SIMULTANEOUS BIDDING ON DAY-AHEAD AUCTION MARKETS FOR SPOT ENERGY AND POWER SYSTEMS RESERVE

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Abstract - In this paper a novel approach for simultaneous bidding on day-ahead auction markets for spot energy and power systems reserve is presented. For the spot market a relatively simple method is considered as a competitive market is assumed. For the reserve market the bidder is considered to follow a Bayes-strategy. Thereby only one bidder is assumed to behave strategically and the behavior of the competitors is summarized in a probability distribution of the market price. This results in a method for simultaneous bidding, where the bidding prices and capacities on the spot and reserve markets are calculated by maximizing a stochastic non-linear objective function of expected profit.

Keywords - Decision support, Simultaneous bidding, Non-linear optimization, Spot market, Reserve market

1 INTRODUCTION

In many liberalized electricity markets a new market segment has recently been established: competitive tendering of power systems reserves (for the German case cf. [1]). Thereby all generation units fulfilling defined requirements are allowed to bid in these markets. This development leads potential bidders to decide whether to bid an available capacity on the already established spot or on these recently commenced reserve markets.

In order to develop appropriate methods for bidding on these markets the respective market design is of special importance. Within this paper and following the operation of the auction markets in Germany the spot market is considered to be a uniform-priced double-sided call auction. The reserve markets are considered to be pay-as-bid one-sided procurement auctions. The markets will be described in more detail in the following.

Decision support for bidding on uniform-priced auction markets for spot electricity has been studied with extended approaches of optimal unit-commitment, cf. [2, 3]. By analysing these approaches may be seen that the liberalization of electricity markets was followed by bidding models focussing on profit maximization. Thereby deterministic and stochastic approaches may be distinguished.

Deterministic approaches have been presented e. g. by Conejo *et al.* [4] and Rodriguez and Andres [5]. Within these papers the *ex ante* unknown market price is considered to follow a stochastic price process described with econometric models. In both papers only the expected value of the price is used, not the price distribution. However, Rodriguez und Andres propose to cover the existing uncertainties of the market price by considering several defined scenarios of the market price. But these scenarios are evaluated separately resulting in a number of separate sub-optimal bidding solutions.

An improved consideration of the uncertain market price is possible by applying methods of stochastic programming, cf. [6]. Such approaches have been proposed for profit maximized bidding on electricity spot markets e. g. by Takriti *et al.* [7] and Ni *et al.* [8]. Within these papers a discrete subset of the uncertain market price is considered. The different scenarios are evaluated simultaneously resulting in just one optimal bidding solution. By applying such an approach one problematic aspect is to guarantee that the considered subset of the stochastic market price sufficiently represents the continuous price distribution. To overcome this problem the continuous probability density function may be considered directly.

The papers discussed so far have been predominantly focussing on spot markets with auction designs hardly comparable to the design of the auction markets for power systems reserve in Germany. Nevertheless, in a few papers methods for bidding on reserve markets have been developed. Allen and Ilic [9] and later Wen and David [10] discuss one-sided procurement auctions for power systems reserve (Wen and David also consider simultaneous bidding on a spot market). In both papers and in difference to the market design considered here a uniform-priced auction is considered. While Allen and Ilic assume the price to be known *ex ante*, Wen and David assume available probability distributions of the bidding price for each rival bidder. The main problem with the latter approach is the availability of detailed price data on historic bids of the rival bidders. A better approach could be to consider only one bidder to behave strategically and summarize the behavior of the remaining in a probability distribution of the market price. Within this paper such an approach is presented for bidding on the considered reserve markets.

The paper is organized as follows: In Section 2 the considered electricity day-ahead spot and reserve markets are described. Novel methodologies for bidding on such markets are presented in Section 3. In Section 4 the results of an exemplary estimation exercise are discussed. Finally, in Section 5 conclusions and indications for further research are given.

2 DAY-AHEAD AUCTION MARKETS

Following the liberalization of the electricity market in Germany several day-ahead auction markets have been established: one spot and four reserve markets.

2.1 Spot market

The spot market in Germany is operated by the European Energy Exchange AG (EEX) and commenced trading for physical contracts in June 2000. Trading is executed day-ahead in a double-sided call auction. Thereby participants submit bids for purchase and sale of hourly contracts for the following day. The bids are collected in a closed order book. Every trading day at noon the individual supply and demand curves are aggregated to a single supply and demand curve. The intersection between the two curves represents the balance between purchase and sale bids and determines the uniform market-clearing price. As long as there are no transmission constraints the spot market price is the same for all Germany.

2.2 Reserve market

As electricity cannot be stored in any major quantities, the amounts of electricity generated and consumed have to match exactly. Within a defined region this system balancing is in the responsibility of a transmission system operator (TSO). The TSO must guarantee to have enough excess generation available for use at all times so that if e. g. one generator fails, all loads may still be served without interruption. The quantity of this power systems reserve is defined *ex ante* and in the system of the Union for the Co-ordination of Transmission of Electricity (UCTE) the quality is differentiated in three qualities.

