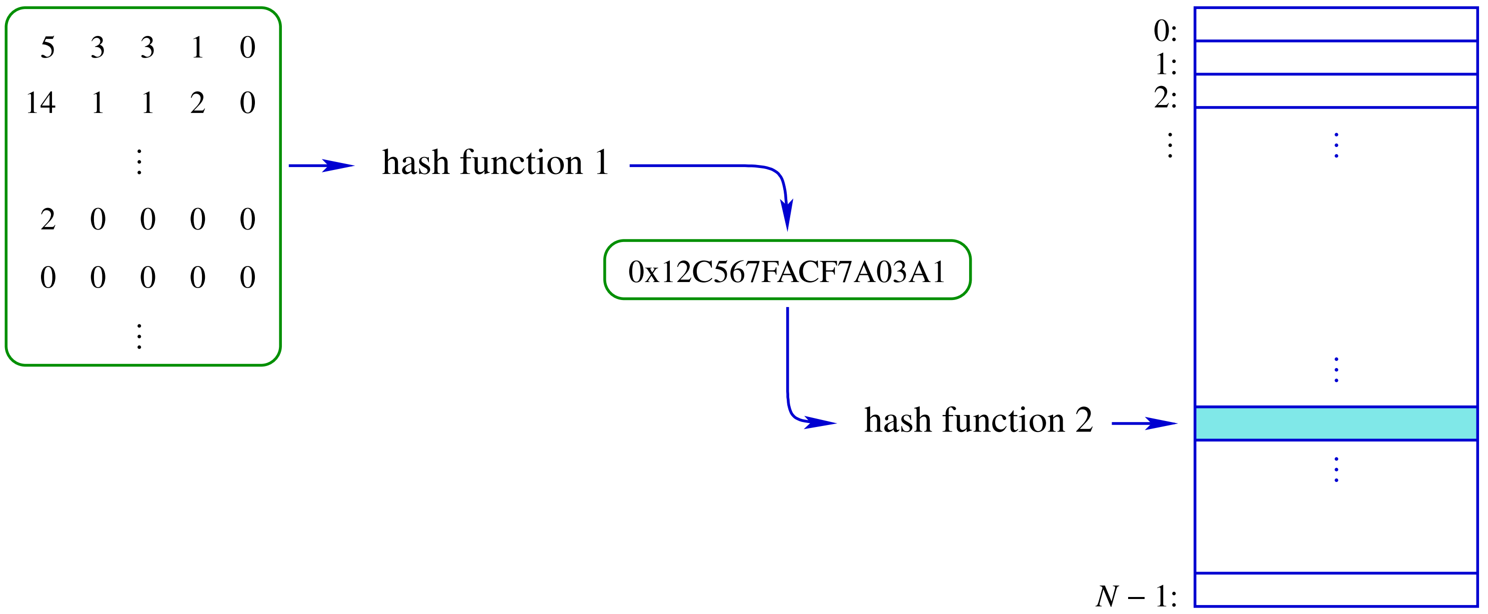
Solution 1: Use multiple independent hash functions (bitstate hashing).



Drawbacks:

- Computation time increases linearly with the number of hash functions.
- The number of hash functions needed to bring the probability of collision sufficiently low can be large.

Solution 2: Compress the state vector into a small-size descriptor, and store this descriptor into a hash table (hashcompact).

