Market dynamics driven by the decision-making power producers

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***Abstract*— In this paper we consider a tool for analyzing the market outcomes when competitive agents (power producers) interact through the market place. The market clearing mechanism is based on the locational marginal price scheme.**

A model of the strategic behavior is formulated for the agents. Each agent chooses its bid in order to maximize its profit by assuming that the other agents will post the same bid as at the previous session of the market, and by knowing the network characteristics.

The income of each agent over a certain temporal horizon for different power system configurations (the addition of new transmission capabilities, new power plants) is evaluated according to this market dynamics and by integrating this dynamics over the chosen temporal horizon.

The mathematical formulation, for the sake of simplicity, is related to a two node power system.

In the simulations, the influence of different conditions (line transfer capacity, the number and size of generators, the presence of portfolio) on market outcomes is analyzed, and interesting and sometimes counterintuitive results are found.

***Index Terms*— Market dynamics, agent-based simulations.**

I. INTRODUCTION

The ideal inspiration for restructuring electricity systems is to achieve highly competitive markets with prices close to short-run marginal costs. The basic economic idea is that with perfect competitive market conditions, market participants maximize their own profits in a decentralized way and bid in the market at a price equal to their marginal cost and so the competitive and efficient market results are attained. In such a case, the prices at the nodes of the network reflect the marginal cost of production and the marginal value for the consumers [1].

However, in most of the deregulated market around the world, the perfect competitive conditions do not hold true. The presence of few power producers that bid strategically, of a highly inelastic electricity demand, and of a transmission network that could be insufficient to accommodate the flows derived from a merit order dispatch are only some of the reasons that move the market results away from the expected optimum.

Several authors have addressed the question of characterizing and computing the market equilibrium point when the market is not perfectly competitive. Their works differ essentially by the type of market considered (Bilateral Market, Uniform Auction or Discriminatory-Price Auction), the network representation adopted (from no transmission constraints to DC or a full AC load flow representation of the network), and the type of approach used to compute these equilibrium points. In particular, Cournot and Bertrand models have been used to mimic the participants behavior and to assess the influence of the transmission network on the market prices [2], [3], [4], [5]. Other approaches in which the Supply Function Equilibrium model is adopted to handle electricity market interactions have been used in [6], [7], to cite only few of them. Furthermore, agent-based models have also been considered to study electricity markets [8], [9], [10]. Within such a framework, the behavior of each market participant is modelled by a set of rules. Once a model is created for each participant, a market dynamics is built and its characteristics are analyzed through simulations. This framework is particularly attractive since it has the potential to deal with detailed market structures (taking into account for example capacity constraints, price caps and other details) and to take into consideration the more complex behaviors the different market participants may have (willingness to form a coalition, participation in negotiation processes, etc).

In this paper, we consider an agent-based approach to electricity markets. Consumers, producers and the Independent System Operator (ISO) are the agents which compose our market model. While the consumer agents are assumed to have a static behavior (inelastic load), the producer agents submit at each session of the market a strategic bid that would maximize their payoff if the rivals' bids were the same with respect to the previous market session. The ISO agent uses the locational marginal price scheme to clear the market. Rather than focusing on what could be the equilibrium point of such a system, we are more interested in observing how the producers' bids evolve during the sessions of the temporal horizon under analysis, and how the agents payoffs vary. Furthermore, we compute the total payoff of each agent by summing the agent payoff over the time horizon and we evaluate the influence of several factors (e.g. transmission capacity limits, presence of portfolio) on this total payoff.

For the sake of simplicity, the approach is explained on a two node power system. It can however be extended in an almost straightforward way to larger power systems.

Section II explains the market dynamics considered here. We suppose a market for which the trading of energy can only be done through a spot market. We describe the clearing mechanism and the strategy used by the power producer agents to determine their bid functions. Section III describes the simulation conditions. In Section IV we simulate the market dynamics and assess the benefits, to the different agents of the

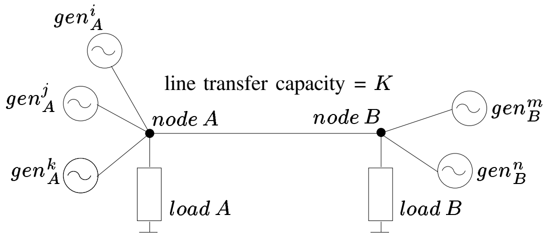


Fig. 1. A simple power system composed of generators and loads connected through a single transmission line.

system, of the building of new transmission capabilities and generation capabilities. In Section V we consider that some generators may belong to the same portfolio and highlight some counterintuitive phenomena that may occur.

