An efficient algorithm for large-scale linear and convex optimization in relative scale

Peter Richtarik, 23/05/2008.

We propose two variants of a single efficient $\mathcal{O}(1/\varepsilon)$ algorithm simultaneously solving a number of related optimization problems in relative scale:

1. Find the intersection of a line and a centrally symmetric convex body Q given as the convex hull of a collection of points.
   Interpretation: Our method produces a sequence of ellipsoids inscribed in Q and "converging" towards the intersection points, and as such can be viewed as a modification of the ellipsoid rounding algorithm of Khachiyan.
2. Maximize a linear function over the polytope polar to Q.
   Interpretation: A "variant" of the ellipsoid method of Nemirovski-Yudin-Shor.
3. Find the minimum l_1-norm solution of a full rank underdetermined linear system.
   Interpretation: An Iteratively Reweighted Least Squares (IRLS) method.
4. Minimize the maximum of absolute values of linear functions on a hyperplane.
   Interpretation: Shor's subgradient method with space dilation.
5. Minimize a (special) smooth convex function on the unit simplex.
   Interpretation: A version of the Frank-Wolfe method with specialized line search.

From among the many possible applications we outline those to truss topology design and optimal design of statistical experiments.