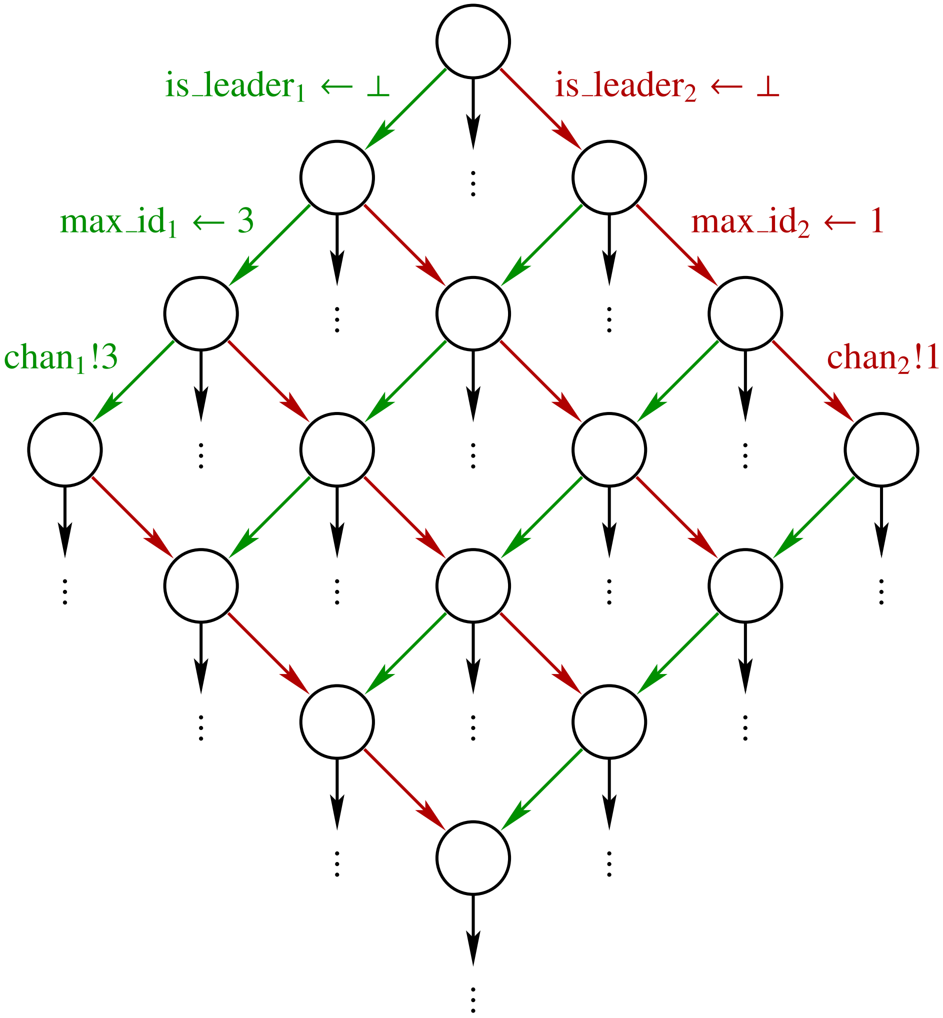


Example: With $m = 10^9$ and 64-bit descriptors ($N = 2^{64}$), one gets $p_c(m, N) \approx 2{,}7\%$. With 80-bit descriptors, $p_c(m, N) \approx 0{,}00004\%$.

