Computing with low-rank positive semidefinite matrices: a geometric approach

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Positive definite matrices have become a fundamental object of modern computational engineering. They appear in many computational problems in the form of variables (convex programming, LMIS problems, Lyapunov equations), covariance matrices (signal processing), diffusion tensors (biomedical imaging), kernels (machine learning), to cite a few. The numerical complexity of associated algorithms is typically $\mathcal{O}(n^3)$ in the problem size, which prevents their use in a growing number of large-scale problems. Working with low-rank approximations of such matrices is a typical solution to reduce the numerical complexity. In this talk, we study the underlying geometry of the set of fixed-rank positive semidefinite matrices. We argue that right geometries are essential to support the development of efficient low-rank versions of many existing algorithms and we illustrate its use in several examples.