Semantic Data

Practice 3 : Description Logics

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Types of exercises

1. Translate from description logic into NL and in FOL.
2. Expressing concepts in description logics.
3. Interpretations / models.

- The example about animals used in the tutorial is based on the following sources:
  - A tutorial from the course *IFT6282 Web Sémantique*, Lapalme, University of Montreal.
  - Using itself a small ontology from *(Antoniou and van Harmelen 2004)*.

1. Translate DL into Natural Language and into FOL

Exercise 1 : translate the following concept descriptions in NL and in FOL.

Concepts

a) $Person \sqcap \neg Woman$

b) $Woman \sqcap \exists hasChild.Person$

c) $Mother \sqcap \forall hasChild.\neg Woman$

a) $Person(x) \land \neg Woman(x)$ (person who is not a woman –male)

b) $Woman(x) \land \exists y(hasChild(x, y) \land Person(y))$
(woman who has at least one child – mother)

c) $Mother(x) \land \forall y(hasChild(x, y) \rightarrow \neg Woman(y))$
(mother without daughter)

Axioms

d) $Person \subseteq \neg Plant$

e) $Parent \equiv Father \sqcup Mother$

f) $MotherWithoutDaughter \equiv Mother \sqcap \forall hasChild.\neg Woman$

d) $\forall x\,(Person(x) \rightarrow \neg Plant(x))$

e) $\forall x\,(Parent(x) \leftrightarrow (Father(x) \lor Mother(x)))$

f) $\forall x\,(MotherWithoutDaughter(x) \leftrightarrow (Mother(x) \land \forall y\,(hasChild(x, y) \rightarrow \neg Woman(y))))$

1. Translate DL into Natural Language and into FOL

Exercise 2 : translate the following concept descriptions in NL and in FOL.

a) $Father \sqcap \forall child.(Rich \sqcup Famous)$

b) $\exists works\_for.(Company \sqcap \exists has.Cafeteria)$

c) $Father \sqcap \forall child.(Doctor \sqcup \exists manages.(Company \sqcap \exists employs.Doctor))$

a) Fathers whose children are either rich or famous.

$Father(x) \land \forall y\, (child(x, y) \rightarrow (Rich(y) \lor Famous(y)))$

b) Those who work for a company that has (at least) a cafeteria.

$\exists y\, (works\_for(x, y) \land Company(y) \land \exists z\, (has(y, z) \land Cafeteria(z)))$

c) Fathers whose children are either doctors or manage a company which employs a doctor.

$Father(x) \land \forall y\, (child(x, y) \rightarrow (Doctor(y) \lor \exists z\, (manages(y, z) \land Company(z) \land \exists u\, (employs(z, u) \land Doctor(u)))))$

1. Translate DL into FOL

- **Suggested exercise 1** : look at the exercises 1, 2 and 3 i, j, k of the section 2 : "*Expressing concepts in description logics*", and try to express the same concepts in FOL.

2. Expressing concepts in description logics

- **Exercise 1:** in description logics $\mathcal{ALC}$ using concepts *Person*, *Happy*, *Pet*, *Cat*, *Old*, *Fish*, and a role *owns*, define the following concepts :

a) happy person
b) happy pet owner
c) person who owns only cats
d) unhappy person who owns an old cat
e) pet owner who only owns cats and fish

a) $\text{Person} \sqcap \text{Happy}$
b) $\text{Person} \sqcap \text{Happy} \sqcap \exists \text{owns.Pet}$
c) $\text{Person} \sqcap \forall \text{owns.Cat}$
d) $\text{Person} \sqcap \neg\text{Happy} \sqcap \exists \text{owns.}(\text{Cat} \sqcap \text{Old})$
e) $\text{Person} \sqcap \exists \text{owns.Pet} \sqcap \forall \text{owns.}(\text{Cat} \sqcup \text{Fish})$
(note the use of $\sqcup$ *!).*

(From An introduction to Description Logic, Baader at al. 2017, exercises chapter 2)