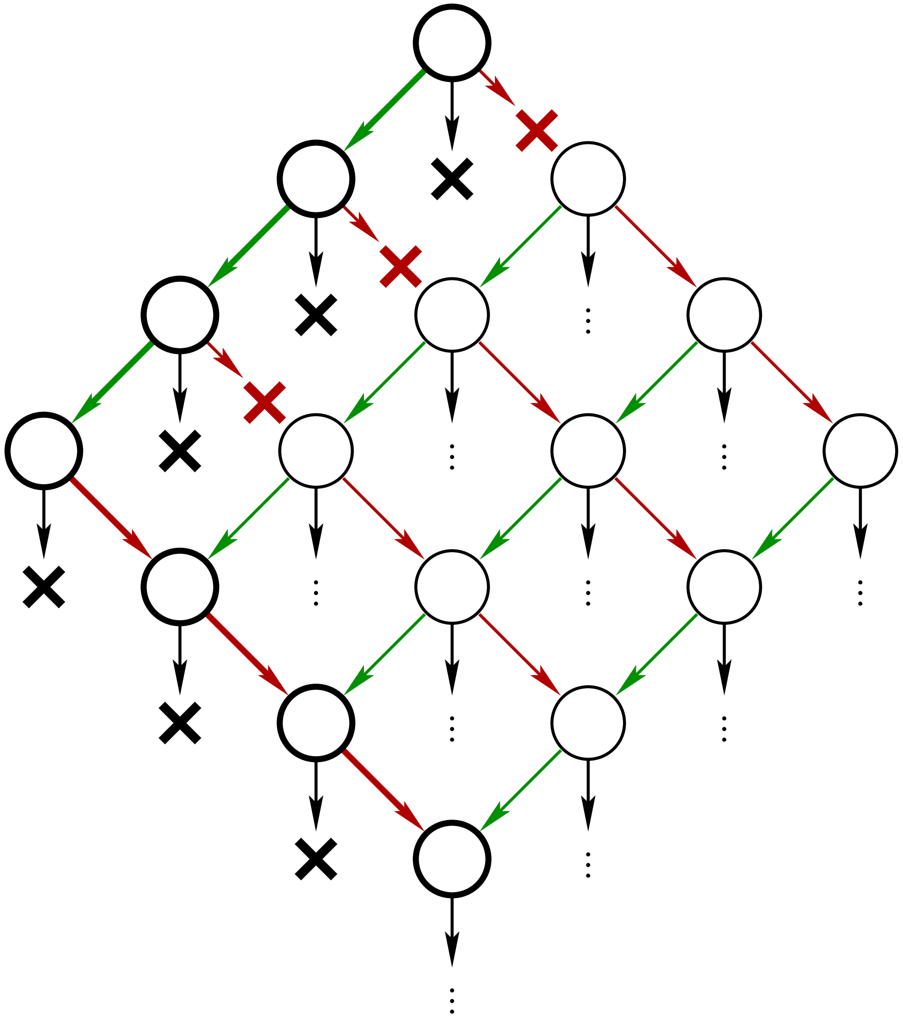
Reducing the search space

Sometimes, exploring all interleavings of the operations performed by processes is highly redundant.

```
process $i$
    is_leader $\leftarrow \bot$
    max_id $\leftarrow$ id$_i$
    next!id$_i$
    while $\top$ do
        prev?$j$
        if $j =$ id$_i$ then
            is_leader $\leftarrow \top$
            next!id$_i$
            prev?$j$
            return
        if $j =$ max_id then
            next!$j$
            return
        if $j >$ max_id then
            max_id $\leftarrow j$
            next!$j$
```



Partial-order methods are able to compute on-the-fly which enabled transitions do not need to be followed, while still guaranteeing that violations of the properties of interest will be detected.



The Spin tool

Spin is a model checker for finite-state concurrent models.

Main features:

- Can check assertion violations, deadlocks, unreachable code, and verify liveness properties expressed in Linear-Time temporal Logic (LTL) (more on that later . . .).
- Several options for storing visited states:
  - hash table.
  - collapsed representation.
  - bitstate hashing and hashcompact (possibly incomplete search).
  - minimized automaton (memory efficient and exact, but much slower).
- Choice of two search strategies:
  - depth-first search (DFS).
  - breadth-first search (BFS).

- Partial-order reductions for reducing the search space.

- Simple graphical front-end: iSpin.

For installation instructions, documentation, and examples: `https://spinroot.com`.

The Promela modeling language

- Processes are defined by `proctype` declarations, and are created dynamically using the `run` statement.

  Note: Processes corresponding to `active` proctypes are automatically created from the beginning.

- Processes communicate via shared variables and communication channels.

- An `init` declaration defines an initial process, that typically
  - initializes global variables, and
  - creates other processes.

