Linear control systems

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Lecture 9. Robustness and performance

1. How to use the theory of feedback systems in design problems?
2. Sensitivity functions
3. Robustness measures
4. Performance limitations

Robustness

1. the ability of the closed-loop system to be insensitive to component variations
2. what makes possible to design feedback system based on strongly simplified systems
3. Is feedback without robustness relevant?

Robustness is not implied by feedback

Caveat: (nave) pole placement can be highly non robust.

Example 9.2.

$$\begin{aligned} \dot{x}_1 &= -x_1 + (a-1)u \\ \dot{x}_2 &= x_1 + u \\ y &= x_2 \end{aligned} \tag{1}$$

$$G(s) = \frac{s+a}{s(s+1)}$$

System is controllable and observable. Minimum phase. $\Rightarrow$ Easy to control.

A seemingly good design

1. Pole placement: $s^2 + 2\zeta_c\omega_c s + \omega_c^2$

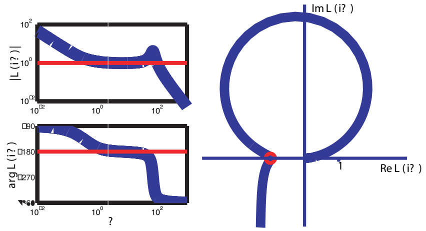
2. Observer design: $s^2 + 2\zeta_o\omega_o s + \omega_o^2$

Choice: $\omega_c = 5$, $\zeta_c = \zeta_o = 0.6$, $\omega_o = 10 = 2\omega_5$.

Observation: 2% process variation causes instability.

What is wrong?

Loop analysis insight

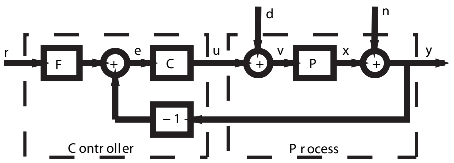


1. no stability margins!

2. pole placement might result in bad stability margins

3. there are ways to guarantee robustness with (advanced) state-space methods

Back to the basic feedback loop



$$Y = \frac{F\,C\,P}{1+PC}\quad R + \frac{P}{1+PC}\quad D + \frac{1}{1+PC}\quad N$$

$$U = \frac{F\,C}{1+PC}\quad R - \frac{C}{1+PC}\quad N + \frac{PC}{1+PC}\quad D$$

Basic specifications

1. Load disturbance rejection $(\frac{Y}{D})$

2. Noise measurement injection $(\frac{U}{N})$

3. Robustness to process variations

4. Response to command signals (tracking) $(\frac{Y}{R})$

Sensitivity functions

$S = \frac{1}{1+PC}$ is the sensitivity function

$T = \frac{PC}{1+PC} = 1 - S$ is the complementary sensitivity function

$\frac{P}{1+PC} = PS = \frac{T}{C}$ is the load sensitivity $(= \frac{Y}{R})$

$\frac{C}{1+PC} = CS = \frac{T}{P}$ is the noise sensitivity $(= \frac{U}{N})$

The gang of six is:

	R	D	N
Y	F T	P S	S
U	F C S	C S	T

Key transfer functions in a two-degree of freedom architecture

Optimize C for the feedback path:

1. S (sensitivity to P variations) and T (=meas. noise to output $\frac{Y}{N}$=load disturbance to control $\frac{-U}{D}$))

2. PS (=disturbance to output $\frac{Y}{D}$) and CS (=meas. noise to control $-\frac{U}{N}$)

Adjust F for the feedforward path:

1. Reference to output: $\frac{Y}{R} = FT$

2. Reference to control value: $\frac{Y}{U} = FCT$

Frequent mistake: Focus on only one transfer function

Example: Cancelling a slow proces pole

$$P(s) = \frac{1}{(s+1)(s+\epsilon)}$$

The PI control $K\frac{s+\epsilon}{s}$ provides the loop transfer $L = \frac{K}{s(s+1)}$

$T(s) = \frac{PC}{1+PC} = \frac{K}{s^2+s+K}$ is "good"

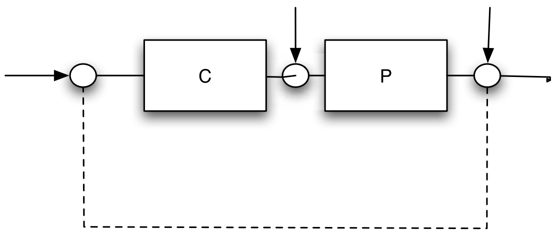
BUT

$\frac{Y}{D} = \frac{T}{C} = \frac{s}{(s+\epsilon)(s^2+s+K)}$ is "bad"

CCL: Plot the step response and frequency response of the six transfer functions!