II. The market dynamics

A. Power system description

We consider here a power system composed of two nodes A and B. A load is connected at each node, *load A* and *load B*, that consume respectively q_A and q_B. These values are supposed to be constant. The line that connects these two nodes is supposed to be a lossless transmission line which can only transfer a limited amount of power (K) which may cause some congestion problems. Generators are connected to each end of the line. We suppose that the marginal cost of each generator is constant and that the generators can only produce a finite amount of power.

Let gen_A^1, gen_A^2, $\cdots$, $gen_A^{n_A}$ (gen_B^1, gen_B^2, $\cdots$, $gen_B^{n_B}$) be the n_A (n_B) generators of the system connected to node A (B) and c_A^i (c_B^i) the marginal cost of generator gen_A^i (gen_B^i) and $q^i_{A_{max}}$ ($q^i_{B_{max}}$) the maximum amount of power it can produce.

B. The market clearing mechanism

We consider here a spot market whose mechanism is the following.

At each market session we suppose that each generator gen_A^i (gen_B^i) submits a bid b_A^i (b_B^i) that represents the price the generator gen_A^i (gen_B^i) asks per MW to produce.

The clearing mechanism for the market consists of finding which quantity of power each generator has to produce in order to generate at the cheapest cost the energy required to cover the load while making sure the line is not overloaded.

This mechanism can be stated as a linear programming problem :

Determine $(q_A^1, \cdots, q_A^{n_A}, q_B^1 \cdots, q_B^{n_B}) \in \mathbb{R}^{n_A+n_B}$ that minimizes

$$\sum_{i=1}^{n_A} b_A^i q_A^i + \sum_{i=1}^{n_B} b_B^i q_A^i \tag{1}$$

under the following constraints

$$\sum_{i=1}^{n_A} q_A^i + \sum_{i=1}^{n_B} q_B^i = q_A + q_B \tag{2}$$

$$\sum_{i=1}^{n_A} q_A^i - q_A \leq K, \quad \sum_{i=1}^{n_B} q_B^i - q_B \leq K \tag{3}$$

$$0 \leq q_A^i \leq q^i_{A_{max}}, \quad 0 \leq q_B^i \leq q^i_{B_{max}} \qquad \forall i, j \tag{4}$$

Once this linear programming problem is solved, i.e. once we have computed $(q_A^1, \cdots, q_A^{n_A}, q_B^1 \cdots, q_B^{n_B})$, we determine the price of the energy at nodes A and B of the system (p_A and p_B).

If no congestion occurs, that is if the energy can not be produced at lower cost by increasing the value of K, the price of the energy at node A and node B is equal to the price of the most expensive units for which the production is greater than 0 :

$$p_A = p_B = \max(\max_{i=1,\cdots,n_A} \mathbf{1}(q_A^i \neq 0) * b_A^i, \tag{5}$$

$$\max_{i=1,\cdots,n_B} \mathbf{1}(q_B^i \neq 0) * b_B^i) \tag{6}$$

If congestion occurs, then the price at each node is equal to the price of the most expensive unit generating power at this node :

$$p_A = \max_{i=1,\cdots,n_A} \mathbf{1}(q_A^i \neq 0) * b_A^i \tag{7}$$

$$p_B = \max_{i=1,\cdots,n_B} \mathbf{1}(q_B^i \neq 0) * b_B^i \tag{8}$$

C. The payoff of the different agents

Once the linear programming problem is solved and the prices at the different nodes are computed, it is possible to determine the payoffs (or rewards) of the different agents of the power system :

Power producer agent : The payoff of a power producer agent is equal to the money it gets for producing the electricity minus the costs of producing it. The payoff of the power producer agents gen_A^i and gen_B^i are :

$$r_{gen_A^i} = q_A^i * (p_A - c_A^i) \tag{9}$$

$$r_{gen_B^i} = q_B^i * (p_B - c_B^i) \tag{10}$$

Load agents : The payoff of the load agents $load\,A$ and $load\,B$ are equal to "minus the money they pay for buying the electricity" :

$$r_{load\,A} = -q_A * p_A \tag{11}$$

$$r_{load\,B} = -q_B * p_B \tag{12}$$

ISO agent : The payoff of the ISO is equal to the difference of the money paid by the load to buy the electricity and the money received by the generators to generate this electricity. It can be defined by the following expression :

$$r_{ISO} + r_{load\,A} + r_{load\,B} + \sum_{i=1}^{n_A} q_A^i * p_A + \sum_{i=1}^{n_B} q_B^i * p_B = 0 \tag{13}$$

D. The bidding strategy of the power producers

The different agents of the system want to maximize their payoffs. Since we are considering a power system with inelastic load and a spot market as the only means of trading electricity, only power producer agents can influence the price at the different nodes and the distribution of the production. These power producer agents will try to submit bids that maximize their payoffs. In particular, we assume that the strategic variable for the agents is the bid price, whereas we suppose that no withholding capacity strategy could be adopted.