The reserve qualities differ in terms of the activation and response speed. Primary and secondary reserve are automatically called while tertiary reserve is called via rescheduling of generation. Primary reserve must be fully provided within 30 seconds, secondary reserve within 5 minutes and tertiary reserve within 15 minutes.

Previous to the liberalization of the electricity market the four TSO in Germany predominantly procured the reserve capacities within the same company. In 2001 the market opened due to requirements during merger control. Today the TSO run auction markets to procure power systems reserve by way of competitive tendering. These procurement auctions are characterized by simultaneous tendering of multiple generation units. Any bid consists of the offered capacity and two prices. One price is for holding the capacity in reserve (capacity price) and the other for delivery in case of actual use (energy price). For the remuneration of the accepted bids the pay-as-bid method is applied. Given the different reserve qualities this paper focuses on incremental tertiary only, as it is traded day-ahead (incremental reserve is procured to balance a shortage of supply).

3 BIDDING METHODOLOGY

The bidding problem in auction markets is defined by imperfections. In a perfect market, any bidder would be a price taker. Following microeconomic theory this would result in an optimal bidding price equal to the marginal costs. As soon as a bidder bids other than marginal costs he tries to exploit the imperfections in the market.

In the following a competitive spot market and a non-competitive reserve market is assumed. Hence, the strategically behaving bidder tries to exploit the imperfections of the considered reserve markets by applying a methodology for profit maximized bidding. On the spot market the bids correspond to the respective marginal costs. Thereby a few assumptions are considered:

A1) The strategically behaving bidder j knows the method for paying the bids. A bid is defined by a bidding capacity L_j^{B} and a bidding price p_j^{B}.

A2) The capacity $L^{\max}$ to be procured by the TSO on the respective reserve market is defined *ex ante*. The capacity is constant, price inelastic and known to the bidders. Each bidder i must at least bid a minimal capacity $L^{\min}$. The TSO must at least procure this minimal capacity from any bidder.

A3) The product, i. e. spot energy or power systems reserve, is homogeneous.

A4) The bidder j is risk neutral.

A5) The publication of the market prices is transparent and characterized by the bidding capacities L_i^{B} and prices p_i^{B} of all offers $i \in \mathbb{I}$ on the reserve markets and of the uniform spot market price p^{S}.

The methodology is based an the assumption that the behavior of the competing bidders can be summarized in a joint probability distribution of the market price. This Bayesian approach is based on the fundamental decision theoretic bidding model presented by Friedman [11] and on extensions for uniform-priced auctions by Hanssmann and Rivett [12] and later by Lavalle [13].

Following this general approach a probability of acceptance can be calculated leading to an expected profit to be maximized. With the bidding price p^{B} this probability of acceptance $\mathrm{P}^{\mathrm{A}}(\chi > p^{\mathrm{B}})$ may generally be calculated by (the index j for the strategically behaving bidder is omitted in the following):

$$\mathrm{P}^{\mathrm{A}}(\chi > p^{\mathrm{B}}) = 1 - F^{\chi}(p^{\mathrm{B}}) = 1 - \int_{-\infty}^{p^{\mathrm{B}}} f^{\chi}(p)\, dp \quad (1)$$

Thereby the market price is assumed to be a stochastic variable χ following the density function $f^{\chi}(p) : \mathbb{R} \rightarrow \mathbb{R}_{+}$ with the probability distribution $F^{\chi}(p) : \mathbb{R} \rightarrow [0, 1]$. Considering the bidding capacity L^{B} and the bidding costs c^{B} the expected profit $\tilde{\Pi}$ may then be calculated by:

$$\max_{\{p^{\mathrm{B}}\}} \quad \tilde{\Pi} = \mathrm{P}^{\mathrm{A}}(\chi > p^{\mathrm{B}})\, L^{\mathrm{B}} \left(p^{\mathrm{B}} - c^{\mathrm{B}}\right) \quad (2)$$

In Subsections 3.1 and 3.2 bidding methodologies based on this general approach are presented for spot and reserve markets respectively. Thereby the methodologies are formulated for bidding a capacity L^{B} provided by a single power plant in a single product on the respective market. In Subsection 3.3 these approaches are extended to consider a power plant portfolio, several products and simultaneous bidding.

3.1 Spot market

The spot market is characterized by a double-sided call auction with a uniform market-clearing price. This price can generally be seen as a stochastic variable allowing to derive a continuous probability distribution. For the uniform spot market price a log-normal distribution with density function $f^{\mathrm{S}}(p) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ may be taken:

$$f^{\mathrm{S}}(p) = \frac{1}{\sqrt{2\pi}\xi p} \exp\left(- \left(\frac{\ln(p) - \varrho}{\sqrt{2}\xi} \right)^2 \right) \quad (3)$$

The decision for the distribution function within this paper is based on an analysis of historic price data. Using time series of the spot market prices the distribution parameters ϱ and ξ in Eq. (3) can be derived using econometric methods. These parameters are directly related to the expected value μ and the standard deviation σ:

$$\xi = \sqrt{\ln((\sigma/\mu)^2 + 1)} \quad \text{and} \quad \varrho = \ln(\mu) - 0.5\xi^2 \quad (4)$$

