

# Logic

## Tutorial 8

07 December 2017

1. What is the link (in terms of logical consequence) between the following couples of formulas?

- (a)  $p(x)$  and  $\forall x p(x)$
- (b)  $p(x)$  and  $\exists x p(x)$
- (c)  $\forall x p(x) \wedge \forall x q(x)$  and  $\forall x [p(x) \wedge q(x)]$
- (d)  $\forall x p(x) \vee \forall x q(x)$  and  $\forall x (p(x) \vee q(x))$
- (e)  $\forall x \forall y p(x, y)$  and  $\forall x \forall y p(y, x)$

2. What is the link between the following formulas?

- (a)  $A \triangleq \forall x P(x) \Rightarrow \forall x Q(x)$
- (b)  $B \triangleq \exists x P(x) \Rightarrow \forall x Q(x)$
- (c)  $C \triangleq \forall x P(x) \Rightarrow \exists x Q(x)$
- (d)  $D \triangleq \forall x [P(x) \Rightarrow Q(x)]$

3. What is the link between the following formulas?

- (a)  $A \triangleq \forall x \exists y [P(x) \Rightarrow Q(x, y)]$
- (b)  $B \triangleq \forall x [P(x) \Rightarrow \exists y Q(x, y)]$
- (c)  $C \triangleq \forall x P(x) \Rightarrow \exists y Q(x, y)$
- (d)  $D \triangleq \forall x [P(x) \Rightarrow \forall y \exists z Q(x, y, z)]$

4. What is the link between the following formulas?

- (a)  $\alpha \triangleq \exists x \exists y \exists z [P(x, y) \Rightarrow [Q(x, z) \Rightarrow R(y, z)]]$
- (b)  $\beta \triangleq \exists x \exists y [P(x, y) \Rightarrow [\forall z Q(x, z) \Rightarrow \exists z R(y, z)]]$
- (c)  $\gamma \triangleq \forall x \forall y P(x, y) \Rightarrow [\forall x \forall z Q(x, z) \Rightarrow \exists y \exists z R(y, z)]$

5. What can you say about the following inference rule?

$$\frac{H \Rightarrow \forall x A(x), H \Rightarrow \exists x [A(x) \Rightarrow \forall y B(x, y)]}{H \Rightarrow \exists x \forall y B(x, y)}$$

6. Using the semantic tableaux method, determine whether the following formulas are valid, consistent or inconsistent.

(a)  $\forall y [p(y) \Rightarrow \forall x p(x)]$

(b)  $\forall x [p(x) \Rightarrow q(x)] \Rightarrow [\forall x p(x) \Rightarrow \forall x q(x)]$

(c)  $[\forall x p(x) \wedge \neg \forall y q(y)] \vee \forall z [p(z) \Rightarrow q(z)]$

(d)  $\forall x \exists y p(x, y) \wedge \forall x \neg p(x, x) \wedge \forall x \forall y \forall z [(p(x, y) \wedge p(y, z)) \Rightarrow p(x, z)]$

7. Consider the following inference rules:

$$\frac{\forall x A \quad \forall x (A \Rightarrow B)}{\forall x B}$$

$$\frac{\exists x A \quad \forall x (A \Rightarrow B)}{\exists x B}$$

$$\frac{\exists x A \quad \exists x (A \Rightarrow B)}{\exists x B}$$

$$\frac{\forall x A \quad \exists x (A \Rightarrow B)}{\exists x B}$$

Are they correct?

If not, do they become correct if one adds restrictions on the occurrences of the variable  $x$  within  $A$  and/or  $B$ ? Motivate your answers.