BEHAVIORAL STATE SYSTEMS

Let \( \Sigma = (\mathbb{R}, \mathcal{W}, \mathcal{X}, \mathcal{B}) \) be a continuous time state system. This means: \( \mathbb{R} = \) time-axis, \( \mathcal{W} = \) space of manifest variables, \( \mathcal{X} = \) state space, \( \mathcal{B} = \) behavior, \( \mathcal{B} \subseteq (\mathbb{W} \times \mathcal{X})^\mathbb{R} \).

External behavior:

\[
\mathcal{B}_{\text{ext}} := \{ w \mid \exists x \text{ such that } (w, x) \in \mathcal{B} \}
\]

\( \sim \quad \Sigma_{\text{ext}} := (\mathbb{R}, \mathcal{W}, \mathcal{B}_{\text{ext}}) \).

In the (limited) classical input/output setting, \( (u, y) = w \).

Assume that \( \Sigma \) is time-invariant, i.e. \( \sigma^t \mathcal{B} = \mathcal{B} \) for all \( t \in \mathbb{R} \), where \( \sigma^t \) denotes the \( t \)-shift, \( (\sigma^t f)(t') := f(t' + t) \).
This state definition is the implementation of the idea:

**The state at time \( t \), \( x(t) \), contains all the information (about \( (w, x) \)!) that is relevant for the future behavior.**

The state = the memory.

The past and the future are ‘independent’, conditioned on (given) the present state.

**Example:** \( \Sigma : \dot{x} = f(x, u), y = h(x, u), w = (u, y) \).
**Basic theorem (general version):** Let $\Sigma$ and $s$ be given. The following are equivalent:

1. $\Sigma$ is dissipative w.r.t. $s$ (i.e. $\exists$ a storage f’n $V$)
2. $\int s(w) \, dt \geq 0$
   for all periodic $(w, x) \in \mathcal{B}$.
3. $V_{\text{available}} < \infty$
4. $V_{\text{required}} > -\infty$

**Proof:**

No changes required from the differential equation case. Verify!

**SYSTEM INTERCONNECTION**

Think of interconnection in terms of physical terminals.

**Before interconnection:**

**Terminal 1**

**Terminal 2**

**after interconnection**

Formalize & prove: interconnection of dissipative systems is dissipative!
Think of interconnection in terms of **physical terminals**.

Variables on such terminals:

<table>
<thead>
<tr>
<th>Type of terminal</th>
<th>Variables</th>
<th>Signal space</th>
</tr>
</thead>
<tbody>
<tr>
<td>electrical</td>
<td>(voltage, current)</td>
<td>$\mathbb{R}^2$</td>
</tr>
<tr>
<td>mechanical (1-D)</td>
<td>(force, position)</td>
<td>$\mathbb{R}^2$</td>
</tr>
<tr>
<td>mechanical (2-D)</td>
<td>((position, attitude),</td>
<td>$(\mathbb{R}^2 \times S^1)$</td>
</tr>
<tr>
<td></td>
<td>(force, torque))</td>
<td>$\times (\mathbb{R}^2 \times T^* S^1)$</td>
</tr>
<tr>
<td>mechanical (3-D)</td>
<td>((position, attitude),</td>
<td>$(\mathbb{R}^2 \times S^2)$</td>
</tr>
<tr>
<td></td>
<td>(force, torque))</td>
<td>$\times (\mathbb{R}^2 \times T^* S^2)$</td>
</tr>
<tr>
<td>thermal</td>
<td>(temp., heat flow)</td>
<td>$\mathbb{R}^2$</td>
</tr>
<tr>
<td>fluidic</td>
<td>(pressure, flow)</td>
<td>$\mathbb{R}^2$</td>
</tr>
<tr>
<td>thermal - fluidic</td>
<td>(pressure, temp., mass flow, heat flow)</td>
<td>$\mathbb{R}^4$</td>
</tr>
</tbody>
</table>

Formalization of interconnection. (Also) this is (by far) easiest in the behavioral setting.

We proceed as if we want to interconnect two terminals of **one and the same** system. It is easy to see that this covers the general situation, even when interconnecting many terminals of many different systems.

Think of interconnection in terms of **physical terminals**.

Imposes laws on the variables that ‘live’ on the terminals.