Following Eq. (1) the probability of acceptance, considering that a bid is accepted only and entirely if the spot market price is higher than the bidding price, is calculated using the primitive of the distribution of the spot market price given in Eq. (3):

$$\mathrm{P}^{\mathrm{A}}(p^{\mathrm{S}} > p^{\mathrm{B}}) = 1 - \int_{-\infty}^{p^{\mathrm{B}}} f^{\mathrm{S}}(p)\, dp \quad (5)$$

As long as a competitive spot market is assumed the bidding price can be set equal to the marginal costs of generation, i. e. $p^{\mathrm{B}} = c^{\mathrm{S}}$. These bidding costs will be discussed in more detail in Subsection 3.3. With the assumption of a competitive market it is advisable for a bidder to offer a supply curve in the market. Here, the discrete steps of such a supply curve are given for each power plant in a given power plant portfolio by the respective marginal costs of generation and available capacities.

On the considered spot market the next-days uniform market price is *ex ante* unknown. In order to estimate the expected profit it is therefore necessary to find a description of the expected spot market price. This is possible by calculating the conditional expected spot market price:

$$\mathrm{E}[p^{\mathrm{S}} | p^{\mathrm{S}} > p^{\mathrm{B}}] = \frac{1}{\mathrm{P}^{\mathrm{A}}(p^{\mathrm{S}} > p^{\mathrm{B}})} \int_{p^{\mathrm{B}}}^{\infty} p\, f^{\mathrm{S}}(p)\, dp \quad (6)$$

With the probability of acceptance, cf. Eq. (5), the conditional expected price, cf. Eq. (6), and $p^{\mathrm{B}} = c^{\mathrm{S}}$ the expected profit on a spot market $\tilde{\Pi}^{\mathrm{S}}$ is given by:

$$\tilde{\Pi}^{\mathrm{S}} = \mathrm{P}^{\mathrm{A}}(p^{\mathrm{S}} > c^{\mathrm{S}})\, L^{\mathrm{B}} \left(\mathrm{E}[p^{\mathrm{S}} | p^{\mathrm{S}} > c^{\mathrm{S}}] - c^{\mathrm{S}} \right) \quad (7)$$

Even though energy is traded on the spot market the bidding capacity is considered in Eq. (7). This is possible as long as hourly products are assumed to be traded.

3.2 Reserve market

In a pay-as-bid procurement auction no uniform market price rather a price range can be observed. In fact the market price can be between the efficiency and the marginal price. The efficiency price p^{E} is thereby set by the less and the marginal price p^{M} by the most expensive accepted bid (as seen from the procurer's perspective). If the accepted offers are transparently published these characteristic market prices can be derived *ex post* by analysing the merit order of accepted offers. However, at the time of submitting a bid the efficiency price, the marginal price and the cascaded merit order curve are *ex ante* unknown. Hence, a solution on how to deal with these uncertainties needs to be found.

Following Assumption A5 historic time series of bidding capacities and prices are assumed to be available to the bidder. Hence, probability distributions of the efficiency and marginal prices, with the prices assumed to be stochastic variables, can easily be derived. For the efficiency price a Gaussian-mixture distribution with density function $f^{\mathrm{E}}(p) : \mathbb{R} \rightarrow \mathbb{R}_+$ may be taken:

$$f^{\mathrm{E}}(p) = \sum_{j=1}^{m} \frac{\lambda_j}{\sqrt{2\pi}\sigma_j} \exp\left(- \left(\frac{p - \mu_j}{\sqrt{2}\sigma_j} \right)^2 \right) \quad (8)$$

Eq. (8) gives a mixture of m normal distributions. The distribution is characterized by the expected values μ_j, the standard deviations σ_j and the probabilities λ_j (for the latter holds $\sum_{j=1}^{m} \lambda_j = 1$). Within this paper a mixture of two normal distributions ($m = 2$) is taken.

To estimate the density function of the marginal price, first a density function of the difference between the marginal and efficiency price needs to be calculated. For this difference an Erlang distribution with density function $f^{\Delta\mathrm{ME}}(p) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ and parameter b may be taken:

$$f^{\Delta\mathrm{ME}}(p) = \frac{p}{b^2} \exp\left(-\frac{p}{b} \right) \quad (9)$$

Knowing these basic distributions the probability density function $f^{\mathrm{M}}(p) : \mathbb{R} \rightarrow \mathbb{R}_+$ of the marginal price can be calculated by applying a single-sided convolution:

$$f^{\mathrm{M}}(p) = \int_{0}^{\infty} f^{\mathrm{E}}(p - u)\, f^{\Delta\mathrm{ME}}(u)\, du \quad (10)$$

Using time series of the efficiency and marginal prices the price distribution parameters in Eq. (8) and (9) can be derived using econometric methods. The decision for the distribution functions within this paper is based on an analysis of available historic price data and the important fact that using these functions the convolution in Eq. (10) can be found analytically, cf. [14].

So far was shown that characteristic market prices, i. e. efficiency and marginal prices, can be seen as stochastic variables and be represented by probability distributions. However, to follow the fundamental approach presented above the probability of acceptance need to be calculated. It may be first noted that one's bidder's offer will only be accepted if the bidding price is lower than a relevant market price. Thereby the relevant market price $p^{\mathrm{R}} \in [p^{\mathrm{E}}, p^{\mathrm{M}}]$ describes the price on the merit order of all offers necessary to displace competing bidders.