To model the behavior of the agents we have made the following assumptions. We suppose that each agent knows the value of the loads (q_A and q_B), the maximal amount of power that can be transmitted (K), the clearing mechanism and the bids the other agents submitted the previous time the market was cleared. In particular, it is useful to remark that both quantities and prices offered by other agents are known. Therefore, each agent submits a bid that will maximize its payoff if the other agents repeat the bids they submitted before.

Let us suppose that B_A^i (B_B^i) is the set of possible bids for generator gen_A^i (gen_B^i) and that $b_{A_t}^i$ ($b_{B_t}^i$) is the bid submitted by gen_A^i (gen_B^i) at time t. By noticing that the value of p_A and p_B at time t are functions of the bids submitted, generator gen_A^i and gen_B^i compute their bids at this time according to the following expressions :

$$b_{A_t}^i = \arg\max_{b_A^i \in B_A^i} \quad r_{gen_A^i}(b_{A_{t-1}}^1, \cdots, b_A^i, \cdots, b_{A_{t-1}}^{n_A}, b_{B_{t-1}}^1, \cdots, b_{B_{t-1}}^{n_B}) \tag{14}$$

$$b_{B_t}^i = \arg\max_{b_B^i \in B_B^i} \quad r_{gen_B^i}(b_{A_{t-1}}^1, \cdots, b_{A_{t-1}}^{n_A}, b_{B_{t-1}}^1, \cdots, b_B^i, \cdots, b_{B_{t-1}}^{n_B}) \tag{15}$$

Results of the simulations will of course depend strongly on the model behavior of the agent adopted. It is however difficult to state what the perfect model is since humans intervene in the bidding process. For more information about agent modelling in electricity spot markets we refer the reader to [11].

Equilibrium point

The system is said to be in an equilibrium point at time t if we have $b_{A_{t+1}}^i = b_{A_t}^i$, $b_{B_{t+1}}^j = b_{B_t}^j$ $\forall i \in \{1, \cdots, n_A\}$ $\forall j\{1, \cdots, n_B\}$. Note that when a system is in an equilibrium point, a power producer agent cannot increase its payoff by being the only one to change its bid function [1].

Extension to portfolio

It may happen that several generators belong to the same agent, named here the *portfolio*. In this case we suppose that the portfolio determines the bids for the generators it owns in order to maximize the sum of their rewards (once again by supposing that the other generators keep the same bid functions as in the previous stage). For example if a portfolio owns gen_A^i and gen_A^j, it will compute $b_{A_t}^i$ and $b_{A_t}^j$ as follows :

$$\begin{aligned}(b_{A_t}^i, b_{A_t}^j) = \arg\max_{(b_A^i, b_A^j) \in B_A^i \times B_A^j} [\\ r_{gen_A^i}(b_{A_{t-1}}^1, \cdots, b_A^i, \cdots, b_A^j, \cdots, b_{A_{t-1}}^{n_A}, b_{B_{t-1}}^1, \cdots, b_{B_{t-1}}^{n_B}) \\ + r_{gen_A^j}(b_{A_{t-1}}^1, \cdots, b_A^i, \cdots, b_A^j, \cdots, b_{A_{t-1}}^{n_A}, b_{B_{t-1}}^1, \cdots, b_{B_{t-1}}^{n_B})]\end{aligned}$$

III. Simulation conditions

In the next two sections we are going to simulate the market dynamics. We describe hereafter the simulation conditions used.

• *Number of stages considered.* The market dynamics is going to be integrated over 25 periods.

• *Values of the loads.* q_A and q_B are considered to be constant and equal to 50 MW.

• *Price Cap.* A price cap p_{cap} of 4 \$/MW for generators' bid price is considered.

• *Bid sets.* For each generator gen_A^i (gen_B^i) we will consider in the next two sections a set of possible bids equal to $\{c_A^i, c_A^i + 0.1, \cdots, c_A^i + (k-1) * 0.1, c_A^i + k * 0.1\}$ ($\{c_B^i, c_B^i + 0.1, \cdots, c_B^i + (k-1) * 0.1, c_B^i + k * 0.1\}$) such that $c_A^i + k * 0.1 \leq p_{cap}$ and $c_A^i + (k+1) * 0.1 > p_{cap}$ ($c_B^i + k * 0.1 \leq p_{cap}$ and $c_B^i + (k+1) * 0.1 > p_{cap}$)[2].

• *Several bids lead to the same reward.* If Eqns (14) and (15) lead to several possible bids, we suppose that the power producer agents submit the least expensive one.