<table>
<thead>
<tr>
<th>Pair of terminals</th>
<th>Terminal 1</th>
<th>Terminal 2</th>
<th>Law</th>
</tr>
</thead>
<tbody>
<tr>
<td>electrical</td>
<td>$(V_1, I_1)$</td>
<td>$(V_2, I_2)$</td>
<td>$V_1 = V_2, I_1 + I_2 = 0$</td>
</tr>
<tr>
<td>1-D mech.</td>
<td>$(F_1, q_1)$</td>
<td>$(F_2, q_2)$</td>
<td>$F_1 + F_2 = 0, q_1 = q_2$</td>
</tr>
<tr>
<td>2-D mech.</td>
<td>$(Q_1, T_1)$</td>
<td>$(Q_2, T_2)$</td>
<td>$Q_1 + Q_2 = 0, T_1 = T_2$</td>
</tr>
<tr>
<td>thermal</td>
<td>$(p_1, f_1)$</td>
<td>$(p_2, f_2)$</td>
<td>$p_1 = p_2, f_1 + f_2 = 0$</td>
</tr>
<tr>
<td>fluidic</td>
<td>m-input u</td>
<td>m-output y</td>
<td>$u = y$</td>
</tr>
<tr>
<td>info processing</td>
<td>etc.</td>
<td>etc.</td>
<td>etc.</td>
</tr>
<tr>
<td>etc.</td>
<td>etc.</td>
<td>etc.</td>
<td>etc.</td>
</tr>
</tbody>
</table>
Recall the definition of a behavioral system: \( \Sigma = (\mathbb{R}, \mathbb{W}, \mathcal{B}) \), with \( \mathbb{R} \) the time-axis, \( \mathbb{W} \) the space of manifest variables, and \( \mathcal{B} \) the behavior, \( \mathcal{B} \subseteq (\mathbb{W})^\mathbb{R} \).

Let
\[
\Sigma = (\mathbb{R}, \mathbb{W} \times \mathbb{V}_1 \times \mathbb{V}_2, \mathcal{B})
\]
be a dynamical system.

The variables \( \mathbf{v}_1 \) and \( \mathbf{v}_2 \) are the variables that ‘live’ on the terminals which will be interconnected. As the idea of what interconnection does, we take: it imposes a static relation among the variables on the interconnected terminals. Interconnections should be ‘trivialities’ that obey all conceivable conservation laws.

Let
\[
\Sigma = (\mathbb{R}, \mathbb{W} \times \mathbb{V}_1 \times \mathbb{V}_2, \mathcal{B})
\]
be a dynamical system.

The interconnection constraint
\[
I(\mathbf{v}_1, \mathbf{v}_2) = 0.
\]
and the interconnected system \( \Sigma_I = (\mathbb{R}, \mathbb{W}, \mathcal{B}_I) \) with
\[
\mathcal{B}_I := \{ w \mid \exists (\mathbf{v}_1, \mathbf{v}_2) \text{ such that } (w, \mathbf{v}_1, \mathbf{v}_2) \in \mathcal{B} \text{ and } I(\mathbf{v}_1(t), \mathbf{v}_2(t)) = 0 \forall t \}.
\]

We will assume that the supply rate is additive among the terminals, i.e., if there are \( n \) terminals, with terminal variables 
\( w_1, w_2, \ldots, w_n \),
leading to the space of manifest variables 
\[
\mathbb{W} = \mathbb{W}_1 \times \mathbb{W}_2 \times \cdots \mathbb{W}_n,
\]
then
\[
s((w_1, w_2, \ldots, w_n)) = s_1(w_1) + s_2(w_2) + \cdots + s_n(w_n).
\]
Consider two terminals with variables $v_1, v_2$ and supply rates $s_1(v_1), s_2(v_2)$. The interconnection constraint

$$I(v_1, v_2) = 0.$$ 

is said to be (supply) neutral $\iff$

$$I(v_1(t), v_2(t)) = 0 \forall t \in \mathbb{R}$$

$$\Rightarrow s_1(v_1(t)) + s_2(v_2(t)) \forall t \in \mathbb{R}$$

**Examples:**

**Mechanical terminals:**
Terminal variables: force ($F$), position ($q$), velocity ($v$).

$$v = \frac{d}{dt}q$$ will be among the behavioral eq’ns.