Hence, given a bidding price p^{B} the capacity procured depends on the bidding capacity L^{B} and the *ex ante* unknown and unpredictable cascaded merit order. For the merit order a linear approximation between the efficiency and marginal price is assumed. For the relevant market price p^{R} then follows:

$$p^{\mathrm{R}} = \left(p^{\mathrm{M}} - p^{\mathrm{E}}\right) k(L^{\mathrm{B}}) + p^{\mathrm{E}} \tag{11}$$

With $k(L^{\mathrm{B}}) \in [0, 1]$ as the index of the merit order:

$$k(L^{\mathrm{B}}) = \frac{L^{\max} - L^{\mathrm{B}}}{L^{\max} - L^{\min}} \tag{12}$$

Following the discussion above, the relevant market price, cf. Eq. (11), needs to be seen as a stochastic variable. Hence, Eq. (10) needs to be extended resulting in a density function of the relevant market price p^{R} depending on the bidding capacity L^{B}:

$$f^{\mathrm{R}}(p^{\mathrm{B}}\,; L^{\mathrm{B}}) = \int_{0}^{\infty} f^{\mathrm{E}}(p^{\mathrm{B}} - k(L^{\mathrm{B}})\, u)\, f^{\Delta \mathrm{ME}}(u)\; du \tag{13}$$

Following Eq. (1) the probability of acceptance, considering that a bid is accepted only and entirely if the relevant market price is higher than the bidding price, is calculated using the primitive of the distribution of the relevant market price given in Eq. (13):

$$\mathrm{P}^{\mathrm{A}}(p^{\mathrm{R}} > p^{\mathrm{B}}\,; L^{\mathrm{B}}) = 1 - \int_{-\infty}^{p^{\mathrm{B}}} f^{\mathrm{R}}(p\,; L^{\mathrm{B}})\; dp \tag{14}$$

Using this approach it is important to note that a single bid is sufficient for optimized bidding. It is hence not necessary to submit a discrete supply curve as often (and within this paper) proposed for a spot market. A bid will only be accepted if it is advantageous to the procurer and displaces the offers of the competing bidders. More than one bid of a single bidder would result in displacing another bid of the bidder's own supply curve.

The analysis so far has dealt with the market prices from a systems perspective with the efficiency and marginal prices described as stochastic processes. With the assumption of a non-competitive market a single bidder with a non-negligible bid quantity has however to take into account that his bid may influence the market price – not only in the bidding period itself but also in subsequent periods. Thereby, most problematic from a bidder's point of view are decreases in the future average price level as a consequence of his own bid. Such a long-term price effect of price dumping is most likely to occur, if the price bid becomes the efficiency price.

If the process of the efficiency price can be described by an ARMA model it can be shown that the long-term unity price effect due to price dumping can be calculated by analysing the step response on this "efficiency price system", cf. [14]. Considering the conditional expected efficiency price

$$\mathrm{E}[p^{\mathrm{E}} | p^{\mathrm{E}} > p^{\mathrm{B}}] = \frac{1}{\mathrm{P}[p^{\mathrm{E}} > p^{\mathrm{B}}]} \int_{p^{\mathrm{B}}}^{\infty} p\; f^{\mathrm{E}}(p)\; dp \tag{15}$$

the probability weighted average of the decrease of the efficiency price through the bidding price $\Delta \tilde{p}^{\nu}(p^{\mathrm{B}})$ can be determined by multiplying the price dumping effect ν with the expected height of the initial shock by:

$$\Delta \tilde{p}^{\nu}(p^{\mathrm{B}}) = \nu\, \mathrm{P}[p^{\mathrm{E}} > p^{\mathrm{B}}] \left(p^{\mathrm{B}} - \mathrm{E}[p^{\mathrm{E}} | p^{\mathrm{E}} > p^{\mathrm{B}}]\right) \tag{16}$$

With the probability of acceptance, cf. Eq. (14), and the long-term price effect of price dumping, cf. Eq. (16), the expected profit $\tilde{\Pi}^{\mathrm{R}}$ is given by:

$$\tilde{\Pi}^{\mathrm{R}} = \mathrm{P}^{\mathrm{A}}(p^{\mathrm{R}} > p^{\mathrm{B}}\,; L^{\mathrm{B}})\, L^{\mathrm{B}} \left(p^{\mathrm{B}} + \Delta \tilde{p}^{\nu}(p^{\mathrm{B}}) - c^{\mathrm{B}}\right) \tag{17}$$

In Eq. (17) also the expected bid costs c^{B}, i. e. the costs occurring when the bid is accepted, are relevant. These bid costs will be discussed in more detail in the following.

3.3 Simultaneous bidding

The methodologies presented so far focus on bidding on one market, a spot or a reserve market. However, in many electricity markets a bidder has the opportunity to decide on the capacity (and price) to offer on several spot and reserve markets. Within this paper trading on these markets is assumed to be simultaneous, i. e. an available capacity can be offered just once. Thereby a bidder will try to maximize his expected overall profit.