• *First bid submitted.* For the first session of the market ($t = 0$) we still use expressions (14) and (15) to determine the bids $b_{A_0}^i$ and $b_{B_0}^j$ by assuming that $b_{A_{-1}}^i$ and $b_{B_{-1}}^j$ are respectively equal to c_A^i and c_B^j. This strategy consists of considering that each generator submits its first bid by assuming that the other generators will submit a bid equal to their marginal costs.

• *The clearing mechanism leads to more than one solution.* The solution of the minimization problem may not be unique if several generators submit the same bids. In this case we have chosen among the set of possible solutions the one that tends to allocate the power between the different generators having the same price proportional to their maximum capacity of production[3].

IV. New generation or transmission capabilities

In this section we study the system represented in Figure 2. This system will be studied for four different configurations : with or without gen_A^2 and in each case with or without congestion.

[1]Note that the market dynamics considered in this paper shares some common characteristics with the so-called diagonalisation procedure of [2]. Roughly speaking, this diagonalisation procedure is used to compute some equilibrium points of the market and consists of simulating the market dynamics proposed here till an equilibrium point is reached.

[2]To be more accurate we should use as a unit of price \$/(MW*duration of a stage). However to lighten the notations we simply use \$/MW.

[3]The solution chosen satisfies $\sum_{i=1}^{n_A}\sum_{j=1}^{n_A} \mathbf{1}(p_A^i = p_A^j)|\frac{q_A^i}{p_{A_{max}}^i} - \frac{q_A^j}{p_{A_{max}}^j}| = 0$, $\sum_{i=1}^{n_B}\sum_{j=1}^{n_B} \mathbf{1}(p_B^i = p_B^j)|\frac{q_B^i}{p_{B_{max}}^i} - \frac{q_B^j}{p_{B_{max}}^j}| = 0$ and leads among the set of possible solutions to the smallest value of $\sum_{i=1}^{n_A}\sum_{j=1}^{n_B} \mathbf{1}(p_A^i = p_B^j)|\frac{q_A^i}{p_{A_{max}}^i} - \frac{q_B^j}{p_{B_{max}}^j}|$.

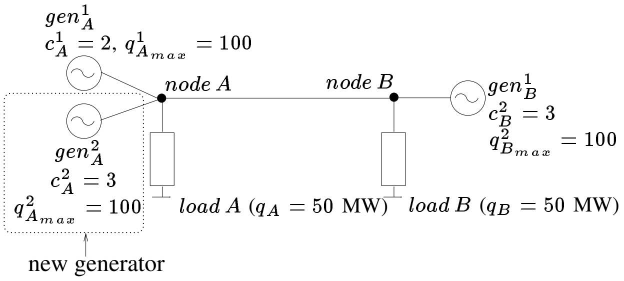


Fig. 2. We evaluate the benefit to the different agents of the system of having a new generator gen_B^2 and/or additional transfer capacity.

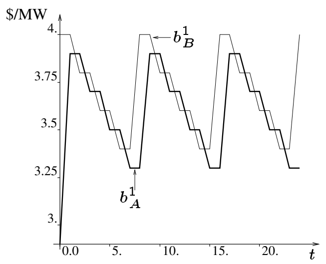
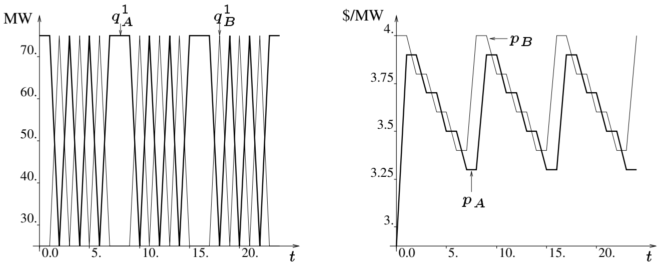


Fig. 3. *Input of the market* : the bids of gen_A^1 and gen_B^2



(a) Production of gen_A^1 and gen_B^1

(b) Prices at node A and node B

Fig. 4. *Output of the market* : production of each generator and nodal prices

A. Two generators in the system and no congestion

Description of the system

The system we consider here has one generator connected to the left side and another generator connected to the right side ($n_A = n_B = 1$). We suppose that each generator can produce 100 MW ($q^1_{A_{\max}} = q^1_{B_{\max}} = 100$) and that gen_A^1 has a marginal cost of 2 ($c_A^1 = 2$) and gen_B^1 a marginal cost of 3 ($c_B^1 = 3$). The line transfer capacity is chosen to be 50 MW. With this line and the production capacities of the generators, each generator can in principle be sufficient to cover the whole load. No congestion can occur since the transfer capacity of the line is not less than q_A^1 or q_B^1.

