

# OPTIMAL COORDINATION OF OVERCURRENT RELAYS IN AN INTERCONNECTED POWER SYSTEM

Javad Sadeh

Assistant Professor

Electrical Department, Faculty of Engineering  
Ferdowsi University of Mashhad, Mashhad, IRAN  
[sadeh@ferdowsi.um.ac.ir](mailto:sadeh@ferdowsi.um.ac.ir)

**Abstract** - The objective of protective relay coordination in an interconnected power system is to achieve selectivity without sacrificing sensitivity and fast fault clearance time. In this paper, a novel method based on a nonlinear optimization algorithm is proposed for optimal coordination of overcurrent relays in an interconnected power system. Most of the previous algorithms proposed for the solution of this problem, supposed that the current setting of relays are known prior and only find the time multiplier setting (TMS) of the relays. In this paper, the current setting and time multiplier setting of all relays are considered as optimization parameters. The proposed algorithm is utilized to obtain the optimal setting of overcurrent relays in a realistic power system in IRAN. The results are presented and discussed.

**Keywords:** *Optimal coordination of overcurrent relay, Nonlinear optimization, Coordination time interval, Time multiplier setting, Current setting*

## 1 INTRODUCTION

Power system protection performs the function of fault detection and clearing as soon as possible, isolating whenever possible only the faulted component or a minimal set of components in any other case. Since the main protection system may fail (relay fault or breaker fault), protections should act as backup either in the same station or in the neighboring lines with time delay according to the selectivity requirement. The determination of the time delays of all backup relays is known as coordination of the protection system.

Coordination of protective relays is necessary to obtain selective tripping. The first rule of protective relaying is that the relay should trip for a fault in its zone. The second rule is that the relay should not trip for a fault outside its zone, except to back up a failed relay or circuit breaker. To coordinate this backup protection with the primary relay characteristic will ensure that the backup relay has sufficient time delay to allow the primary relay (and its breaker) to clear the fault.

Several methods have been proposed for the coordination of overcurrent relays. These methods can be classified into three classes: trail and error [1],

topological analysis method [2,3], and optimization method [4-7]. However, the solutions found by the first two classes, are not optimal in any strict sense, but simply the best of the tried possible solution. In other words, relay time multiplier settings are relatively high. In the optimal methods, the operating times of the relays are minimized, subject to the so-called coordination constraints, relays characteristic curves and the limits of the relays settings.

In the optimization method, some researchers used nonlinear programming for determining the optimal setting of pickup current and a linear programming for optimizing the time multiplier settings of the relays [4-5]. Other researchers [6] applied the linear programming technique only to minimize operating time while the pickup currents are selected based on experience.

Due to the complexity of nonlinear optimal programming techniques, the coordination of overcurrent relays is commonly performed by linear programming techniques, including the simplex, two-phase simplex and dual simplex methods. In these methods the current setting of the relays are assumed to be determined prior, and only find the time multiplier setting of the relays. Generally this is not the global optimum solution of the problem

In this paper, an optimal coordination method for overcurrent relays is proposed. The current setting and time multiplier setting of all relays are considered as optimization parameters and they are obtained simultaneously in an optimal manner. It is shown that lower protection operating time is achievable if the pickup current of the relays are determined in the optimization procedure. The proposed method is tested in an 8-bus test case and also in a realistic test case, that is consists of the 63kV power system located in the Eastern zone of Iran with 27 buses, 38 lines and 77 overcurrent relays.