The assumption of competitive spot markets leads to consider just one such market (more than one would result in bidding on the market with the highest expected profit). The assumption of non-competitive reserve markets on the other hand leads to consider several such markets as strategic bidding is likely to occur.

For the spot and reserve markets different products are distinguished, one-hour products $o \in \mathbb{O}^{\mathrm{S}}$ and several-hour products $o \in \mathbb{O}^{\mathrm{R}}$ respectively (in the following also $\mathbb{O} = \mathbb{O}^{\mathrm{S}} \cup \mathbb{O}^{\mathrm{R}}$ is used). The duration of the products is defined using the binary variable $H_{o,h} \in \{0, 1\}$. This binary variable takes the value 1 if the product o is defined in the respective hour h. For each considered product separate and uncorrelated stochastic price processes are assumed. This allows to derive the probability density functions as given by Eq. (3), (8), (9) and (13).

In the following the available bidding capacities $L^{\mathrm{B}}_{o,k}$ are defined by a portfolio of power plants $k \in \mathbb{K}$. The maximal producing capacity indexed by power plant k and hour h is represented with $L^{\mathrm{P}_2}_{k,h}$. To maintain a certain degree of simplicity no complete unit-commitment is considered within this paper. Rather a preplanning is assumed resulting in a producing capacity $L^{\mathrm{P}_0}_{k,h} \geq 0$ of the available power plants in each hour.

The product specific bidding capacities on the reserve markets are defined to be the sum over the power plants,

i. e. $L_o^{\mathrm{B}} = \sum_{k \in \mathbb{K}} L_{o,k}^{\mathrm{B}}$. The boundaries of the bidding capacities are not only given by the available capacities but are also subject to the reserve market products and given by the minimal and maximal capacities to be procured, $L_o^{\min}$ and $L_o^{\max}$ respectively. This would result in a mixed-integer problem that is omitted within this paper by setting the lower boundary to be zero. Whereas the upper boundary for the spot market products equals infinity the lower boundary is also set to be zero.

In the following Eq. (7) need to be extended to consider the spot market products o and power plants k to calculate one's bidders expected profit $\tilde{\Pi}_{o,k}^{\mathrm{S}}$ on the spot market. Therefore, the following variables have to be replaced: $p^{\mathrm{S}} = p_o^{\mathrm{S}}$, $L^{\mathrm{B}} = L_{o,k}^{\mathrm{B}}$ and $c^{\mathrm{S}} = c_{o,k}^{\mathrm{S}}$. The marginal generation costs can be calculated by:

$$c_{k,h}^{\mathrm{S}}(L_{k,h}^{\mathrm{P}_1}, L_{k,h}^{\mathrm{P}_0}) = \frac{Q(L_{k,h}^{\mathrm{P}_1}) - Q(L_{k,h}^{\mathrm{P}_0})}{L_{k,h}^{\mathrm{P}_1} - L_{k,h}^{\mathrm{P}_0}} \cdot c_k^{\mathrm{F}} \qquad (18)$$

Thereby, the producing capacity $L_{k,h}^{\mathrm{P}_0}$ after preplanning and, considering accepted offers on the spot market, the producing capacity $L_{k,h}^{\mathrm{P}_1} = L_{k,h}^{\mathrm{P}_0} + L_{o,k}^{\mathrm{B}} H_{o,h}$ is needed. The non-linear fuel consumption curve, multiplied by the fuel costs c_k^{F}, can be given by ($\varsigma_i = \mathrm{const}$):

$$Q(L_{k,h}^{\mathrm{P}}) = \varsigma_0 + \varsigma_1 L_{k,h}^{\mathrm{P}} + |\varsigma_2| \left(L_{k,h}^{\mathrm{P}}\right)^2 \qquad (19)$$

In the following Eq. (17) need to be extended to consider the reserve market products o and power plants k to calculate one's bidders expected profit $\tilde{\Pi}_{o,k}^{\mathrm{R}}$ on the reserve markets. Therefore, the following variables have to be replaced: $p^{\mathrm{R}} = p_o^{\mathrm{R}}$, $p^{\mathrm{B}} = p_o^{\mathrm{B}}$ and $L^{\mathrm{B}} = L_o^{\mathrm{B}} = \sum_{k \in \mathbb{K}} L_{o,k}^{\mathrm{B}}$. Contrarily to the marginal generation costs considered for bidding on the spot market any bid costs on the reserve markets are neglected as the procured reserve is seldom actually used, cf. [1, 14].