2. Expressing concepts in description logics

- **Exercise 2 :** in description logics : using concepts highlighted *in italic* and roles *partOf*, *eats*, *isEatenBy*, define the following concepts (this exercise will be reused in practice 4 *Ontology editing*) :

a) A *Tree* is a subclass of *Plant*.
b) A *Branch* is (only) part of a *Tree*.
c) A *Leaf* is (only) part of a *Branch*.
d) An *Herbivore* is exactly an *Animal* eating only *Plants* or part of *Plants*.
e) A *Carnivore* is exactly an *Animal* eating *Animals*.
f) A *Giraffe* is an *Herbivore* and it is eating only *Leaves*.
g) A *Lion* is a *Carnivore* eating only *Herbivores*.
h) A *TastyPlant* is a *Plant* eaten by *Herbivores* and by *Carnivores*.
i) *eats* is the inverse of *isEatenBy*.
j) indicate which of the above cannot be expressed in $\mathcal{ALC}$?

a) $\text{Tree} \subseteq \text{Plant}$.
b) $\text{Branch} \subseteq \forall \text{partOf}.\text{Tree}$
c) $\text{Leaf} \subseteq \forall \text{partOf}.\text{Branch}$
d) $\text{Herbivore} \equiv \text{Animal} \sqcap \forall \text{eats}.(\text{Plant} \sqcup \forall \text{partOf}.\text{Plant})$
e) $\text{Carnivore} \equiv \text{Animal} \sqcap \exists \text{eats}.\text{Animal}$
f) $\text{Giraffe} \subseteq \text{Herbivore} \sqcap \forall \text{eats}.\text{Leaf}$
g) $\text{Lion} \subseteq \text{Carnivore} \sqcap \forall \text{eats}.\text{Herbivore}$
h) $\text{TastyPlant} \subseteq \text{Plant} \sqcap \exists \text{isEatenBy}.\text{Herbivore} \sqcap \exists \text{isEatenBy}.\text{Carnivore}$
i) $\text{Eats} \equiv \text{isEatenBy}^-$
j) i) is not in $\mathcal{ALC}$ but in $\mathcal{ALCI}$

2. Expressing concepts in description logics

- **Exercise 3 :** Build an ALC knowledge base: capture each of the following statements in a suitable GCI, equivalence axioms, or assertion, using only the concept names *Vehicle, Boat, Bicycle, Car, Device, Wheel, Engine, Axle , Rotation, Water Human, Driver, Adult, Child* and the role names *hasPart, poweredBy, capableOf , travelsOn, controls*. A driver is not necessarily a car driver.

a) Cars are exactly those vehicles that have wheels and are powered by an engine.
b) Bicycles are exactly those vehicles that have wheels and are powered by a human.
c) Boats have no wheels.
d) Cars and bicycles do not travel on water.
e) Wheels are exactly those devices that have an axle and are capable of rotation.
f) Drivers of cars are adults.
g) Humans are not vehicles.
h) Humans are either adults or children.
i) Bob controls QE2.
j) Bob controls a car.
k) QE2 is a vehicle that travels on water.

a) $\text{Car} \equiv \text{Vehicle} \sqcap \exists \text{hasPart.Wheel} \sqcap \exists \text{poweredBy.Engine}$
b) $\text{Bicycle} \equiv \text{Vehicle} \sqcap \exists \text{hasPart.Wheel} \sqcap \exists \text{poweredBy.Human}$
c) $\text{Boat} \sqsubseteq \forall \text{hasPart.}\neg\text{Wheel}$
d) $\text{Car} \sqcup \text{Bicycle} \sqsubseteq \forall \text{travelsOn.}\neg\text{Water}$
e) $\text{Wheel} \equiv \text{Device} \sqcap \exists \text{hasPart.Axle} \sqcap \exists \text{capableOf.Rotation}$
f) $\text{Driver} \sqcap \exists \text{controls.Car} \sqsubseteq \text{Adult}$
g) $\text{Human} \sqsubseteq \neg\text{Vehicle}$
h) $\text{Human} \sqsubseteq \text{Adult} \sqcup \text{Child}$
i) (Bob, QE2) : controls
j) Bob : ($\exists$controls.Car)
k) QE2 : (Vehicle $\sqcap$ $\exists$travelsOn.Water)

(From An introduction to Description Logic, Baader at al. 2017, exercises chapter 2)

2. Expressing concepts in description logics

Exercise 4 :

i. Which of the following statements belong to the TBox, and which to the ABOX ?
ii. Which statements are equivalence axioms, GCIs ?
iii. Which statements are concept assertions, role assertions?
iv. Is the TBox of the knowledge base an acyclic terminology?