Market dynamics

By using Eqns (14) and (15) to determine the bid evolution we observe that the bids of generators gen_A^1 and gen_B^1 are constant whatever the value of t. An equilibrium point is reached the first time the generators bid.

Generator gen_A^1 bids at 2.9 $/MW, just below the marginal cost of generator gen_B^1 and produces all the power requested by the loads. Generator gen_B^1 bids at its marginal cost (3 $/MW) but is unable to be selected for producing power because of its higher marginal cost. It can not afford to compete with gen_A^1.

The two nodal prices are equal to 2.9 $/MW and the flow from node A to node B is equal to 50 MW. This solution corresponds to the classical Bertrand solution, obtained when a competition in price is studied.
The total amount of money paid by the load over 25 cycles is $7,250$ $, the net income of gen_A^1 is $2,250$ $, whereas ISO and gen_B^1 do not earn anything. It should be noted that the ISO earns money only if there exists a difference between the two nodal prices.

B. Two generators in the system and congestion

Description of the system

The system is exactly the same as in Section IV-A except that the transfer capacity of the line is assumed to be 25 MW.

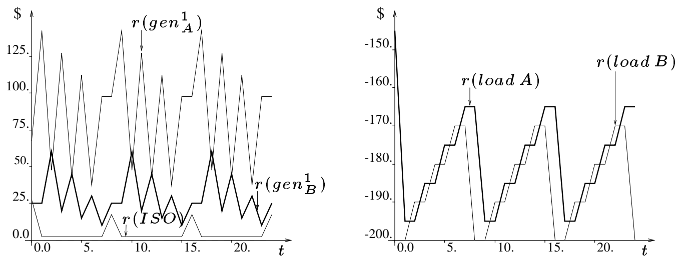
Market dynamics

In this case the generator bidding strategy is heavily influenced by the congestion that may occur on the line. Figure 3 illustrates the bidding strategy of the two generators. During the first stage, gen_A^1 offers 2.9 $/MW since it makes its bid by assuming that gen_B^1 bids at marginal cost. At the same time gen_B^1 bids at the price cap. The initial bid price of gen_B^1 is accountable on the basis of the following remarks : (i) gen_A^1 has a lower marginal cost than gen_B^1; (ii) if gen_A^1 bids at its marginal cost, it will be selected to produce all the power needed at its own node (node A) and the maximum power that it can produce to serve the load at the other node (node B) subject to the line limit; (iii) some of the power offered by gen_B^1 is needed at node B because of the inelastic demand of the load at that node and the limited capacity of the line.

In the following stages, each generator assumes that the rival keeps the same bid and chooses to bid just below as long as the net profit arising from this type of bid becomes less than the net profit arising from a bid price equivalent to the price cap. It is interesting to notice that the market dynamics does not converge here to an equilibrium point. Note that each time gen_A^1 bids below gen_B^1 the power transferred in the line is equal to 25 MW and goes from left to right while the same magnitude flow but in the opposite direction occurs when gen_B^1 bids below gen_A^1. The productions of gen_A^1 and gen_B^1 are sketched in Figure 4a.

Unlike in the case analyzed in Section IV-A where no congestion occurred, the presence of congestion gives rise to different nodal prices. These are represented in Figure 4b. Remark that the nodal prices at node A and node B (p_A and p_B) are identical to the bid prices of gen_A^1 and gen_B^2, respectively.

It is interesting to notice that this type of cyclic behavior of generators' bid prices and consequently of nodal prices is closely related to the Edgeworth Cycle. Edgeworth (1925) analyzed the static Bertrand price equilibrium and showed that when firms face capacity constraints, there exists generally a cyclic price behavior rather than an equilibrium point. He showed that this cycle is composed of two phases : the *price war* phase and the *relenting* phase. During the first one, the firms undercut each other successively to increase their market



(a) Rewards of gen_A^1, gen_B^1 and ISO (b) Rewards of $load\,A$ and $load\,B$

Fig. 5. Rewards obtained by the different agents of the system

	Rewards obtained by the different agents of the system over 25 stages				
K	gen_A^1	gen_B^1	ISO	$load\,A$	$load\,B$
25	2,182.5	715	132.5	-4,465	-4,640
50	2,250	0	0	-3,625	-3,625

TABLE I

INFLUENCE OF THE LINE TRANSFER CAPACITY ON THE REWARDS.

share, until the war becomes too costly. At that point it starts the second phase, when some firm increases its price and the other firms follow. After that, a new cycle of price war starts.

However, while in the Edgeworth Cycle it is a constraint on the capacity of the strategic agents (producers) that induces this dynamics, here it is the presence of a capacity constraint on the transmission line that gives rise to this result. In fact, either firm tries to increase profit by undercutting the rival price and thereby selling all the quantity at its own node and a quantity equal to K at the other node. This type of behavior has been observed also in [4].