$$s_1(F_1, q_1, v_1) = F_1 * v_1,$$
$$s_2(F_2, q_2, v_2) = F_2 * v_2,$$

$$I(F_1, q_1, v_1, F_2, q_2, v_2) : q_1 = q_2, F_1 + F_2 = 0.$$
Consider two terminals with variables $v_1, v_2$ and supply rates $s_1(v_1), s_2(v_2)$. The interconnection constraint

$$I(v_1, v_2) = 0.$$ 

is said to be (supply) neutral if

$$I(v_1(t), v_2(t)) = 0 \forall t \in \mathbb{R}$$

$$\Rightarrow s_1(v_1(t)) + s_2(v_2(t)) \forall t \in \mathbb{R}$$

Examples: input/output connection:
Terminal variables: terminal 1: $y_1$, terminal 2: $u_2$

$$s_1(y_1) = -||y_1||^2, \quad s_2(u_2) = ||u_2||^2, \quad I(y_1, u_2) : u_2 = y_1.$$ 

So with these supply rates, SIMULINK®’s connections are neutral.

This theorem has a number of interesting applications.

1. Feedback and passivity. Consider the feedback system

$$\Sigma = (\mathbb{R}, \mathbb{W} \times \mathbb{V}_1 \times \mathbb{V}_2, \mathbb{X}, \mathbb{B})$$

is dissipative w.r.t.

$$s((w, v_1, v_2)) = s'(w) + s_1(v_1) + s_2(v_2)$$

with storage function $V$. Assume furthermore that the interconnection constraint $I(v_1, v_2) = 0$ is neutral w.r.t. $s_1 + s_2$.

Then the interconnected system $\Sigma_I = (\mathbb{R}, \mathbb{W}, \mathbb{X}, \mathbb{B}_I)$ is dissipative w.r.t. $s'$ with storage function $V$.

Proof: trivial

Decompose this as (the notation reflects the interconnection constraints):

CLOSED LOOP SYSTEM
Now verify:

- System 3 is dissipative w.r.t. $\mathbf{u}^T \mathbf{y} - \mathbf{u}_1^T \mathbf{y}_1 - \mathbf{u}_2^T \mathbf{y}_2$.
- The interconnections are neutral.

Conclude that if

1. System 1 is diss. (passive) w.r.t. $\mathbf{u}_1^T \mathbf{y}_1$ with st. f'n $V_1(x_1)$
2. System 2 is diss. (passive) w.r.t. $\mathbf{u}_2^T \mathbf{y}_2$ with st. f'n $V_2(x_2)$

⇒ feedback system dissipative (passive) w.r.t. $\mathbf{u}^T \mathbf{y}$, storage function $V_1(x_1) + V_2(x_2)$.

Taking $\mathbf{u} = 0$, yields $V_1(x_1) + V_2(x_2)$ as a Lyapunov f'n. This is at the basis of many stability criteria.

Other situations:

1. The Popov criterion

   System 1: SISO LTI diff., diss. w.r.t. $\mathbf{u}_1^T (\mathbf{y}_1 + \alpha\mathbf{y}_1)$ with st. f'n $V(x)$ (i.e., $G(\xi)(1 + \alpha\xi)$ p.r.)
   System 2: a memoryless nonlinearity $u_2 \mapsto y_2 = f(u_2)$, with $\sigma f(\sigma) \geq 0 \ \forall \sigma \in \mathbb{R}$. This system is diss. w.r.t.
   $\mathbf{y}_2^T (\mathbf{u}_2 + \alpha \mathbf{u}_2)$ with st. f'n $\alpha\mathbf{F}(\mathbf{u}_2)$, $\mathbf{F}(\sigma) := \int_0^\sigma (\nu) \ d\nu$.

   ⇒ feedback system dissipative w.r.t. $\mathbf{u}^T (\mathbf{y} + \alpha\mathbf{y})$, with storage function $V(x) + \alpha\mathbf{F}(y)$.

   Taking $\mathbf{u} = 0$, yields $V(x) + \alpha\mathbf{F}(y)$ as a Lyapunov f'n.