a) $\text{Car} \equiv \text{Vehicle} \sqcap \exists\text{hasPart.Wheel} \sqcap \exists\text{poweredBy.Engine}$
b) $\text{Bicycle} \equiv \text{Vehicle} \sqcap \exists\text{hasPart.Wheel} \sqcap \exists\text{poweredBy.Human}$
c) $\text{Boat} \sqsubseteq \forall\text{hasPart.}\neg\text{Wheel}$
d) $\text{Car} \sqcup \text{Bicycle} \sqsubseteq \forall\text{travelsOn.}\neg\text{Water}$
e) $\text{Wheel} \equiv \text{Device} \sqcap \exists\text{hasPart.Axle} \sqcap \exists\text{capableOf.Rotation}$
f) $\text{Driver} \sqcap \exists\text{controls.Car} \sqsubseteq \text{Adult}$
g) $\text{Human} \sqsubseteq \neg\text{Vehicle}$
h) $\text{Human} \sqsubseteq \text{Adult} \sqcup \text{Child}$
i) $(\text{Bob}, \text{QE2}) : \text{controls}$
j) $\text{Bob} : (\exists\text{controls.Car})$
k) $\text{QE2} : (\text{Vehicle} \sqcap \exists\text{travelsOn.Water})$

Solution :

i. The TBox of the knowledge base are the statements (a) to (h); the statements (i) to (k) constitutes its ABox.

ii. The statements (a), (b), (e) are equivalence axioms, and statements (c), (d), (f), (g), (h) are GCIs.

iii. The statements (j) and (k) are concept assertions, and statement (i) is a role assertion.

iv. The TBox of our knowledge base is not an acyclic terminology : it contains GCIs, and in particular GCIs (d) and (f) with compound concept descriptions on their left hand side.

(From An introduction to Description Logic, Baader at al. 2017, exercises chapter 2)

Suggested exercises

- **Suggested exercise 1 :** in description logics $\mathcal{ALCN}$ with concepts *Male*, *Female*, and a role *hasChild* define the following concepts :

a) Person
b) Mother
c) Parent
d) Childless
e) Grandfather
f) ParentOfSons (a parent with at least one son)
g) ParentOfOnlySons
h) MotherWithManyChildren (a mother with more than three children)
i) GrandfatherOfOnlyGrandsons

- **Suggested exercise 2 :**

a) in description logic $\mathcal{ALC}$ using concepts *Male*, *Doctor*, *Rich*, *Famous* and roles *hasChild*, *hasFriend*, define a concept HappyFather being a *father whose all children are doctors and have rich or famous friends*.

b) in description logic $\mathcal{ALC}$ using concepts *Female*, *Diplomat*, *StudyingAtUniversity*, *Working* and roles *married*, *hasChild*, define a concept SuccessfulMan being a *man who married a diplomat and who has a child at university or having a job*.

3. Interpretations / models

- **Exercise 1** : Given the following TBox $\mathcal{T}$:

  $A \sqsubseteq B$
  $B \sqsubseteq C$
  $C \sqsubseteq \exists r.D$
  $D \sqsubseteq \neg A$

  1. Tell whether the TBox $\mathcal{T}$ is satisfiable, and if so, show a model for $\mathcal{T}$;
  2. Tell whether the concept D is satisfiable with respect to $\mathcal{T}$, and if so, show a model for $\mathcal{T}$ where the interpretation of D is non-empty;
  3. Tell whether the concept expression $A \sqcap D$ is satisfiable with respect to $\mathcal{T}$, and if so, show a model for $\mathcal{T}$ where the interpretation of $A \sqcap D$ is non-empty.

(after an exercise from R. Rosati, University di Roma)

- SOLUTION

1. Let $\mathcal{I}$ be the interpretation over the domain $\Delta^{\mathcal{I}} = \{d\}$ such that $A^{\mathcal{I}} = B^{\mathcal{I}} = C^{\mathcal{I}} = D^{\mathcal{I}} = r^{\mathcal{I}} = \emptyset$.

   All the axioms of $\mathcal{T}$ are satisfied in $\mathcal{I}$: e.g., since $A^{\mathcal{I}}$ is empty, it is obviously true that $A^{\mathcal{I}} \subseteq B^{\mathcal{I}}$. Hence $\mathcal{I}$ is a model for $\mathcal{T}$.