In Figures 5a-b we have drawn the rewards obtained by the different agents of the system. The spikes for the ISO rewards occur when the difference between the two nodal prices is maximum. This occurs when gen_B^1 bids at the price cap and gen_A^1 bids at its minimum strategic price.

C. Who benefits from a larger line transfer capacity ?

In the last two subsections, we have studied the market dynamics for different values of the line transfer capacity. Suppose now that the line transfer capacity is equal to 25 MW and that we explore the possibility of building new transmission devices to bring this transfer capacity to 50 MW. Agents willing to pay for these new investments are agents who will benefit from them. Table I gathers the sum of the rewards obtained over 25 stages by each agent for the different transfer capacities. As we observe, load agents get better rewards with a larger transfer capacity, which is normal since this increases the competition between the different generators. If gen_A^1 slightly increases its revenue with a larger transfer capacity, the revenue of gen_B^1 drops to zero. By analyzing this table one may conclude that three agents would be willing to pay for an increase of transfer capacity : $load\,A$, $load\,B$ and gen_A^1.

D. Three generators and no congestion

Description of the system

The system analyzed here is the same as in Section IV-A except that one generator (gen_A^2) has been added to node A. The generator has the same marginal cost and the same production capacity as gen_A^1 ($c_A^1 = c_A^2 = 2$ and $p^1_{A_{max}} = p^2_{A_{max}} = 100$).

Market dynamics

As in the case treated in Section IV-A, an equilibrium point is reached the first time the generators bid ($b^1_{A_0} = 2$, $b^2_{A_0} = 2$ and $b^1_{B_0} = 3$). However, this time, the increase of competition between power producers caused by the appearance of a new power producer agent leads to a drop of the nodal prices from 2.9 \$/MW to 2 \$/MW.

E. Three generators and congestion

Description of the system

The system is exactly the same as in the previous subsection except that the line transfer capacity is now equal to 25 MW.

Market dynamics

An equilibrium point is reached the first time the generators bid ($b^1_{A_0} = 2$, $b^2_{A_0} = 2$ and $b^1_{B_0} = 4$). To this equilibrium point corresponds the following nodal prices : $p_A = 2$ and $p_B = 4$. The competition between gen_A^1 and gen_A^2 prevents a nodal price at node A larger than the marginal cost of the generators. However, gen_B^2 takes advantage of the congestion to sell at the price cap the 25 MW the other generators cannot produce.

F. Who benefits from a larger line capacity and from new generation ?

	Rewards obtained by the different agents of the system over 25 stages					
K	gen_A^1	gen_A^2	gen_B^1	ISO	$load\,A$	$load\,B$
25	0	0	625	1,250	-2,500	-5,000
50	0	0	0	0	-2,500	-2,500

TABLE II

INFLUENCE OF THE LINE TRANSFER CAPACITY ON THE REWARDS.

We have drawn in Table II the rewards accumulated over 25 stages by the different agents for the systems treated in the two previous subsections. We can observe that an increase in the line capacity is only favorable to $load\,B$ because it can now buy its power at the marginal cost of gen_A^1 rather than at the price cap. By comparing Tables I and II, we can see that an increase in competition among the generators decreases their payoffs and is favorable to the load.

Note that when congestion exists (line transfer capacity = 25) the building of gen_A^2 increases the competition at node A and tends therefore to decrease the price at node A while the price at node B remains unchanged. This causes an increase in the nodal price difference and therefore an increase of the ISO revenue. The ISO revenue increases from 132.5 \$ (see Table I) to 1,250 \$.

V. PORTFOLIO

The system we consider now is represented in Figure 6. Two machines are connected to the left node and three machines to the right node. The machines on the left side have a lower marginal cost than the machines on the right side.

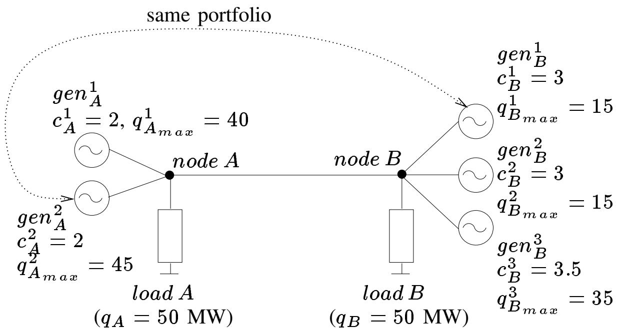


Fig. 6. We evaluate the impact of the portfolio and/or of the increase of the line transfer capacity on the benefits of the different agents of the system

In the previous section one machine could, in the absence of congestion, generate the energy for all the system, but that's not so here. And even together the machines located on one side can not generate all the power for the system. In order to show how the presence of the portfolio, together with the transmission capacity limit, can give rise to different market outcomes, we study four different cases. In the first one, the line transfer capacity is assumed to be 50, and no presence of portfolio is considered. In the second one, the line transfer capacity is the same, but we assume gen_A^2 and gen_B^1 belong to the same portfolio. The third and fourth cases are respectively analogous to the first and second ones, but with a line transfer capacity of 25 MW.