This discussion finally results in a method for simultaneous bidding, where the bidding prices on the reserve markets and the capacities on the spot and reserve markets are calculated by maximizing a stochastic non-linear objective function of expected profit given by:

$$\begin{aligned} &\max_{\{p_o^{\mathrm{B}},\, L_{o,k}^{\mathrm{B}}\}} \quad \tilde{\Pi} = \sum_{k \in \mathbb{K}} \left(\sum_{o \in \mathbb{O}^{\mathrm{S}}} \tilde{\Pi}_{o,k}^{\mathrm{S}} + \sum_{o \in \mathbb{O}^{\mathrm{R}}} \tilde{\Pi}_{o,k}^{\mathrm{R}} \right) \\ &\text{s.t.} \quad L_o^{\min} \le \sum_{k \in \mathbb{K}} L_{o,k}^{\mathrm{B}} \le L_o^{\max} \;\; ; \;\; L_{o,k}^{\mathrm{B}} \ge 0 \\ &\qquad \sum_{o \in \mathbb{O}} L_{o,k}^{\mathrm{B}} H_{o,h} \le L_{k,h}^{\mathrm{P}_2} \;\; ; \;\; p_o^{\mathrm{B}} \ge 0 \; (p_o^{\mathrm{B}} \in \mathbb{O}^{\mathrm{R}}) \end{aligned} \qquad (20)$$

4 EXEMPLARY APPLICATION

It has been mentioned that no complete unit-commitment is considered within this paper in order to maintain a certain degree of simplicity. The exemplary application is therefore simplified by assuming a preplanning of a defined exemplary power plant portfolio. With this simplification it is possible to neglect e. g. availabilities, start-up costs and minimum operation times.

The results of the non-linear objective function in Eq. (20) are derived with Matlab® and the `fmincon` function, based on a sequential quadratic programming approach, of the optimization toolbox.

To be able to show the applicability of the methods in Subsection 4.1 the markets and products to be considered and in Subsection 4.2 a power plant portfolio is described. This is followed by a discussion of the main results in Subsection 4.3.

4.1 Markets and products

The application is based on real-world markets operating in Germany. Thereby one spot market, operated by the European Energy Exchange AG (EEX), and two reserve markets, respectively operated by the RWE Net AG and the E.ON Netz GmbH, are considered. On these markets different product times are distinguished, one-hour products on the spot and several-hour products on the reserve markets. On the RWE market five products represented by $\mathbb{O}^{\mathrm{R}_1}$ are traded. Except product o_3 all products cover time periods of four hours: product o_1 from 0-4 am, product o_2 from 4-8 am, product o_3 from 8 am-4 pm, product o_4 from 4-8 pm and product o_5 from 8-12 pm. On the E.ON market two products represented by $\mathbb{O}^{\mathrm{R}_2}$ are traded: one peak product o_6 covering the time period from 6 am-10 pm and one base product o_7 covering all other hours.

For these products the parameters of the respective probability density functions as needed in Eq. (3), (8), (9) and (13) are given in Table 1 and 2, respectively for the spot and reserve market products. The parameters have been estimated using publicly available historic time series of the relevant market prices. The estimation of the parameters and the day-ahead forecasts of the expected prices are based on econometric methods applying the classic ARMA approach, cf. [14].

	μ	σ		μ	σ
o_1	14.5	7.3	o_{13}	30.8	19.4
o_2	10.0	7.5	o_{14}	28.8	16.9
o_3	9.1	7.7	o_{15}	28.2	15.3
o_4	8.2	6.5	o_{16}	26.0	12.4
o_5	9.9	7.4	o_{17}	22.8	9.4
o_6	15.6	7.7	o_{18}	22.5	9.1
o_7	18.2	8.2	o_{19}	22.1	9.3
o_8	27.4	14.2	o_{20}	21.2	9.3
o_9	28.2	14.2	o_{21}	21.8	9.8
o_{10}	28.8	16.6	o_{22}	20.5	10.4
o_{11}	32.3	18.9	o_{23}	18.5	8.4
o_{12}	44.2	30.1	o_{24}	13.7	5.6

Table 1: Parameters of the EEX spot market probability density functions for the products $o \in \mathbb{O}^{\mathrm{S}}$, μ and σ in (€/MWh)

The reserve markets are characterized by the minimal bidding and maximal procuring capacities. Here the minimal capacities are set to be $L_o^{\min} = 0$ MW $\forall\, o \in \mathbb{O}^{\mathrm{R}}$. The maximal capacities are set to be $L_o^{\max} = 750$ MW $\forall\, o \in \mathbb{O}^{\mathrm{R}_1}$ and $L_o^{\max} = 1100$ MW $\forall\, o \in \mathbb{O}^{\mathrm{R}_2}$.

	o_1	o_2	o_3	o_4	o_5	o_6	o_7
λ_1	0.2	0.3	0.2	0.2	0.3	0.3	0.4
λ_2	0.8	0.7	0.8	0.8	0.7	0.7	0.6
μ_1	6.5	13.7	108.2	27.7	9.4	126.2	15.5
μ_2	7.1	14.1	108.8	28.3	9.8	126.6	15.6
σ_1	3.6	2.7	27.3	4.9	2.3	17.4	1.7
σ_2	0.1	0.1	1.1	0.7	0.1	2.6	0.3
ν	11.3	10.8	13.4	12.6	10.9	2.6	3.4
b	0.4	0.6	0.6	0.4	0.5	0.8	0.6

Table 2: Parameters of the RWE and E.ON reserve market probability density functions for the products $o \in \mathbb{O}^{\mathrm{R}}$, μ_j and σ_j in (€/MW)

