ANALYSIS OF FEEDBACK SYSTEMS: THEORY AND COMPUTATION HOMEWORK 1

RODOLPHE SEPULCHRE

(1) Let $H(s)$ be a (SISO) transfer function with no poles in the right half plane and H the (causal) operator associated to $H(s)$.
Show that the $\mathcal{L}_2$-gain of H satisfies

$$\gamma_2 = \sup_{\omega \in \mathbb{R}} |H(j\omega)|$$

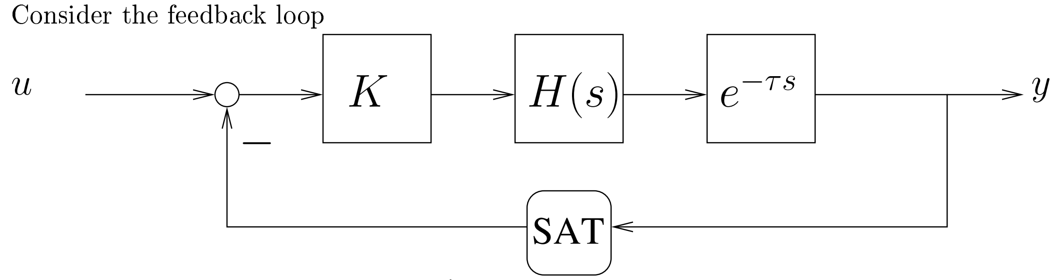
and that the $\mathcal{L}_\infty$-gain of H is

$$\gamma_\infty = \|h\|_1$$

where h is the impulse response associated to $H(s)$. (Hint: Establish first the inequality $\leq$. Then construct a sequence of signals for which the upper bound is attained in the limit).
Estimate the difference between the two gains for $H(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$ as $\zeta \to 0$.

(2) Under the same assumption, consider the feedback interconnection of H with a time-invariant sector nonlinearity $\phi(\cdot) \in$ Sector $[-1, 1]$.
 (a) Derive a stability criterion from the small-gain theorem.
 (b) Derive a stability criterion from the passivity theorem (hint: to reduce the conservatism, consider applying first a loop transformation that maps $\phi(\cdot)$ in the sector $(0, \infty)$).
 (c) Interpret the criteria (a) and (b) as a graphical conditions on the Nyquist curve of H.
 (d) Show the equivalence of the two conditions using the conformal mapping $\frac{z-1}{z+1}$ in the complex plane.

(3) Consider the feedback loop



with $H(s) = \frac{1}{s^2+0.5s+1}$ and $\text{sat}(s) = \begin{cases} s & , -1 \leq s \leq 1 \\ \text{sign}(s) & , |s| > 1 \end{cases}$
 (a) Estimate the maximal gain K_{max} that ensures stability of the feedback system.
 (b) Can you obtain a less conservative estimate if $\tau = 0$ (no delay)?