2. We define a new interpretation $\mathcal{J}$, over the domain $\Delta^{\mathcal{J}} = \{d\}$, such that $A^{\mathcal{J}} = B^{\mathcal{J}} = C^{\mathcal{J}} = r^{\mathcal{J}} = \emptyset$ and $D^{\mathcal{J}} = \{d\}$.

   All axioms of $\mathcal{T}$ are satisfied in $\mathcal{J}$. In particular, $D \sqsubseteq \neg A$ is satisfied since $(\neg A)^{\mathcal{J}} = \Delta^{\mathcal{J}} = \{d\}$. Hence, $\mathcal{J}$ is a model for $\mathcal{T}$.

3. Since the TBox $\mathcal{T}$ contains the axiom $D \sqsubseteq \neg A$, it follows that every model $\mathcal{I}$ for $\mathcal{T}$ is such that $D^{\mathcal{I}} \subseteq (\neg A)^{\mathcal{I}}$, or $D^{\mathcal{I}} \cap A^{\mathcal{I}} = \emptyset$.

   Hence no model $\mathcal{I}$ for $\mathcal{T}$ exists such that $(A \sqcap D)^{\leftarrow}$ is non empty.

3. Interpretations / models

Exercise 2 : given the interpretation below and its graphical representation,

Interpretation :

Δ^I = {John, David, Fred, Maths, ML}

Teacher = {John, David}

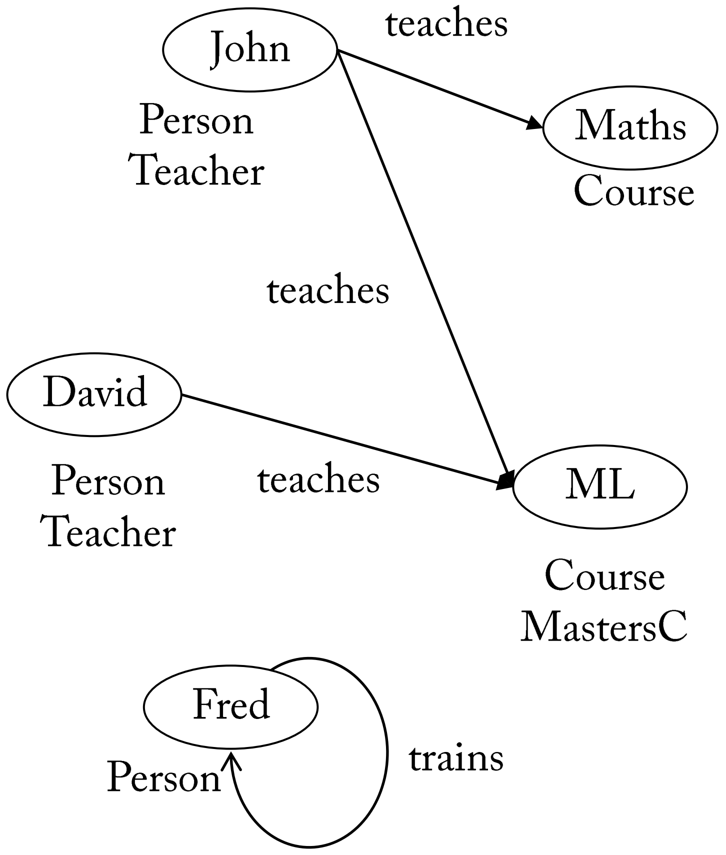
Person = {John, David, Fred}

Course = {Maths, ML}

teaches = {(John, Maths), (John, ML), (David, ML)}

trains = {(Fred, Fred)}

MastersC = {ML}



Is Math in the extension of Course ⊓ ¬Person ?

Answer : yes.

Is John in the extension of Person ⊓ ∃teaches.(Course ⊓ ¬MasterC) ?