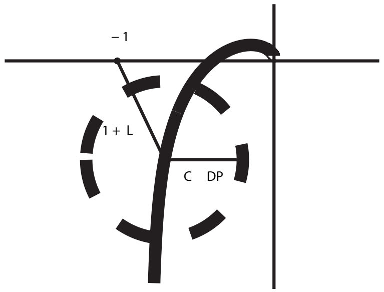
The sensitivity function

$$Y_{cl} = S(s)Y_{ol}$$



Feedback attenuates external signals when $\mid S(j\omega) \mid < 1$ and amplifies signals when $\mid S(j\omega) \mid > 1$

The complementary sensitivity function



Stability if

$$| \frac{\Delta P(j\omega)}{P(j\omega)} |< \frac{1}{| T(j\omega) |}$$

A fundamental trade-off

$$S(s) + T(s) = 1$$

Load disturbance attenuation $\Rightarrow$ $| S(j\omega) | << 1$ at low frequencies

Robustness to unmodelled dynamics $\Rightarrow$ $| T(j\omega) | << 1$ at high frequencies

Robustness measures: sensitivity peaks

Minimize the sensitivity peaks:

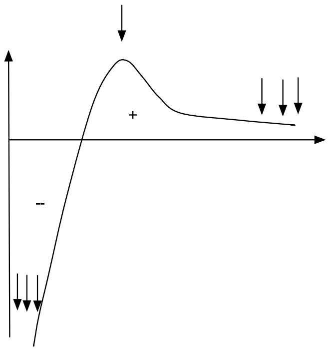
$M_s = \| S \|_\infty = \sup_\omega | S(j\omega) |$ Maximal sensitivity to external disturbance

$M_t = \| T \|_\infty = \sup_\omega | T(j\omega) |$ Maximal sensitivity to process variations

Rule of thumb: it is good to keep $M_t \leq 1.1 - 1.2$ and $M_s \leq 1.1 - 1.2$

Performance limitation

For minimum phase systems: $\int_0^{+\infty} \log | S(j\omega) | d\omega = 0$



Pushing the low frequency spec (for load disturbance attenuation) and the high frequency spec (for robustness against unmodelled dynamics) causes a higher peak of sensitivity : waterbed effect

Consequences for modeling

The sensitivity peaks arise near ω_c. They are related to the stability margins.

1. Good process knowledge is critical around ω_c

2. it is hard to reject external disturbances around ω_c.

Consequences for design

$S = \frac{d_p d_c}{d_p d_c + n_p n_c}$, $\quad T = \frac{n_p n_c}{d_p d_c + n_p n_c}$

$T \approx 1$ at low frequencies and increases for frequencies close to process zeros unless there is a closed-loop pole nearby.

$\Rightarrow$ match slow zeros with slow closed-loop poles

$S \approx 1$ at high frequencies and increases for frequencies close to process poles unless there is a closed-loop pole nearby.

$\Rightarrow$ match fast open-loop poles with fast closed-loop poles

Back to the specifications

1. Load disturbance rejection ($\frac{Y}{D} = PS = \frac{T}{C}$)

2. Noise measurement injection ($\frac{U}{N} = CS = \frac{T}{P}$)

3. Robustness to
   - process variations $\mid \frac{\Delta P(j\omega)}{P(j\omega)} \mid < \frac{1}{|T(j\omega)|} \leq M_t^{-1}$
   - external disturbances $\mid S(j\omega) \mid < M_s$

$+$ Response to signal commands: $\frac{Y}{R} = TF$

Load disturbance attenuation: $\frac{T}{C}$

At low frequencies, $\mid T \mid \approx 1$.

Therefore $\mid \frac{Y}{D} \mid \approx \frac{1}{|C|}$

It the controller contains integral action, then $C(s) = \frac{1}{sT_i} + \dots = \frac{k_i}{s} + \dots$

Consequence:

$$\mid \frac{Y}{D} \mid \approx \frac{\mid \omega \mid}{k_i}$$

Noise measurement attenuation: $\frac{U}{N} = CS$

At high frequencies, $\mid S \mid \approx 1$.

Therefore $\mid \frac{U}{N} \mid \approx \mid C \mid$

For instance, the phase lead $\frac{s+t_d}{1+\frac{st_d}{N}}$ amplifies the noise by a factor N at high frequencies.

Summary of lecture

- A good design of feedback system integrates performance and robustness considerations
- Loop analysis is useful to asses robustness and performance: the gang of six
- the sensitivity functions capture much of the design trade-offs.
- With this material, you should be able to design good controllers for "easy control systems" and to recognize when a model is difficult to control.