4.2 *Power plant portfolio*

The exemplary power plant portfolio is defined according to reflect a small generation company. In the portfolio two coal fired (k_1 and k_2), one pumped hydro (k_3) and two gas fired power plants (k_4 and k_5) are considered. The characteristics of the plants are given in Table 3. Thereby the maximal producing capacity, the parameters of the fuel consumption curve, cf. Eq. (19), and the fuel costs, needed for calculating the marginal costs of generation cf. Eq. (18), are given. The latter have been derived using statistics of the German electricity system, cf. [14].

		k_1	k_2	k_3	k_4	k_5
L^{P_2}	(MW)	150	100	100	150	75
ς_0	(MW)	28.0	31.7	5.3	19.5	60.3
ς_1	(–)	2.18	2.13	0.98	2.30	2.19
ς_2	(MW^{-1} 10^{-3})	1.56	3.76	1.63	0.58	4.00
c^{F}	(€/MWh)	5.3	5.3	15.0	16.4	16.4

Table 3: Exemplary power plant portfolio

It is assumed that the bidder needs to cover a contracted and inflexible demand. The demand is partly covered by the exemplary power plant portfolio following a least-cost preplanning unit-commitment not described in this paper. Following this assumption some of the power plants are expected to operate in part-load in different moments of time with the remaining capacities available to bid on the considered markets. The resulting available capacities for simultaneous bidding are given in Figure 1.

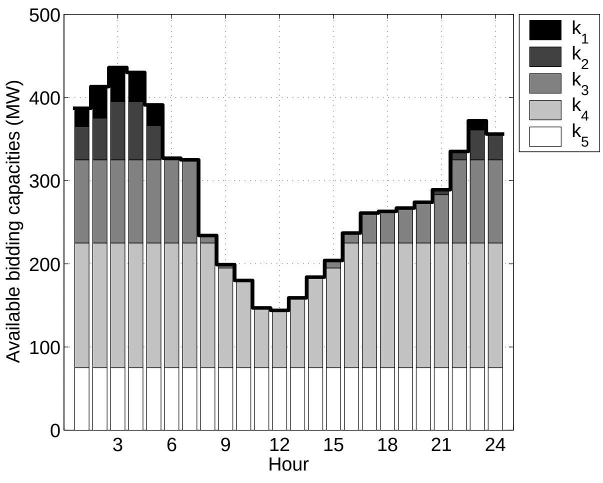

Figure 1: Available bidding capacities after preplanning

4.3 *Results*

In an analysis of historic price data of the considered spot and reserve markets may be seen that the expected profits on the reserve markets are generally higher than on the spot market, cf. [1]. Thereby the expected profits on the E.ON market are higher than on the RWE market, cf. Table 2. This may lead to the initial assumption that nearly all of the available bidding capacities will be offered on the E.ON reserve market. However, this assumption neglects the reserve markets to be non-competitive.

Hence, optimized bidding on the reserve markets will not always result in offering the available capacity entirely on the market with the highest expected profit. This is due to the presented methodology with the probability of acceptance depending on the bidding capacity, cf. Eq. (14). With an in-depth analysis of the methodology may be seen that the function of expected profit over the bidding capacity is concave and monotonic increasing, cf. [14]. From this follows *a priori* that the optimal bidding capacity equals the maximal one. However, considering the function being concave a non-proportional increase of the expected profit with increasing bidding capacity can be observed. This implies that it might be advisable to bid only a part of the total available bidding capacity in one market when simultaneous trade on several markets is possible.

Nevertheless, by applying the methodology for simultaneous bidding and considering the described products and the defined power plant portfolio it can be shown that high capacities are offered on the E.ON reserve market, cf. Table 4 and Figure 2. However, it can be seen that in several hours bidding on the spot market is superior to bidding on the reserve markets. This is partly due to the already discussed concave function of expected profit. Another reason is a relatively low difference between the expected spot market prices and the marginal generation costs in these hours. It can, hence, be shown that the methodology reflects the *a priori* expectation that power plants with high marginal generation costs (k_4 and k_5) are predominantly offered on the reserve markets.

	o_1	o_2	o_3	o_4	o_5	o_6	o_7
p^{E}	7.0	14.0	108.7	28.2	9.7	126.5	15.6
p^{M}	7.8	15.2	109.9	29.0	10.7	128.1	16.8
p^{B}	7.2	14.2	–	28.2	9.9	122.9	15.5
L^{B}	81	81	–	81	81	144	192

Table 4: Expected efficiency and marginal and optimized bidding prices (€/MW) and capacities (MW) as estimated for the products $o \in \mathbb{O}^{\mathrm{R}}$ on the RWE and E.ON reserve markets

Next to the optimized bidding capacities also the expected efficiency and marginal as well as the optimized bidding prices are given in Table 4. It may be seen that the optimized bidding prices on the RWE market are between the expected efficiency and marginal prices. The optimized bidding prices on the E.ON market on the other hand may be seen to be lower. This difference in bidding is due to the estimated value of the price dumping effect ν that has been estimated to be higher for the products on the RWE than on the E.ON market, cf. Table 2. Here may be worth to note that the expected efficiency and marginal prices do not necessarily equal to the actual values on the respective trading day. The latter depend on the bidding behavior of the competitors. However, the presented methodology accounts for deviations between the expected values and the actual prices by considering the complete price distribution, i. e. not only the expected values also the standard deviations are considered.

