Abstract

The present dissertation studies specific issues related to the coordination of a set of "agents" evolving on a nonlinear manifold, more particularly a homogeneous manifold or a Lie group. The viewpoint is somewhere between control algorithm design and system analysis, as algorithms are derived from simple principles — often retrieving existing models — to highlight specific behaviors.

With a fair amount of approximation, the objective of the dissertation can be summarized by the following question: *Given a swarm of identical agents evolving on a nonlinear, nonconvex configuration space with high symmetry, how can you define specific collective behavior, and how can you design individual agent control laws to get a collective behavior, without introducing hierarchy nor external reference points that would break the symmetry of the configuration space?*

Maintaining the basic symmetries of the coordination problem lies at the heart of the contributions. The main focus is on the global geometric invariance of the configuration space. This contrasts with most existing work on coordination, where either the agents evolve on vector spaces — which, to some extent, can cover *local* behavior on manifolds — or coordination is coupled to external reference tracking such that the reference can serve as a beacon around which the geometry is distorted towards vector space-like properties. A second, more standard symmetry is to treat all agents identically.

Another basic ingredient of the coordination problem that has important implications in this dissertation is the reduced agent interconnectivity: each agent only gets information from a limited set of other agents, which can be varying.

In order to focus on issues related to geometry / symmetry and reduced interconnectivity, individual agent dynamics are drastically simplified to simple integrators. This is justified at a "planning" level. Making the step towards realistic dynamics is illustrated for the specific case of rigid body attitude synchronization.

The main contributions of this dissertation are

I. an extensive study of synchronization on the circle, (a) highlighting difficulties encountered for coordination and (b) proposing simple strategies to overcome these difficulties;

II. (a) a geometric definition and related control law for "consensus" configurations on compact homogeneous manifolds, of which synchronization — all agents at the same point — is a special case, and (b) control laws to (almost) globally reach synchronization and "balancing", its opposite, under general interconnectivity conditions;

III. several propositions for rigid body attitude synchronization under mechanical dynamics;

IV. a geometric framework for "coordinated motion" on Lie groups, (a) giving a geometric definition of coordinated motion and investigating its implications, and (b) providing systematic methods to design control laws for coordinated motion.

Examples treated for illustration of the theoretical concepts are the circle S^1 (sometimes the sphere S^n), the rotation group $SO(n)$, the rigid-body motion groups $SE(2)$ and $SE(3)$ and the Grassmann manifolds $Grass(p, n)$. The developments in this dissertation remain at a rather theoretical level; potential applications are briefly discussed.