## 2 OPTIMAL COORDINATION OF OVERCURRENT RELAY

### 2.1 Modeling of Overcurrent Relay Characteristic

Over-current relay generally include an instantaneous unit and inverse time equipment. The inverse time

operation characteristic can be provided in terms of a family of curves depending on a parameter usually referred as the time multiplier setting. The mathematical modelization of this family of curves can be performed using multiple regression techniques in order to obtain an expression giving the operating time in function of time multiplier and the current flowing through the relay. In general, overcurrent relays respond to a characteristic function of the type,

$$T = f(TMS, I_p, I) \quad (1)$$

where  $T$  is the operation time,  $TMS$  is time multiplier setting,  $I_p$  is the pickup current and  $I$  is the current flowing through the relay. Under simplistic assumption, the above equation can be approximated by the following equation [8]:

$$T = K_1 \frac{TMS}{(I/I_p)^{K_2} + K_3} \quad (2)$$

where  $K_1$ ,  $K_2$  and  $K_3$ , are constants that depend upon the specific device being considered. A more precise formula for approximating the relay characteristics is as follows [2]:

$$T = P(TMS)P(I_p) \quad (3)$$

where:

$$P(TMS) = K_{10} + K_{11}TMS + K_{12}TMS^2 + K_{13}TMS^3 \quad (4)$$

$$P(I_p) = A_0 + \frac{A_1}{(M-1)} + \frac{A_2}{(M-1)^2} + \frac{A_3}{(M-1)^3} + \frac{A_4}{(M-1)^4} \quad (5)$$

$M$  is the ratio of relay current ( $I$ ) to the pickup current ( $I_p$ ) and  $K_{10}$ ,  $K_{11}$ ,  $K_{12}$ ,  $K_{13}$ ,  $A_0$ ,  $A_1$ ,  $A_2$ ,  $A_3$ , and  $A_4$  are scalar quantities which characterize the particular device being simulated.

The calculation of two settings,  $TMS$  and  $I_p$ , is the essence of the overcurrent relay coordination study. It is very important to mention that in general these two parameters are discrete. In this study, however, these parameters were assumed to be continuous variables. The discrete solution are obtained by rounding-off the continuous solution to the nearest discrete values.

## 2.2 Problem Formulation

The general relay coordination problem can be stated as a parametric optimization problem. The objective function of operating time of the primary relays is minimized subject to keeping the operation of the backup relays coordinated. The objective function  $J$  to be minimized can be expressed as follow:

$$J = \sum_{i=1}^n T_{ii} \quad (6)$$

where  $T_{ii}$  is the operating time of the primary relay  $R_i$  for a close fault to relay  $i$ . It is assumed that the network consisting of  $n$  relays.

The operating time of the backup relay must be greater than the sum of the operating time of its primary relay and the coordination margin. This can be expressed as:

$$T_{ji} \geq T_{ii} + CTI \quad (7)$$

where  $T_{ji}$  is the operating time of the backup relay  $R_j$  for the same near-end fault at  $i$ , and  $CTI$  is the coordination time interval. There are many equations such eq. (7) for any pair of primary/backup relays for a given fault. Generally as shown in eq. (1), the relation between the operation time  $T$  of the time overcurrent unit, and the pickup current  $I_p$ , and time multiplier setting is a nonlinear function. As a consequence, in general this problem is a nonlinear optimization problem, but if the pickup current are determined prior and considering the eq. (2) as a relay characteristic, the objective function can be represented by a linear function of  $TMS$  and can be solved by linear programming methods. But in this paper, we try to solve this problem in general form, that means we consider the pickup current ( $I_p$ ) and the time multiplier setting ( $TMS$ ) as the optimization parameters and find the optimal value of them by solving the nonlinear optimization problem.

The other constraints in this optimization problem are the limitation of the variables as follows:

$$TMS_{i \min} \leq TMS_i \leq TMS_{i \max} \quad (8)$$

$$I_{p_i \min} \leq I_{p_i} \leq I_{p_i \max}$$

Equation (6) is optimized using the Nelder-Mead simplex (direct search) method [9] subject to that the operation of the backup relays remains properly coordinated and all of the constraints are satisfied.

In order to explain the proposed method, two case studies are presented in the next section. At first an 8-bus system and then a realistic test case, that is consists of the 63kV power system located in the Eastern zone of IRAN with 27 buses, 38 lines and 77 overcurrent relays, are considered.