A. *Line transfer capacity equal to 50*

We consider here a transmission capacity of 50 MW.

When no portfolio is considered, an equilibrium point is reached at the first session of the market. In particular, the bids at $t = 0$ are equal to the marginal production cost both for the power producers at node A ($b_{A_0}^1 = 2$, $b_{A_0}^2 = 2$) and for the power producers at node B ($b_{B_0}^1 = 3$, $b_{B_0}^2 = 3$, $b_{B_0}^3 = 3.5$). To this equilibrium point corresponds the generation dispatches $q_A^1 = 40$, $q_A^2 = 45$, $q_B^1 = 7.5$, $q_B^2 = 7.5$, $q_B^3 = 0$ and the nodal prices $p_A = p_B = 3$.

Note that, even if the generators on the left side submit a price equal to their marginal cost 2, they get paid a price of 3 for the energy since it is the bid price on the units of the right that generates power ($q_B^i \neq 0$).

An equilibrium point is also reached at the first session of the market when the portfolio is considered. However, while the bids are the same for the generators gen_A^1, gen_B^2 and gen_B^3, the bids of the generators belonging to the portfolio are different. Their bids are now : $b_A^2 = 3.4$, $b_B^1 = 3.5$. To this equilibrium point corresponds the generation dispatches $q_A^1 = 40$, $q_A^2 = 45$, $q_B^1 = 0$, $q_B^2 = 15$, $q_B^3 = 0$ and the nodal prices $p_A = p_B = 3.4$.

Table III gathers the total rewards of each agent over the whole temporal horizon for the two cases analyzed.

It should be noted that the presence of a portfolio strategy penalizes strongly the loads but benefits generator gen_B^2. The latter can indeed take advantage of the portfolio strategy to sell more power and at a higher price.

B. *Line transfer capacity equal to 25*

We consider here a transmission capacity of 25 MW.

Rewards obtained by the different agents of the system over 25 stages								
gen_A^1	gen_A^2	gen_B^1	gen_B^2	gen_B^3	$Port.$	ISO	$load\ A$	$load\ B$
1000	1125	0	0	0	—	0	-3750	-3750
1400	1575	0	150	0	1575	0	-4250	-4250

TABLE III

REWARDS IN THE ABSENCE AND IN THE PRESENCE OF A PORTFOLIO COMPOSED OF gen_A^2 AND gen_B^1. THE LINE TRANSFER CAPACITY IS EQUAL TO 50 MW.

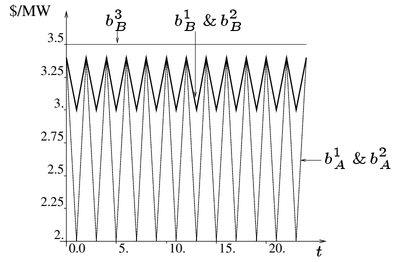


Fig. 7. *Input of the market* : the bids of gen_A^1, gen_A^1, gen_B^1, gen_B^2 and gen_B^3. No portfolios are considered.

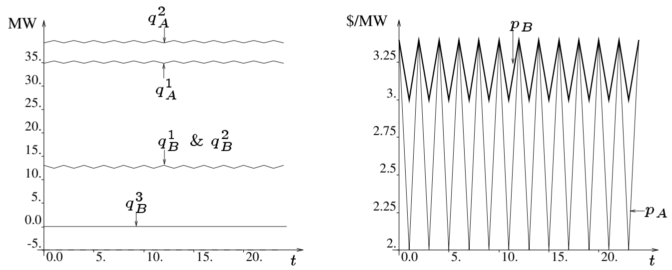
While an equilibrium point was reached with a line transfer capacity of 50 MW, that doesn't happen here. In fact, both in the case where each power plant belongs to a different owner and the case where the portfolio is considered, no equilibrium points are reached.

First, we consider the case where no portfolio is present. The evolutions of the generator bids are represented in Figure 7. It can be noted that $b_{A_t}^1$ and $b_{A_t}^2$ have the same behavior. The same observation holds for $b_{B_t}^1$ and $b_{B_t}^2$. It is also useful to remark that these four bids are below the marginal cost of $c_B^3 = 3.5$. The b_B^3 acts as a sort of *virtual price cap* for the other generators.