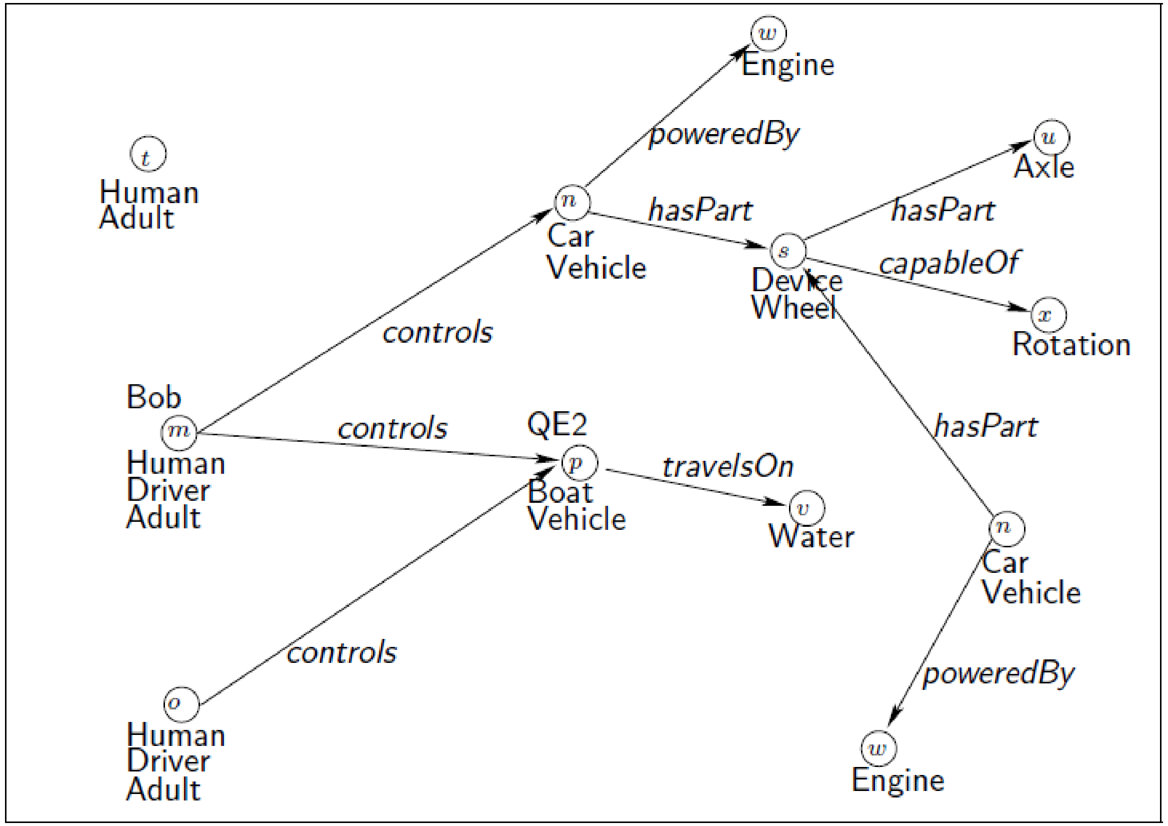
Answer : yes.

(after Baader et al. 2017)

3. Interpretations / models

Suggested exercise 1 : is the following interpretation a model of the knowledge base from section 2 – exercise 4, repeated below ? Justify.

a) Car ≡ Vehicle ⊓ ∃hasPart.Wheel ⊓ ∃poweredBy.Engine
b) Bicycle ≡ Vehicle ⊓ ∃hasPart.Wheel ⊓ ∃poweredBy.Human
c) Boat ⊆ ∀hasPart.¬Wheel
d) Car ⊔ Bicycle ⊆ ∀travelsOn.¬Water
e) Wheel ≡ Device ⊓ ∃hasPart.Axle ⊓ ∃capableOf.Rotation
f) Driver ⊓ ∃controls.Car ⊆ Adult
g) Human ⊆ ¬Vehicle
h) Human ⊆ Adult ⊔ Child
i) (Bob, QE2) : controls
j) Bob : (∃controls.Car)
k) QE2 : (Vehicle ⊓ ∃travelsOn.Water)



(after Baader et al. 2017)

3. Interpretations / models

- **Exercise 3** : which concept is satisfiable ?

a) $A \sqcap \neg A$
b) $A \sqcup \neg A$
c) $A \sqcap \exists r.B \sqcap \exists r.\neg B$
d) $A \sqcap \exists r.B \sqcap \forall s.\neg B$
e) $A \sqcap \exists r.B \sqcap \forall r.\neg B$
f) $A \sqcap \exists r.B \sqcap \forall r.(\neg B \sqcup \exists r.A)$
g) $A \sqcap \exists r.(B \sqcap C) \sqcap \forall r.\neg B$

a) No
b) Yes
c) Yes
d) Yes
e) No
f) Yes
g) No

(after Baader et al. 2017)