## 3 CASE STUDIES

### 3.1 8-bus system

In this section the proposed method will be illustrated using the 8-bus, 9-branch network, taken from [10,11] and shown in figure 1. This figure also specifies the location of directional overcurrent relays. Table 1 presents the line data; detailed information is accessible in [10]. Using graph theory, the primary/backup relay pairs are identified and depicted in table 2. In this case study, the number of optimization parameters and constraints are 28 and 20 respectively.

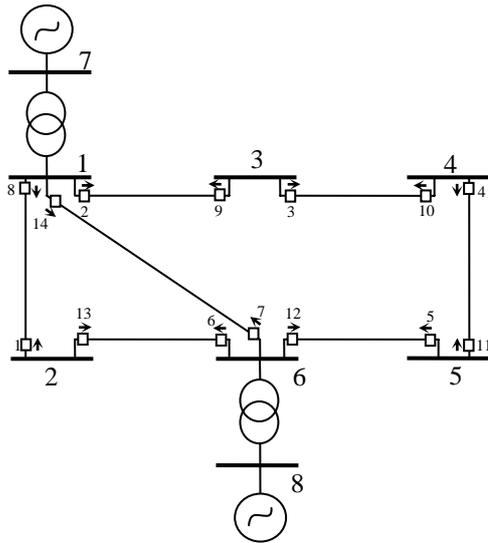


Figure 1: Single line diagram of 8-bus system

Table 1: Lines characteristics

Extreme nodes	R( $\Omega$ /km)	X( $\Omega$ /km)	Y(S/km)	L(km)
1 2	0.004	0.05	0.0	100
1 3	0.0057	0.0714	0.0	70
3 4	0.005	0.0563	0.0	80
4 5	0.005	0.045	0.0	100
5 6	0.0045	0.0409	0.0	110
2 6	0.0044	0.05	0.0	90
1 6	0.005	0.05	0.0	100

Table 2: List of primary/backup relay pairs

Pair no.	Primary relay	Backup relay	Pair no.	Primary relay	Backup relay
1	1	6	11	14	9
2	7	13	12	8	9
3	12	13	13	5	4
4	2	7	14	9	10
5	8	7	15	4	3
6	6	14	16	3	2
7	12	14	17	10	11
8	13	8	18	2	1
9	6	5	19	14	1
10	7	5	20	11	12

At first the current settings of the relays are supposed to be known (**case 1**). The values of these currents are given in [10] and are shown in third column of table 3. In the second column of table 3, the current transformer ratios (CTRs) are illustrated. Assuming a 0.3 sec. for CTI the results of optimization problem (linear programming) are shown in forth column of table 3. In this case the optimal value of objective function is 18.2043 seconds. If the current settings of the relays are considered as the optimization problem (**case 2**), we have 28 parameters which will be obtained from optimization, e.g. 14 TMSs and 14 current settings. Solving this problem using optimization toolbox of MATLAB<sup>®</sup>, the results are presented in fifth and sixth columns of table 3. In this case the optimal value of objective function is 10.0688 seconds. Comparing the results, it can be seen that considerable reduction in objective function is occurred, that means that the protection system is selective and as fast as possible.

Table 3: Optimal values of relays' TMSs and Pick-up taps

Relay no.	CT Ratio	Pick up Tap (case 1)	TMS (case 1)	Pick up Tap (case 2)	TMS (case 2)
1	240	0.5	0.4840	1.8	0.1000
2	240	2.0	0.2801	2.2	0.2482
3	160	1.5	0.2657	1.9	0.2145
4	240	1.5	0.1380	2.0	0.1000
5	240	1.0	0.1048	0.8	0.1000
6	240	2.0	0.3436	2.2	0.1621
7	160	1.5	0.4205	2.2	0.2222
8	240	1.5	0.4807	1.8	0.1873
9	160	1.5	0.6671	0.5	0.6007
10	240	1.5	0.5776	2.5	0.2218
11	240	1.0	0.8015	2.5	0.2810
12	240	2.0	0.6803	2.5	0.3575
13	240	0.5	0.6208	2.1	0.1000
14	160	1.5	0.6036	2.5	0.2781

