Non-linear observers (estimators) and symmetries : theory and examples.

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One way to build non-linear estimators in control theory is to make observers. They use a model of the system and noisy measurements to provide a real-time estimation of its internal state. When the system has symmetries, the usual observers (Luenberger observer, extended Kalman filter) break these symmetries. We developed a method which provides a way to modify the usual observer equations so that they respect the symmetries of the system. Once this is done, one can define a new estimation error (which is the "difference" between the estimated state and the true state of the system) which relies on symmetries. The whole convergence analysis of the observer (i.e. how to make the estimation error tend to zero) is then based on the symmetries.

We will not spend much time on the theoretical developments and we will rather focus on some examples.

The first example is tutorial. We consider a non-holonomic car equipped with a GPS device (position measurement) and we want to find its orientation. The symmetries are associated to the invariance of the equations under the action of the group $SE(2)$ (translations and rotations in the plane).

The second example deals with the estimation of a 2-level quantum system. Its state is a superposition of two states. The state space is a two-dimensional space whose underlying geometry is that of the surface of a sphere (Bloch sphere). The Schrödinger equation is invariant under the action of $SU(2)$ — or in the Bloch sphere representation, the dynamics is invariant under the action of the rotation group $SO(3)$ acting on the sphere.

Finally, we will present a preliminary work on data assimilation in oceanography (nudging) where the model is invariant under the action of $SE(2)$.