Logic

Tutorial 9

12 December 2019

Prenex, Skolem and Clausal forms and Syllogism

1. Give the prenex, Skolem and clausal form of the following formulas:

   - $p(a) \wedge \exists x \neg p(x)$
   - $\forall x \left[p(x) \Rightarrow \forall y \left[\forall z \, q(x,y) \Rightarrow \neg \forall z \, r(y,x) \right] \right]$
   - $\forall x \, p(x) \Rightarrow \exists x \left[\forall z \, q(x,z) \vee \forall z \, r(x,y,z) \right]$
   - $\exists x \, p(x,z) \Rightarrow \forall z \left[\exists y \, p(x,z) \Rightarrow \neg \forall x \exists y \, p(x,y) \right]$
   - $\left[\exists x \, p(x) \vee \exists x \, q(x) \right] \Rightarrow \exists x \left[p(x) \vee q(x) \right]$

2. Determine the predicates and the formulas of the following syllogisms, state their mode and figure. Using a Venn diagram, determine whether these syllogisms are valid, quasi-valid, ...

$$\begin{array}{c} \forall x \, (Q(x) \Rightarrow R(x)) \\ \forall x \, (P(x) \Rightarrow Q(x)) \\ \hline \forall x \, (P(x) \Rightarrow R(x)) \end{array} \qquad \begin{array}{c} \forall x \, (A(x) \Rightarrow B(x)) \\ \exists x \, (B(x) \wedge C(x)) \\ \hline \exists x \, (C(x) \wedge \neg A(x)) \end{array}$$

3. Is the following rule a syllogism? Can it be transformed into a syllogism? Is it correct?

$$\begin{array}{c} \forall x \exists y \left[\neg Q(x,y) \vee R(x) \right] \\ \exists x \forall y \left[P(x) \wedge Q(x,y) \right] \\ \hline \exists y \left[R(y) \wedge P(y) \right] \end{array}$$