### 3.2 A realistic power system

The second system on which the proposed method will be analyzed is the realistic 63kV sub-transmission system, located in the Eastern zone of IRAN (Khorasan province) with 27 buses, 38 lines and 77 overcurrent relays. The single line diagram of the network is shown in figure 2. In this system, five types of overcurrent relays are used. The characteristics of all relays are normally inverse type and these characteristic are modeled by suitable formula, applying curve fitting method and they used in the optimization algorithm. The time multiplier setting (TMS) ranges from 0.1 to 1 and pick-up taps range from 0.5 to 2.5.

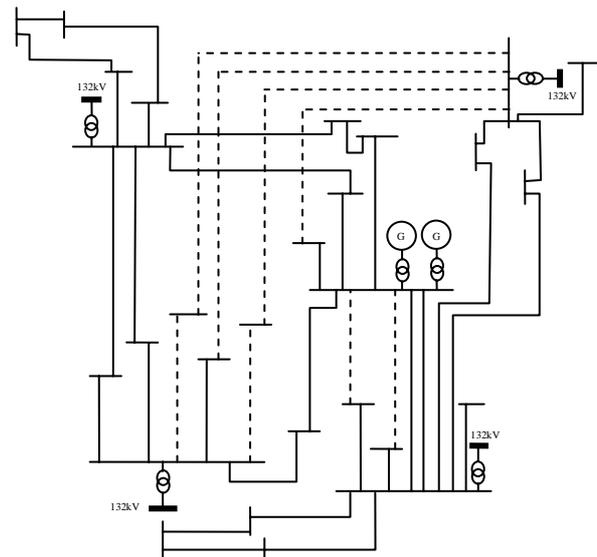


Figure 2: Single line diagram of the 63kV Khorasan network

Using the network topology the primary/backup relay pairs are identified. For this system an optimization problem are formulated integrating an objective function of operating time of the primary relays, 250

constraints, one for each primary/backup relay pairs, 154 (2×77) constraints representing the admissible range of the time multiplier setting of each relay and 154 constraints representing the admissible range of the pickup tap of each relay. The number of parameters which will be determined in optimization process is 154.

As the previous case, at first the current settings of the relays are supposed to be known (**case 3**). These values are shown in third column of table 4. In this case the number of optimization parameters is 77, e.g. TMS of each relay. Solving this problem using optimization toolbox of MATLAB<sup>®</sup> (linear programming), the optimal time multiplier settings of the relays are obtained and are shown in forth column of table 4. In this case the optimal value of objective function is 40.7697 seconds. If the current settings of the relays are considered as the optimization problem (**case 4**), we have 154 parameters which will be obtained from optimization, e.g. 77 TMSs and 77 current settings. As previous cases, assuming a 0.3 sec. for CTI, the results of optimization problem (Non-linear optimization) are shown in fifth and sixth columns of table 4. In this case the optimal value of objective function is 34.1883 seconds that means that a considerable reduction in operation time of the relays is achieved.