Example:

```
int x1, x2;

proctype p1()
{
  printf("p1, x1: %d\n", x1)
}

proctype p2()
{
  printf("p2, x2: %d\n", x2)
}

init
{
  x1 = 1;
  x2 = 1;

  run p1();
  run p2()
}
```

Data types and variable declarations

Primitive:

`bool` or `bit`	1 bit	$[0, 1]$
`byte`	8 bits	$[0, 255]$
`short`	16 bits	$[-2^{15}, 2^{15} - 1]$
`int`	32 bits	$[-2^{31}, 2^{31} - 1]$

```
byte c1, c2 = 2, c3;
```

Arrays

```
bool table[16];
int v[8] = 1;
```

Symbolic constants

```
mtype = { MSG, ACK, NACK };
```

```
mtype message = MSG;
```

Communication channels:

```
chan xmit = [3] of { mtype, short };
```

```
xmit!MSG,10;
xmit?MSG,nb;
```

Structures:

```
typedef message
{
  bit b[8];
  int nb
};
```

```
message m;
```

Statements

- The body of a proctype takes the form of a sequence of statements. Statements are separated by semicolons (“`;`”).
- An any moment during execution, a statement is either
  - executable, or
  - blocked.
- An assignment (e.g., `x = a + b`) is always executable.
- An expression (e.g., `a + b`) can also be used as a statement. It is executable if its evaluation returns a non-zero value.

Special statements

- `skip` is always executable, and does nothing.
- `run` instantiates a new process. Such a statement is executable only if the maximum number of processes has not yet been reached.
- `printf` is always executable, and displays debugging information. This statement has no effect during verification.

Example:

```
int x1 = 0;

init
{
  run p1(10);
  x1 != 0;  // to become executable, another process must modify x1
  skip
}
```

- `assert` is always executable. It evaluates an expression, and generates an error if the returned value is equal to 0.

  Example: `assert(nb <= 10)`

The `if` statement

Syntax:

```
if
  ::  guard1 -> instructions1;
  ::  guard2 -> instructions2;
   ⋮
fi
```

Notes:

- This statement selects non-deterministically one sequence

  `guard`i `->` `instructions`i

  among those for which `guard`i is executable.

- If no guard is executable, then the `if` statement itself is not executable.

- There exists a special guard `else` that becomes executable only if none of the other guards is executable.

- The “`->`” symbol is equivalent to “`;`”. It is used by convention for separating the guards from the instructions.
- There is no need for the guards to be mutually exclusive.
- There is also a special guard `timeout` that only becomes executable if no other process is executable in the current state.

The do statement

Syntax:

```
do
  ::  guard1 -> instructions1;
  ::  guard2 -> instructions2;
   ⋮
od
```

Notes:

- The modalities are similar to those of the `if` statement. The difference is that the `do` statement repeats the operation after each execution of a sequence `guard`i `->` `instructions`i.
- The instruction `break` makes it possible to exit the loop. This instruction is always executable.
- Another possibility of exiting the loop is to use the `goto` instruction.

Atomicity

In Promela, every individual statement is executed atomically. Sequences of operations, however, can be interleaved with operations performed by other processes. There are two ways to modify this default mode of execution:

- The statement

  ```
  atomic { instructions }
  ```

  attempts to execute `instructions` without interleaving operations from other processes.

  Notes:

  - This statement is executable if the first statement of `instructions` is executable.
  - If a subsequent statement of `instructions` becomes blocked, atomicity is lost.

- The statement

  `d_step { instructions }`

  is similar to an `atomic` block, but

  - executes `instructions` in a single step, without generating intermediate states,
  - imposes that the block `instructions` is executed deterministically,
  - does not allow to jump in or out of `instructions`,
  - does not allow any statement inside `instructions` except the first one to become blocked.

Note: Very often, the use of `atomic` and `d_step` blocks makes it possible to greatly reduce the search space.