2. The circle criterion exercise

2. Feedback and contractivity. Consider the feedback system

Taking $\mathbf{u} = 0$, yields $V(x) + \alpha\mathbf{F}(y)$ as a Lyapunov f'n.
Decompose this as (the notation reflects the interconnection constraints):

\[ y_1 u_1 \]
\[ y_2 u_2 \]
\[ y_1 u_2 \]

**INTERCONNECTION and DISSIPATIVITY**

Now verify:

1. System 3 is dissipative w.r.t. \( ||y_1||^2 - ||y_2||^2 \).
2. The interconnections are neutral.

Conclude that if

1. System 1 is dissipative w.r.t. \( ||u_1||^2 - ||y_1||^2 \) with storage function \( V_1(x_1) \) and
2. System 2 is dissipative w.r.t. \( ||u_2||^2 - ||y_2||^2 \) with storage function \( V_2(x_2) \),

\[ \Rightarrow \text{feedback system dissipative w.r.t. } s = 0, \text{storage f'n } V_1(x_1) + V_2(x_2). \]

This yields \( V_1(x_1) + V_2(x_2) \) as a Lyapunov f'n.

This is at the basis of many stability criteria.

**RECAP**

- The basic th'm on dissipative systems holds for general state systems.
- System interconnection is readily formalized in the setting of behavioral systems.
- Under reasonable assumptions:
  - interconnection of dissipative systems is dissipative.
  - Essential for preservation of dissipativity by interconnection:
    - interconnection constraints that are ‘supply neutral’. 
- Important application: the construction of Lyapunov functions for feedback systems with passivity or contractivity conditions on the open loop systems.

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Refinement:

Let \( |\rho| \leq 1 \). System 3 is dissipative w.r.t.

\[ ||y_1||^2 - |\rho|^2 ||u_2||^2 - (1 - |\rho|^2) ||y||^2. \]

Conclude that if

1. System 1 diss. w.r.t. \( ||u_1||^2 - ||y_1||^2 \) st. f’n \( V_1(x_1) \)
2. System 2 diss. w.r.t. \( |\rho|^2 ||u_2||^2 - ||y_2||^2 \) st. f’n \( V_2(x_2) \),

\[ \Rightarrow \text{feedback system dissipative w.r.t. } -(1 - |\rho|^2) ||y||^2 \]

with storage f’n \( V_1(x_1) + V_2(x_2) \).

\[ \nrightarrow \text{ } V_1(x_1) + V_2(x_2) \text{ as a Lyapunov f'n, with strictness on } V^\Sigma. \]

This is at the basis of many asymptotic stability criteria.
THE REALIZATION PROBLEM

Given a set of building blocks, and a way to interconnect these building blocks, what behaviors can be obtained?

Example 1: State representation algorithms. Building blocks: adders, amplifiers, forks, integrators (as in analog computers)

\[ \dot{x} = Ax + Bu, \quad y = Cx + Du. \]


BUILDING BLOCKS

Module Types:

Resistors, Capacitors, Inductors, Transformers, Connectors.

All terminals are of the same type: electrical, and there are 2 variables associated with each terminal,

- \((V, I)\)
- \(V\) the potential,
- \(I\) the current (counted \(> 0\) when it flows into the module).

\[ \sim \] signal space of each terminal: \(\mathbb{R}^2\).

Resistor: 2-terminal module.

Parameter: \(R > 0\) (resistance in ohms, say).

Device laws:

\[ V_1 - V_2 = RI_1; \quad I_1 + I_2 = 0. \]
**Capacitor:** 2-terminal module.

Parameter: $C > 0$ (capacitance in farads, say).

Device laws:

$$C \frac{d}{dt}(V_1 - V_2) = I_1 ; \quad I_1 + I_2 = 0.$$  

**Inductor:** 2-terminal module.

Parameter: $L > 0$ (inductance in henrys, say).

Device laws:

$$L \frac{d}{dt}I_1 = V_1 - V_2 ; \quad I_1 + I_2 = 0.$$  

**Transformer:** 4-terminal module; terminals (1,2): primary; terminals (3,4): secondary.

Parameter: $N \in \mathbb{R}$ (the turns ratio, $\in (0, \infty)$).

Device laws:

$$V_3 - V_4 = N(V_1 - V_2) ; \quad I_1 = -NI_3 ; \quad I_1 + I_2 = 0; \quad I_3 + I_4 = 0.$$  

**Connector:** many-terminal module.

Parameter: $n$ (number of terminals, an integer).

Device laws:

$$V_1 = V_2 = \cdots = V_n ; \quad I_1 + I_2 + \cdots + I_n = 0.$$
Assume that terminal 1, with terminal variables $V_1, I_1$, is connected to terminal 2, with terminal variables $V_2, I_2$. Interconnection constraint:

$$I(V_1, I_1, V_2, I_2) : V_1 = V_2, I_1 + I_2 = 0.$$ 

Call the ‘unconnected’ terminals, the external terminals. Number them: $(1, 2, \ldots, |E|)$. Take as manifest variables of the circuit, the external terminal voltages and currents: $\prod_{k \in |E|} (V_k, I_k)$.