**Table 4:** Optimal values of relays' TMSs and Pick-up taps

Relay no.	CT Ratio	Pick up Tap (case 3)	TMS (case 3)	Pick up Tap (case 4)	TMS (case 4)
1	600	1.00	0.2831	1.5419	0.1819
2	600	1.00	0.2907	2.5000	0.1300
3	600	1.00	0.3856	1.3998	0.2792
4	600	1.00	0.3827	1.1727	0.2984
5	600	1.00	0.3556	1.7728	0.2721
6	600	1.00	0.2897	1.0590	0.3009
7	600	1.00	0.1000	0.8921	0.1021
8	600	1.00	0.1000	0.7595	0.1000
9	600	1.00	0.1000	0.7805	0.1000
10	600	1.00	0.1000	0.6898	0.1000
11	80	5.00	0.1982	1.2488	0.3396
12	80	5.00	0.1000	1.3723	0.1000
13	80	5.00	0.2018	2.5000	0.2557
14	80	5.00	0.2306	2.5000	0.2937
15	80	5.00	0.2368	2.5000	0.3005
16	80	5.00	0.1924	2.5000	0.2570
17	80	5.00	0.1924	2.5000	0.2570
18	80	5.00	0.1832	2.5000	0.1724
19	80	5.00	0.1885	1.4634	0.2530
20	600	0.50	0.1000	0.5000	0.1000
21	600	1.00	0.1000	0.5066	0.1000
22	600	1.00	0.1000	0.9073	0.1000
23	600	1.00	0.1631	1.0539	0.1589
24	600	0.50	0.1000	0.5000	0.1000
25	600	1.00	0.1000	0.5000	0.1000
26	600	1.00	0.1277	1.5032	0.1000
27	600	1.00	0.1269	1.2100	0.1000
28	600	1.00	0.2012	1.1916	0.1529
29	600	1.00	0.1230	1.1025	0.1058
30	600	1.00	0.1387	1.1106	0.1021
31	600	1.00	0.1959	1.4085	0.1500

**Table 4:** continue ...

Relay no.	CT Ratio	Pick up Tap (case 3)	TMS (case 3)	Pick up Tap (case 4)	TMS (case 4)
32	600	1.00	0.2825	2.0006	0.1513
33	600	1.00	0.2698	1.5303	0.1716
34	600	1.00	0.1916	1.0764	0.1775
35	600	1.00	0.1027	1.1120	0.1000
36	600	1.00	0.2245	1.2498	0.1558
37	600	1.00	0.2552	1.4352	0.1591
38	600	1.00	0.2151	1.2153	0.1680
39	600	1.00	0.2934	1.5563	0.2208
40	400	0.80	0.1000	0.6529	0.1046
41	400	1.00	0.1070	0.8941	0.1011
42	300	0.80	0.1090	0.7950	0.1000
43	300	0.80	0.1000	0.7318	0.1000
44	600	1.00	0.1533	1.4273	0.1000
45	600	1.00	0.1869	1.5218	0.1000
46	80	5.00	0.2758	2.5000	0.3316
47	80	5.00	0.2701	2.5000	0.3114
48	80	5.00	0.2629	2.5000	0.3249
49	80	5.00	0.1904	2.5000	0.2552
50	80	1.00	0.3864	2.5000	0.2435
51	80	1.00	0.3512	2.5000	0.2225
52	600	1.00	0.1279	1.4797	0.1000
53	600	1.00	0.1284	1.1704	0.1000
54	600	1.00	0.2729	1.8314	0.1548
55	300	1.75	0.2892	2.2523	0.2219
56	300	1.70	0.2943	2.1287	0.2306
57	600	1.00	0.2716	1.0764	0.2513
58	600	1.00	0.2643	1.0623	0.2466
59	600	1.00	0.1521	1.1864	0.1402
60	600	1.00	0.1521	1.1864	0.1402
61	600	1.00	0.2879	1.4861	0.1762
62	600	1.00	0.2053	1.7094	0.1000
63	600	1.00	0.1396	1.3388	0.1000
64	600	1.00	0.2571	1.0605	0.2242
65	600	1.00	0.2465	1.1120	0.2023
66	600	1.40	0.3717	1.7885	0.2544
67	600	1.00	0.3965	1.4841	0.2712
68	600	1.00	0.3365	1.1769	0.3047
69	600	1.00	0.2248	1.1285	0.1901
70	600	1.00	0.1396	1.3274	0.1000
71	600	1.00	0.1624	1.2497	0.1035
72	300	1.00	0.1119	0.9360	0.1094
73	300	1.00	0.1000	0.6603	0.1000
74	450	1.00	0.1323	1.3694	0.1000
75	450	1.00	0.1000	0.8327	0.1000
76	600	1.00	0.1677	1.1576	0.1356
77	600	1.00	0.1691	1.1475	0.1432