Example: Simple mutual exclusion

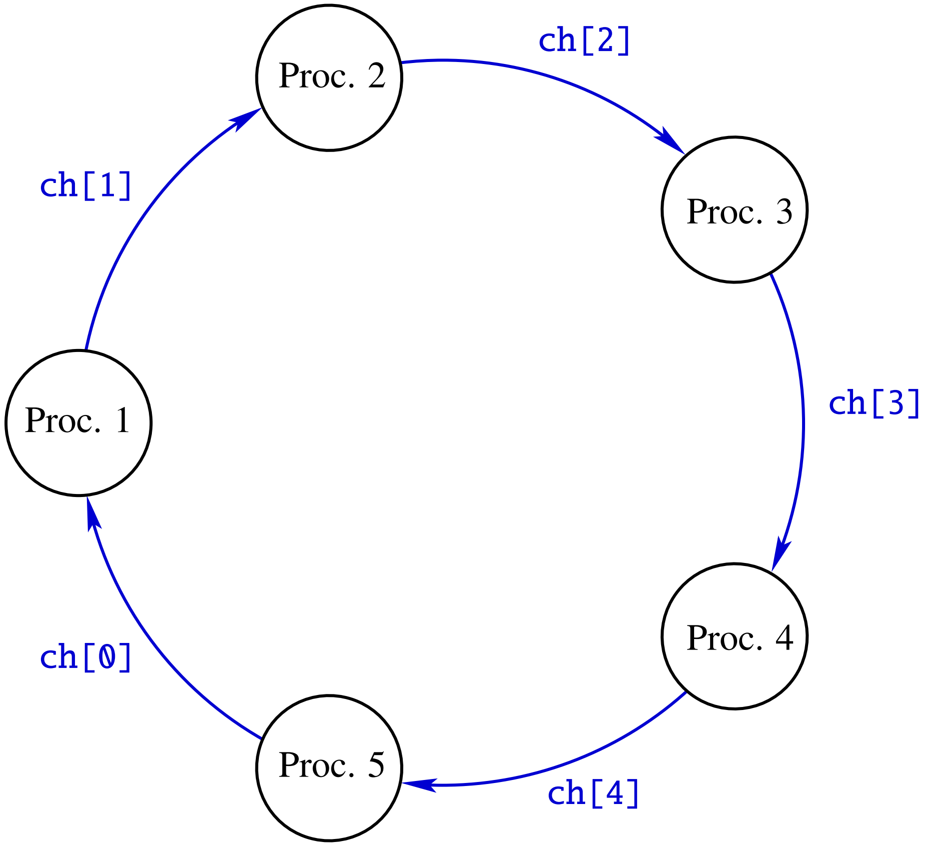
```
int s = 1;
int nb = 0;

proctype p()
{
  do
    :: skip ->
       atomic { s > 0; s-- }
       nb++;
       assert(nb == 1);
       nb--;
       s++
  od
}

init
{
  run p();
  run p();
  run p()
}
```

Modeling LeLann-Chang-Roberts's algorithm

Principles:

- Fixed number of processes, each running the same code.
- The communication channels are stored in a global array.

```
#define NB_NODES   5
#define CAPACITY   5

chan ch[NB_NODES] =
    [CAPACITY] of { byte };
```



Instantiating the processes

Problem: Assigning sequential numbers to the processes (which they need to know to access their input and output channels).

Solution:

```
init
{
  atomic
    {
      byte i = 0;

      do
        :: i < NB_NODES ->
               i++;
               run node(i)

        :: i == NB_NODES -> break
      od
    }
}
```

Assigning process IDs

Problem: How can we make sure that all possible assignments of process IDs are considered?

Solution:

- Let each process choose its own ID non-deterministically.
- Use a global bitfield for enforcing that all process IDs are distinct.

```
bit id_taken[NB_NODES] = 0;

proctype node(byte num)
{
  byte id = 1;

  do
    :: id < NB_NODES -> id++
    :: id > 1 -> id--
    :: atomic { !id_taken[id - 1] -> id_taken[id - 1] = 1 ; break }
  od

  ...
}
```

Specifying the property to verify

Problem: How can we check that the elected leader is always unique?