To these bids are associated the generation dispatches and the nodal prices displayed in Figure 8(a) and Figure 8(b). It can be noted that the production of each generator does not change remarkably during the whole time horizon even if the nodal prices vary greatly.

When a line capacity equal to 25 MW is considered, the presence of the portfolio does not influence at all the value of the generator bids.

By analyzing the case where gen_A^2 and gen_B^1 belong to the same portfolio, we have observed that the generator bids are the same as when no portfolio is considered. Therefore,



(a) Production of gen_A^1 and gen_B^1

(b) Prices at node A and node B

Fig. 8. *Output of the market* : production of each generator and nodal prices. No portfolios are considered.

	Rewards obtained by the different agents of the system over 25 stages							
K	gen_A^1	gen_A^2	gen_B^1	gen_B^2	gen_B^3	ISO	$load\,A$	$load\,B$
25	633	712	68	68	0	300	-3410	-4010
50	1000	1125	0	0	0	0	-3750	-3750

TABLE IV

INFLUENCE OF THE LINE TRANSFER CAPACITY ON THE REWARDS. 5 GENERATORS AND NO PORTFOLIO.

	Rewards obtained by the different agents of the system over 25 stages						
K	gen_A^1	$Port.$ ($r_{gen_A^2} + r_{gen_B^1}$)	gen_B^2	gen_B^3	ISO	$load\,A$	$load\,B$
25	633	780 (712 + 68)	68	0	300	-3410	-4010
50	1400	1575 (1575 + 0)	150	0	0	-4250	-4250

TABLE V

INFLUENCE OF THE LINE TRANSFER CAPACITY ON THE REWARDS. 5 GENERATORS WITH gen_A^2 AND gen_B^1 BELONGING TO THE SAME PORTFOLIO.

all the market outcomes (nodal prices, generation productions, and agent rewards) do not vary.

The limited transmission capacity hinders the portfolio from exercising its market power. In fact, when no congestion can occur (transmission capacity is 50 MW) the portfolio produces less (45 MW rather than 52.5 MW) but at a higher price. In fact, it has foreseen that the high bid price it posts for gen_B^2 gives rise to an increase of market price (from 3 to 3.4) and, since this price is paid for the dispatched quantity (45 MW produced only by the cheapest portfolio unit), it yields a large total profit. Whereas, when a transmission constraint exists, the cheapest portfolio unit (gen_A^2) can not produce its total capacity without entering into direct competition with gen_A^1, so a situation analogous to the case for which no portfolio was considered, occurs. Therefore, the limited transmission capacity prevents the portfolio from using the bids of the generators it owns at the different locations of the system to exercise its market power.

C. Some final remarks

Tables IV and V summarize the agent rewards evaluated in Sections V-A and V-B. In particular, the first table refers to the cases where no portfolio was considered, whereas the latter summarizes the payoffs obtained when gen_A^1 and gen_B^2 belong to the same portfolio.

It should be noted that, in the absence of a portfolio, the $load\,B$ as well as the two generators located at node A benefit from an increase in transmission capacity. The ISO, $load\,A$, and the generators at node B are, instead, penalized. The main reason for $load\,A$ being penalized by additional transfer capacities is that the congestion allows it to pay on average a cheaper price for the energy (2.728 \$/MW) than the marginal cost of any generator located on the right side (3 \$/MW at minimum).

The presence of a portfolio (Table V) reverses some of the observations made in Table IV. The average prices during the 25 periods are 2.728 \$/MW for $load\,A$ and 3.208 \$/MW for $load\,B$ when the available transmission capacity is 25 MW while, when the available transmission capacity is 50 MW, both loads experience an average nodal price of 3.4 \$/MW. These values give us a clear measure of how much worse off each load is when a greater transmission capacity is available and the portfolio is in place.

It is interesting to point out, once again, that when the portfolio is considered, an increase in transmission capacity *penalizes* considerably the two loads by allowing the portfolio to actively exercise its market power.

VI. Conclusions

We have considered in this paper an electricity market composed of a set of agents that compete under certain market rules, and have proposed a way to model the different agents of the market. The models have been used to build a market dynamics that has been analyzed through simulations.

By integrating the market dynamics for different configurations of the power system, we have been able to assess the influence of some factors, like the building of new generators, new transmission capabilities, and the presence of a portfolio, on the payoffs obtained by the different agents of the system. We have also observed that this market dynamics has a cyclic behavior when the transmission network faces congestions.

The approach we have developed here to analyze the characteristics of the electricity market may have several applications. It could be used for example to understand the complex phenomena that occur in the market. Furthermore, the different agents of a power system could also use it as a tool to decide which investments they should make to maximize their payoffs.

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