#### 4 CONCLUSION

In this paper, an optimization methodology is presented to solve the problem of coordinating directional overcurrent relays in an interconnected power system. Most of the previous algorithms, supposed that the current settings of relays are known prior and try to find the time multiplier setting of the relays. In this paper, the current setting and time multiplier setting of all relays were considered as optimization parameters and were obtained utilizing a non-linear optimization. The propose algorithm were used to obtain the optimal setting of overcurrent relays

in two case study systems, an 8-bus and a realistic power system in IRAN. Comparison the results, it can be seen that a considerable reduction in operation time of the relays is achieved.

#### REFERENCES

- [1] R.E. Albercht, et al, "Digital Computer Protection Device Coordination Program –I– General Program Description", *IEEE Trans. on Power Apparatus and Systems*, vol. PAS 83 (1964), no. 4, pp. 402-410
- [2] M.J. Damborg, et al, "Computer Aided Transmission Protection System Design, Part I – Algorithms", *IEEE Trans. on Power Apparatus and Systems*, vol. PAS 103 (1984), no. 1, pp.51-59
- [3] L. Jenkines, et al, "An Application of Functional Dependencies to the Topological Analysis of Protection Schemes", *IEEE Trans. on Power Delivery*, vol. 7 (1992), no. 1, pp. 77-83
- [4] A. Urdenta, et al, "Optimal Coordination of Directional Overcurrent Relays in Interconnected Power Systems", *IEEE Trans. on Power Delivery*, vol. 3 (1988), no. 3, pp. 903-911
- [5] N.A. Laway, H.O. Gupta, "A Method for Coordination of Overcurrent Relays in Interconnected Power System", *IE J*, vol. 74 (1993), pp. 59-65
- [6] B. Chattopadhyay, et al "An on-line Relay Coordination Algorithm for Adaptive Protection Using Linear Programming Technique", *IEEE Trans. on Power Delivery*, vol. 11 (1996), no. 1, pp. 165-173
- [7] A. Urdenata, et al "Optimal Coordination of Directional Overcurrent Relays Considering Dynamic Changes in the Network Topology",

*IEEE Trans. on Power Delivery*, vol. 12 (1997), no. 4, pp. 1458–1463

- [8] A.R.C. Warrington "The Protective Relays, Theory and Practice", *John Wiley & Son*, New York, 1969
- [9] J.C. Lagarias, et al, "Convergence Properties of the Nelder-Mead Simplex Algorithm in Low Dimensions", *SIAM Journal of Optimization*, May (1997)
- [10] A.S. Baraga and J.T. Saraiva, "Coordination of Overcurrent Directional Relays in Meshed Networks Using Simplex Method", *IEEE MELECON Conference*, vol. 3, (1996), pp. 1535-1538
- [11] H.A. Asgarian, et al, "A New Optimal Approach for Coordination of Overcurrent Relays in Interconnected Power Systems", *IEEE Trans. on Power Delivery*, vol. 18 (2003), no. 2, pp. 430–435

#### BIOGRAPHY



**JAVAD SADEH** was born in Mashhad, IRAN in 1968. He received the B.Sc. and M.Sc. in electrical engineering from Ferdowsi University of Mashhad in 1990 and 1994 respectively and the Ph.D from Sharif University of

Technology, Tehran Iran with the collaboration of the electrical engineering laboratory of the National Polytechnic Institute of Grenoble (INPG), France in 2000. Since then he served as an assistant professor at the Ferdowsi University of Mashhad. His research interests are Power System Protection, Restructuring, and Electromagnetic Transients in Power System.