Solution:

- Define a global variable containing the ID of the elected leader.
- Before assigning a value to this variable, assert that this has not yet been done.

```
byte leader = 0;

proctype node(byte num)
{
  ...

              atomic
                {
                   assert (leader == 0);
                   leader = id
                }
  ...
}
```

LeLann-Chang-Roberts's: Full Promela model

```
#define NB_NODES  5
#define CAPACITY  5

chan ch[NB_NODES] = [CAPACITY] of { byte };
bit id_taken[NB_NODES] = 0;
byte leader = 0;

proctype node(byte num)
{
  byte id = 1;

  do
    :: id < NB_NODES -> id++
    :: id > 1 -> id--
    :: atomic { !id_taken[id - 1] -> id_taken[id - 1] = 1 ; break }
  od

  byte max_id = id;
  byte j;
```

```
  ch[num % NB_NODES] ! id;

  do
    :: ch[num - 1] ? j;
       if
         :: j == id ->
              atomic { assert (leader == 0); leader = id }
              ch[num % NB_NODES] ! id;
              ch[num - 1] ? j;
              goto end
         :: else -> skip
       fi;

       if
         :: j == max_id -> ch[num % NB_NODES] ! j; goto end
         :: else -> skip
       fi;

       if
         :: j > max_id ->
              atomic { max_id = j; ch[num % NB_NODES] ! j }
         :: else -> skip
       fi
  od;

  end: skip
}
```

```
init
{
  atomic
    {
      byte i = 0;

      do
        :: i < NB_NODES ->
             i++;
             run node(i)

        :: i == NB_NODES -> break
      od
    }
}
```

Experiments with Spin

NB_NODES = 5:

```
State-vector 112 byte, depth reached 4768, errors: 0
  2063539 states, stored
  2615680 states, matched
  4679219 transitions (= stored+matched)
   129362 atomic steps
hash conflicts:    151699 (resolved)

Stats on memory usage (in Megabytes):
  275.512 equivalent memory usage for states (stored*(State-vector + overhead))
  220.768 actual memory usage for states (compression: 80.13%)
          state-vector as stored = 84 byte + 28 byte overhead
  128.000 memory used for hash table (-w24)
    0.534 memory used for DFS stack (-m10000)
  349.140 total actual memory usage

pan: elapsed time 0.89 seconds
No errors found -- did you verify all claims?
```

NB_NODES = 6:

(Note: The amount of usable memory and the maximum search depth have to be increased from the default values.)

```
State-vector 128 byte, depth reached 66116, errors: 0
 95050831 states, stored
1.3719902e+08 states, matched
2.3224985e+08 transitions (= stored+matched)
  6260876 atomic steps
hash conflicts: 1.30046e+08 (resolved)

Stats on memory usage (in Megabytes):
14141.016 equivalent memory usage for states (stored*(State-vector + overhead))
11715.116 actual memory usage for states (compression: 82.84%)
          state-vector as stored = 101 byte + 28 byte overhead
  512.000 memory used for hash table (-w26)
    5.341 memory used for DFS stack (-m100000)
12232.352 total actual memory usage

pan: elapsed time 78.2 seconds
No errors found -- did you verify all claims?
```

With collapse compression:

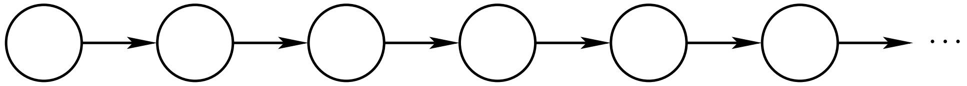
```
State-vector 128 byte, depth reached 66116, errors: 0
 95050831 states, stored
1.3719902e+08 states, matched
2.3224985e+08 transitions (= stored+matched)
  6260876 atomic steps
hash conflicts: 1.0634934e+08 (resolved)

Stats on memory usage (in Megabytes):
14141.016 equivalent memory usage for states (stored*(State-vector + overhead))
 4493.540 actual memory usage for states (compression: 31.78%)
          state-vector as stored = 22 byte + 28 byte overhead
  512.000 memory used for hash table (-w26)
    5.341 memory used for DFS stack (-m100000)
 5009.989 total actual memory usage

pan: elapsed time 103 seconds
No errors found -- did you verify all claims?
```

Note: Our model is far from being optimal! For instance, the state space could be reduced by a factor equal to NB_NODES by exploiting symmetry: the ID of the first process could be set to 1 without loss of generality.

