Exercise session 8. Integration

2015-2016

Question 1

1. Find an integration formula for $\int_0^1 f(x)dx$ which uses $f(0)$, $f(\frac{1}{2})$ and $f(1)$.
2. With $\int_0^1 e^x dx$, compare with the composite trapezes (use the same points).
3. Perform the Romberg algorithm with $h = 1, \frac{1}{2}, \frac{1}{4}$.

Question 2

(*January 2009*) Consider a symmetric ellipse with respect to both axes, as illustrated on Figure 1. We know three points $(0,1), (\sqrt{3}, \frac{1}{2}), (2,0)$. Find the formula to integrate any function with three points, then compute the area of the ellipse.

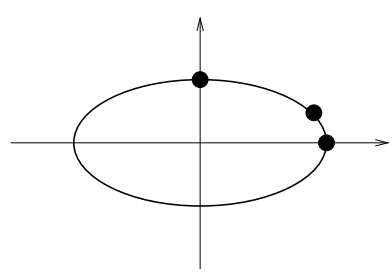

FIGURE 1 – Ellipse

Question 3

(*January 2010*) One wants to know a patient's average blood pressure during an operation. However, the pressure can only be measured at specific times. The measurements for a five-hour long operation are in the following table.

Time after the beginning of the operation	1 hour	3 hours	4 hours
Blood pressure (mm Hg)	130	140	150

(a) Compute the average blood pressure.

(b) With 3 measurements and a five-hour long operation, use the Gaussian quadrature rule to determine the 3 points leading to the most precise measurement of the average blood pressure.

(c) Obtain those three points with known roots of Legendre polynomials.

Additional exercises

Question 4

A radar measures the instantaneous speed of a car at time $0 : 20m/s$. Another radar 250 metres away measures the speed of the same car at time $t = 10\ sec$: $20m/s$.

1. If we suppose that the maximum speed of the car between 0 and 10 sec is achieved at time $t = 5\ sec$, find an approximation of the speed with Simpson's rule.
2. If we suppose that the maximum acceleration of the car is $3m/s$ in absolute value, find a piecewise linear distribution of the speed which minimizes the maximum speed.

Question 5

(*January 2008*) The Royal Meteorological Institute wishes to evaluate the average daily temperature from several measurements at different moments of the day.

Hour	0	3	6	9	12	15	18	21	0
Temp ($^\circ$ C)	3	2	1	3	5	5	5	3	1

Use the Romberg algorithm to obtain the most precise measurement possible of the daily average temperature.