3. Interpretations / Models

Suggested exercise 2 : consider the following interpretation :

$\mathcal{I}$: $\Delta^{\mathcal{I}}$ = {t1, t2, f1, f2, p1, p2, j, k, l, m, n}
$Animal^{\mathcal{I}}$ = {j, k, l, m, n}
$Plant^{\mathcal{I}}$ = {t1, t2, f1, f2, p1, p2}
$Fern^{\mathcal{I}}$ = {f1, f2}
$Tulip^{\mathcal{I}}$ = {t1, t2}
$eats^{\mathcal{I}}$ = {(j, f1), (k, f1), (k, t2), (l, p1), (l, p2), (m, p1), (m, t2), (n, f2), (n, p2)}

Find the interpretation in $\mathcal{I}$ of the following concepts:

a) $\exists eats.Fern \sqcap \exists eats.Tulip$

b) $\exists eats.Fern \sqcap \forall eats.Fern$

c) $\exists eats.Fern \sqcap \exists eats.\neg Fern$

d) $\exists eats.Plant \sqcap \forall eats.\neg(Tulip \sqcup Fern)$

3. Interpretations / Models

- **Exercise 4 :** suppose *Manager* and *Project* are concept names and *manages* is a role name. For each of the following expressions φ:

i. state whether it is an $\mathcal{ALC}$ concept, an $\mathcal{ALC}$ concept inclusion or none of the above.

ii. if φ is a concept inclusion, check whether φ follows from the empty TBox (i.e., $\emptyset \models \varphi$)(*). If this is not the case, define an interpretation $\mathcal{I}$ such that $\mathcal{I} \not\models \varphi$.

iii. if φ is a concept, check whether φ is satisfiable. If so, define an interpretation $\mathcal{I}$ such that $\varphi^{\mathcal{I}} \neq \emptyset$.

(: A GCI follows from the empty TBox if it is true under all interpretations (valid))*

a) $\top \sqsubseteq \bot$

b) ∃manages.Project ⊑ Manager

c) ∀manages.Project

d) ∃Project.manages

a) This says the domain is empty. i) concept inclusion.
ii) does not follow from the empty TBox as any $\mathcal{I}$ is such that $\Delta^{\mathcal{I}} \neq \emptyset$.

b) i) concept inclusion. ii) does not follow from the empty TBox :
$\Delta^{\mathcal{I}}$={a}; manages$^{\mathcal{I}}$ = {(a,a)}; Project$^{\mathcal{I}}$ = {a}; Manager$^{\mathcal{I}}$ = ∅.

c) i) $\mathcal{ALC}$ concept. iii) satisfiable : $\Delta^{\mathcal{I}}$ = {a, b}; manages$^{\mathcal{I}}$ = {(a, b)}; Project$^{\mathcal{I}}$ = {b}.

d) i) not syntactically correct (concept name after ∃).

(after an exercise from F. Wolter, University of Liverpool)

3. Interpretations / Models

- **Suggested exercise 3 :** Suppose *Manager* and *Project* are concept names and *manages* is a role name. For each of the following expressions φ:

i. state whether it is an $\mathcal{ALC}$ concept, an $\mathcal{ALC}$ concept inclusion or none of the above.

ii. if φ is a concept inclusion, check whether φ follows from the empty TBox (i.e., is valid, $\emptyset \models \varphi$). If this is not the case, define an interpretation $\mathcal{I}$ such that $\mathcal{I} \not\models \varphi$.

iii. if φ is a concept, check whether φ is satisfiable. If so, define an interpretation $\mathcal{I}$ such that $\varphi^{\mathcal{I}} \neq \emptyset$.

a) $Manager \sqsubseteq \exists\, manages.\bot$

b) $(\geq 7\ manages.\top) \sqsubseteq Manager$

c) $(\geq 8\ manages.Project) \sqsubseteq Manager$

d) $\forall manages.\top \ \sqsubseteq \exists\, manages.Project$

e) $\exists manages.\top \sqsubseteq (\geq 4\ manages.\top)$

f) $(\geq 4\ manages.\top) \sqsubseteq \exists manages.\top$

(after an exercise from F. Wolter, University of Liverpool)

THANK YOU