Denote $\prod_{k \in |E|} (V_k, I_k)$ as $(V, I) \in \mathbb{R}^{2|E|}$. By carrying out the interconnections, we end up with a system

$$(\mathbb{R}, \mathbb{R}^{2|E|}, \mathcal{B}),$$

with external behavior: $\mathcal{B} \subseteq (\mathbb{R}^{2|E|})^\mathbb{R}$.

Assume that terminal 1, with terminal variables $V_1, I_1$, is connected to terminal 2, with terminal variables $V_2, I_2$. Interconnection constraint:

$$I(V_1, I_1, V_2, I_2) : V_1 = V_2, I_1 + I_2 = 0.$$ 

Now interconnect terminals of a (finite) number of building blocks. The result is called a(n electrical) circuit.
The electrical circuit synthesis problem can be stated as follows:

**Realizability:** Which external behaviors can be obtained by interconnecting a finite number of R’s, C’s, L’s, and T’s?

**Synthesis:** If a behavior is realizable, give a wiring diagram (an architecture) that leads to the desired external behavior.

This problem is of great importance (historical and otherwise) in electrical engineering. Important names:
- Otto Brune
- R.M. Foster
- W. Cauer
- E.A. Guillemin
- Sidney Darlington
- A.D. Fialkow
- B.D.H. Tellegen
- Dante Youla
- Vitold Belevitch
- etc., etc.

We list seven necessary conditions!

We now discuss these conditions, aiming at demonstrating

- the relevance of passivity and positive realness
- the ease of analysis provided by the behavioral approach

1. $\mathcal{B} \in \mathbb{L}^{2|E|}$

i.e., $\Sigma = (\mathbb{R}, \mathbb{R}^{2|E|}, \mathcal{B})$ is a LTIDS. There are $\infty$ ways of stating what this means.

For example, there exists a polynomial matrix $R^{\bullet \times 2|E|} \in \mathbb{R}[\xi]$ such that $\mathcal{B}$ consists of the solutions of

$$R\left(\frac{d}{dt}\right)\begin{bmatrix} V \\ I \end{bmatrix} = 0.$$

Proof: Elimination th’m.
We list seven necessary conditions!

1. \( \mathcal{B} \subseteq \mathcal{L}^{2|E|} \)
2. KVL

\[(V, I) \in \mathcal{B} \text{ and } \alpha \in \mathcal{C}^{\infty}(\mathbb{R}, \mathbb{R}) \Rightarrow (V + \alpha e, I) \in \mathcal{B} \]

with

\[ e = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \]

**Proof:** Verify for each of the modules, and for the int. constraint.

In other words, there exist a partition of \((V, I)\) in \(|E|\) inputs and \(|E|\) outputs, with, if you insist, a proper transfer function.

Consider this together with the next property.

---

We list seven necessary conditions!

1. \( \mathcal{B} \subseteq \mathcal{L}^{2|E|} \)
2. KVL
3. KCL
4. The input cardinality, \(N(\mathcal{B}) = |E|\)

In other words, there exist a partition of \((V, I)\) in \(|E|\) inputs and \(|E|\) outputs, with, if you insist, a proper transfer function.

There exists an I/O repr. for which the input and output var. \((u_1, u_2, \ldots, u_{|E|}), (y_1, y_2, \ldots, y_{|E|})\)

pair as follows:

\[ \{u_k, y_k\} = \{V_k, I_k\} \]

In other words, each terminal is either current controlled or voltage controlled.
We list seven necessary conditions!

1. $\mathcal{B} \in \mathcal{L}^2[|E|]$
2. KVL
3. KCL
4. The input cardinality, $\Pi(\mathcal{B}) = |E|$
5. Hybridicity

6. Passivity. From hybridicity, $\mathcal{B}$ admits a representation as
   \[
   \dot{x} = Ax + Bu, \quad y = Cx + Du
   \]
   This system is dissipative w.r.t. the supply rate $u^T y = V^T I$,
   and with a quadratic positive definite storage $f$'
   \[
   V(x) = x^T K x, \quad K = K^T > 0.
   \]
   Without loss of generality, $K = I$.
   This states that the net electrical energy goes into the circuit.