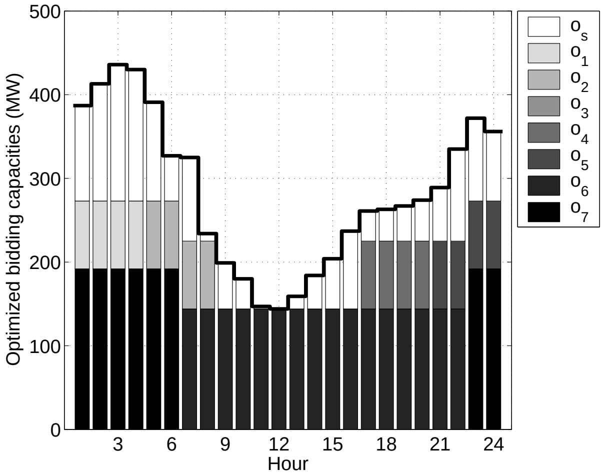

Figure 2: Optimized spot and reserve market bidding capacities

The maximized expected overall profit of 23726 € is dominated by the reserve market products and especially by the peak-product o_6 on the E.ON market. Therefore, the expected profit on the reserve markets is higher than on the spot market, 18778 € and 4948 € respectively. The low fraction of the expected spot market profit is due to relatively high marginal generation costs compared to the expected spot market prices (even though the power plants with the lowest marginal generation costs are offered on the spot market). In this case the respective offers are characterized by relatively low probabilities of acceptance and this hence results in minor expected profits.

5 CONCLUSIONS

In this paper a methodology is presented that enables a strategically behaving bidder to estimate the profit maximizing offers on simultaneously traded day-ahead auction markets for spot energy and power systems reserve. The methodology is based on deriving appropriate probability density functions of the relevant market prices. Thereby special focus is given on bidding on non-competitive reserve markets designed as pay-as-bid procurement auctions. The applicability is discussed using exemplary data. It is shown that the methodology accounts for the interdependencies between diverse products on several markets, helps to manage existing price uncertainties and may hence support the trading decisions of a bidder. Based on the presented methodology further development may omit one or more of the considered assumptions, i. e. homogeneous products, risk neutral bidder, transparent historic bidding data. Further research may lead to implement the developed methods in unit-commitment approaches and may consider today's practice of sequential trading.

REFERENCES

[1] D. J. Swider and C. Weber, "Ausgestaltung des deutschen Regelenergiemarktes" (in German), *Energiewirtschaftliche Tagesfragen*, 53(7):448-453, 2003.

[2] A. G. Kagiannas, D. T. Askounis and J. Psarras, "Power Generation Planning: A Survey From Monopoly to Competition", *International Journal of Electrical Power and Energy Systems*, 26(6):413-421, 2004.

[3] H. Yamin, "Review on Methods of Generation Scheduling in Electric Power Systems", *Electric Power Systems Research*, 69(2-3):227-248, 2004.

[4] A. J. Conejo, F. J. Nogales and J. M. Arroyo, "Price-Taker Bidding Strategy Under Price Uncertainty", *IEEE Transactions on Power Systems*, 17(4):1081-1088, 2002.

[5] C. P. Rodriguez and G. J. Anders, "Bidding Strategy Design for Different Types of Electric Power Market Participants", *IEEE Transactions on Power Systems*, 19(2):964-971, 2004.

[6] J. R. Birge and F. Louveaux, *Introduction to Stochastic Programming*, New York, Springer, 1997.

[7] S. Takriti, B. Krasenbrink and L. S. Y. Wu, "Incorporating Fuel Constraints and Electricity Spot Prices into the Stochastic Unit Commitment Problem", *Operations Research*, 48(2):268-280, 2000.

[8] E. Ni, P. B. Lu and S. Rourke, "Optimal Integrated Generation Bidding and Scheduling with Risk Management Under a Deregulated Power Market", *IEEE Transactions on Power Systems*, 19(1):600-609, 2004.

[9] E. H. Allen and M. D. Ilic, "Reserve Markets for Power Systems Reliability", *IEEE Transactions on Power Systems*, 15(1):228-233, 2000.

[10] F. Wen and A. K. David, "Coordination of Bidding Strategies in Day-Ahead Energy and Spinning Reserve Markets", *International Journal of Electrical Power and Energy Systems*, 24(1):251-261, 2002.

[11] L. Friedman, "A Competitive-Bidding Strategy", *Operations Research*, 4(1):104-112, 1956.

[12] F. Hanssmann and B. H. P. Rivett, "Competitive Bidding", *Operational Research Quarterly*, 10(1):49-55, 1959.

[13] I. H. Lavalle, "A Bayesian Approach to an Individual Player's Choice of Bid in Competitive Sealed Auctions", *Management Science*, 13(7):584-597, 1967.

[14] D. J. Swider, *Handel an Regelenergie- und Spotmärkten: Methoden zur Entscheidungsunterstützung für Netz- und Kraftwerksbetreiber* (in German), University of Stuttgart, Ph.D. thesis, 2005.