Beyond reachability properties

To specify liveness properties, one uses temporal logics, which express properties of infinite sequences of states.

- They extend propositional or first-order logic.
- In this introduction, we present Linear-time Temporal Logic (LTL) on discrete time.
  - A LTL formula is interpreted on states belonging to a sequence:

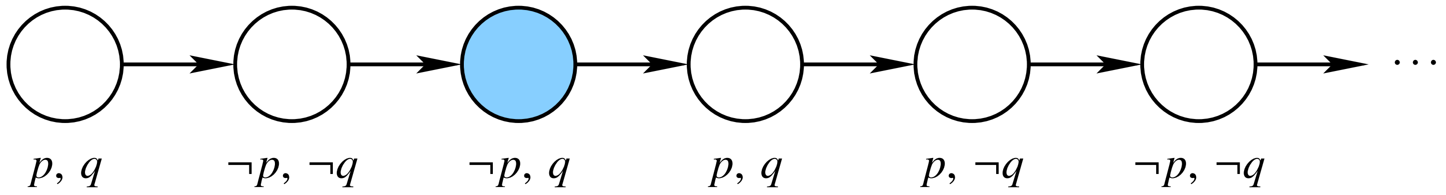


  - Each state assigns a truth value to atomic propositions.
  - Temporal operators indicate in which states the components of formulas should be interpreted.

Some temporal operators

- $\bigcirc \varphi$ (Next): φ is true in the next state of the sequence.
- $\Box \varphi$ (Always): φ is true in the current and all future states of the sequence.
- $\Diamond \varphi$ (Eventually): φ is true in the current or some future state of the sequence.
- $\varphi_1 \, U \, \varphi_2$ (Until): φ_1 is true in the current and all future states until φ_2 becomes true, which must occur.

Examples



In the colored state, the following formulas are true:

- $\neg p \wedge q$
- $\bigcirc(p \wedge q)$
- $\Diamond \neg q$

In that state, the following formulas are false:

- $\Box q$
- $\bigcirc \Box p$

More examples

- $\Box(p \Rightarrow \bigcirc q)$ is satisfied in all states where, for this state and all future ones, each state in which p is true is immediately followed by a state in which q is true.

- $\Box \Diamond p$ is satisfied in all states for which p is true infinitely often in the future.

- $\Diamond \Box p$ is satisfied in all states for which, from some point on in the future, p becomes and remains true.

Verifying LTL properties with Spin

Goal: Checking that LeLann-Chang-Roberts's algorithm always manages to elect a leader.

Solution: Add the following LTL property to the model:

```
ltl { eventually (leader != 0) }
```

Notes:

- To be able to verify LTL properties, the options "acceptance cycles" and "use claim" of Spin must be selected.
- For technical reasons, the temporal operator ○ cannot be used with partial-order reductions.

Result

```
Warning:  Search not completed

State-vector 120 byte, depth reached 23, errors:  1
        6 states, stored
        0 states, matched
        6 transitions (= stored+matched)
       12 atomic steps
hash conflicts:         0 (resolved)

Stats on memory usage (in Megabytes):
    0.001 equivalent memory usage for states (stored*(State-vector + overhead))
    0.286 actual memory usage for states
  128.000 memory used for hash table (-w24)
    5.341 memory used for DFS stack (-m100000)
  133.536 total actual memory usage

pan: elapsed time 0 seconds
To replay the error-trail, goto Simulate/Replay and select "Run"
```

→ Spin finds an execution that violates the LTL property.