7. Reciprocity. The transfer f'n $G$ is signature symmetric, i.e.
   \[
   \Sigma G = G^T \Sigma.
   \]
   $\Sigma$ is the signature matrix $\Sigma = \text{diag}(s_1, s_2, \ldots, s_{|E|})$,
   with $s_k = +1$ if the terminal $k$ is voltage controlled,
   and $s_k = -1$ if the terminal $k$ is current controlled.
We list seven necessary conditions!

1. $\mathfrak{B} \in \mathcal{L}^2[E]$
2. KVL
3. KCL
4. The input cardinality, $m(\mathfrak{B}) = |E|$
5. Hybridicity
6. Passivity
7. Reciprocity

This curious properties may be translated as:

The influence of terminal $k'$ on terminal $k''$ is equal to the influence of terminal $k''$ on terminal $k'$.

Proof: Show that each of the modules satisfy properties (1) to (7). Show that these properties remain valid after interconnection, i.e., proceed one interconnection at the time. The difficult part here is (4).

If $\mathfrak{B}$ is controllable then these conditions are also sufficient for realizability. However, in order to obtain a 'clean' statement, it is convenient to eliminate $I_{|E|} = -I_1 - I_2 - \cdots - I_{|E|-1}$, and look at the behavior of $(V_3 - V_{|E|}, V_4 - V_{|E|}, \ldots, V_{|E|-1} - V_{|E|}, I_1, I_2, \ldots, I_{|E|-1})$.

The transfer function $G \in \mathbb{R}^{(|E|-1) \times (|E|-1)}$ is realizable if and only if it is signature symmetric and positive real.
Consider a 2-terminal circuit

\[ \text{Interconnected RLCT's} \]

\[ \begin{align*}
V_1 & \quad I_1 \\
V_2 & \quad I_2
\end{align*} \]

KCL \Rightarrow I_1 + I_2 = 0. \quad \text{Set} \quad I := I_1 = -I_2.

KVL \Rightarrow \text{the beh. eq'ns involve only } V_1 - V_2. \quad \text{Set} \quad V := V_1 - V_2.

The behavior of \((V, I)\) is called the \textit{port description}.

**Port description:**

\[ \begin{align*}
+ & \quad I \\
V & \quad \quad \\
- & \quad I
\end{align*} \]

\(Z\), the transfer f'n \(I \mapsto V\) is called the \textit{driving point impedance}.

Note that \(Z\) need not be proper.

Which driving point impedances are realizable?

\(Z \in \mathbb{R}(\xi)\) is the driving point impedance

of an electrical circuit that consists of an interconnection

of a finite number of

positive \(R\)'s, positive \(L\)'s, positive \(C\)'s, and transformers

if and only if \(Z\) is positive real.

This result led to the introduction of \textit{positive real functions}.

First proven by Otto Brune in his M.I.T. Ph.D. dissertation (see O. Brune, \textit{Synthesis of a finite two-terminal network whose driving point impedance is a prescribed function of frequency}, Journal of Mathematics and Physics, volume 10, pages 191-236, 1931).
SYNTHESIS of DRIVING POINT IMPEDANCES

Which driving point impedances are realizable?

\[ Z \in \mathbb{R}(\xi) \] is the driving point impedance of an electrical circuit that consists of an interconnection of a finite number of positive \( R \)'s, positive \( L \)'s, positive \( C \)'s, and transformers if and only if \( Z \) is positive real.

Are transformers needed?

In 1949, Bott and Duffin proved ‘no’ in a one-page (!) paper (see R. Bott and R.J. Duffin, Impedance synthesis without transformers, Journal of Applied Physics, volume 20, page 816, 1949). However, their synthesis has common factors, non-controllability!

TERMINALS versus PORTS

Note that we have used throughout the terminal description of circuits. It is simply more appropriate and more general (even when using only ‘port’ devices).

Example:

However, port descriptions are more parsimonious in the choice of variables (it halves their number).

RECAP

- Realizability theory: an important engineering oriented problem area.
- The analysis and synthesis of RLCT circuits is an important application of passive systems.
- 7 necessary conditions for realizability by passive \( R,L,C,T \)’s: differential system, KVL, KCL, input cardinality, hybridicity, passivity, and reciprocity.
- In the controllable case these conditions are also sufficient.
- It is the circuit synthesis problem that led to positive realness.