Indeed, our mechanism for selecting node IDs allows infinite executions in which such IDs are never assigned:

```
bit id_taken[NB_NODES] = 0;

proctype node(byte num)
{
  byte id = 1;

  do
    :: id < NB_NODES -> id++
    :: id > 1 -> id--
    :: atomic { !id_taken[id - 1] -> id_taken[id - 1] = 1 ; break }
  od

  ...
}
```

Solution:

- Add a counter keeping track of how many IDs have been assigned:

```
bit id_taken[NB_NODES] = 0;
byte nb_ids = 0;

proctype node(byte num)
{
  byte id = 1;

  do
    :: id < NB_NODES -> id++
    :: id > 1 -> id--
    :: atomic { !id_taken[id - 1] -> id_taken[id - 1] = 1 ; nb_ids++; break }
  od

  ...
}
```

- Modify the LTL property accordingly:

```
ltl { always ((nb_ids == NB_NODES) -> eventually (leader != 0)) }
```

Result

```
State-vector 120 byte, depth reached 11282, errors: 0
 15528757 states, stored (2.16152e+07 visited)
1.0021168e+08 states, matched
1.2182692e+08 transitions (= visited+matched)
  7538332 atomic steps
hash conflicts:  25327747 (resolved)

Stats on memory usage (in Megabytes):
 2191.788 equivalent memory usage for states (stored*(State-vector + overhead))
 1659.886 actual memory usage for states (compression: 75.73%)
          state-vector as stored = 84 byte + 28 byte overhead
  128.000 memory used for hash table (-w24)
    5.341 memory used for DFS stack (-m100000)
 1792.618 total actual memory usage

pan: elapsed time 22.6 seconds
No errors found -- did you verify all claims?
```

→ Now the LTL property is validated.

Further exercises

- In his lectures, Pascal Fontaine asked a few questions about LeLann-Chang-Roberts's algorithm. Can you answer them using Spin?

```
    process $i$
1       is_leader $\leftarrow \bot$
2       max_id $\leftarrow$ id$_i$
3       next!id$_i$
4       while $\top$ do
5           prev?$j$
6           if $j =$ id$_i$ then
7               is_leader $\leftarrow \top$
8               next!id$_i$
9               prev?$j$
10              return
11          if $j =$ max_id then
12              next!$j$
13              return
14          if $j >$ max_id then
15              max_id $\leftarrow j$
16              next!$j$
```

What happens if:

- lines 11-13 are swapped with 14-16?
- line 9 is removed?
- line 12 is removed?

- Can you harness the search power of Spin to solve the following puzzle (Rush Hour)?

(Source: Samuel Hiard)

Hints:

- Each car can be modeled by a separate process, trying non-deterministically and repeatedly all possible moves.

- The simplest approach is to define one `proctype` for each car type.

- Two-dimensional arrays cannot be directly defined in Promela. To represent the free cells on the grid, you can
  - define a `typedef` corresponding to a row (as an array of Booleans), and
  - represent the grid as an array of rows.

- You will probably have to modify the default parameters (such as the maximum exploration depth).

Other approaches to state-space exploration

- Instead of exploring states one at a time, symbolic state-space exploration techniques handle sets of reachable states, represented using dedicated data structures.

  This makes it possible to explore very large, and even infinite, state spaces.

- Bounded model checking proceeds by setting a bound on the exploration depth, which makes the model finite-state, and increasing this bound until a bug is found.
- Counter-Example-Guided Abstraction Refinement (CEGAR) starts from a simple model that over-approximates the behaviors of the analyzed system. When a bug is found, the error trace is replayed against the actual system and, if needed, the abstraction is iteratively refined, until either it is validated, or a real bug is found.
- There exist modeling formalisms (and associated exploration algorithms) for dealing with timed and hybrid systems.
- . . .