1 Constants

\[ q = 1.602 \times 10^{-19} \text{C} \]
\[ k = 1.38 \times 10^{-23} \text{JK}^{-1} \]
\[ n_i = 1.1 \times 10^{16} \text{carriers/m}^3 @ T = 300 \text{K} \]
\[ n_i \text{ doubles for every 11}^\circ \text{C increase in temperature} \]
\[ n \times p = n_i^2 \]
\[ \varepsilon_0 = 8.854 \times 10^{-12} \text{Fm}^{-1} \]
\[ K_{ox} \cong 3.9 \]
\[ K_s \cong 11.8 \]

2 Diode

\[ V_T = \frac{kT}{q} \cong 26 \text{mV} @ 300K \]

2.1 Reverse-Biased

\[ Q = 2C_{j0} \Phi_0 \sqrt{1 + \frac{V_A}{\Phi_0}} \]
\[ C_j = \frac{C_{ox}}{\sqrt{1 + \frac{2V}{\Phi_0}}} \]
\[ C_{j0} = \sqrt{\frac{qK_{ox}N_AN_D}{\Phi_0Q}} \] if \( N_A \gg N_D \)
\[ \Phi_0 = V_T \ln \left( \frac{N_AN_D}{n_i^2} \right) \]

2.2 Forward-Biased

\[ I_D = I_S \exp \left( \frac{V_D}{V_T} \right) \]
\[ I_S = ADqni \left( \frac{D_p}{L_nN_A} + \frac{D_n}{L_pN_D} \right) \]

Small-Signal Model

\[ r_d = \frac{V_T}{I_D} \]
\[ C_T = C_d + C_j \]
\[ C_d = \tau d \frac{L}{V_T} \]
\[ C_j = 2C_{j0} \]

3 N-channel MOSFET

For p-channel MOSFET, use the same equations as for the n-channel, with negative signs in front of all voltages.

\[ V_{eff} = V_GS - V_{th} \]
\[ V_{th} = V_{tn} - \gamma \left( \sqrt{V_{SB} + 2\Phi} - \sqrt{2\Phi} \right) \]
\[ \Phi_F = V_T \ln \left( \frac{N_A}{n_i} \right) \text{ (see diode equations for } V_T) \]
\[ \gamma = \sqrt{2qK_{ox}N_A} \]
\[ C_{ox} = \frac{K_{ox}}{\tau_{ox}} \]

3.1 Triode region (\( V_{GS} > V_{th}, V_{DS} \leq V_{eff} \))

\[ I_D = \mu_nC_{ox} \left( \frac{W}{L} \right) \left[ (V_{GS} - V_{th})V_{DS} - \frac{V_{DS}^2}{2} \right] \]

Small-Signal Model (\( V_{DS} \ll V_{eff} \))

\[ r_{ds} = \frac{\partial V_{DS}}{\partial I_D} = \frac{1}{\mu_nC_{ox} \left( \frac{W}{L} \right) V_{eff} - V_{DS}} \cong \frac{1}{\mu_nC_{ox} \left( \frac{W}{L} \right) V_{eff}} \]
\[ C_{gd} = C_{gs} = \frac{1}{2}WLC_{ox} + WL_{ov}C_{ox} \]
\[ C_{sb} = C_{db} = \frac{C_{10}(A_s + W/L)}{\sqrt{1 + \frac{4V_D}{\Phi_0}}} \]

3.2 Active (Pinch-Off) Region (\( V_{GS} > V_{th}, V_{DS} \geq V_{eff} \))

\[ I_D = \frac{1}{2}\mu_nC_{ox} \left( \frac{W}{L} \right) (V_{GS} - V_{th})^2 \left[ 1 + \lambda (V_{DS} - V_{eff}) \right] \]
\[ \lambda = \frac{k_{ds}}{2L\sqrt{V_{DS} - V_{eff} + \Phi_0}} \]
\[ k_{ds} = \frac{\sqrt{2K_{ox}}}{qN_A} \]
\[ V_{eff} = V_{GS} - V_{th} = \frac{2I_D}{\mu_nC_{ox}W/L} \]

Small-Signal Model

\[ g_m = \frac{\partial I_D}{\partial VGS} = \mu_nC_{ox} \left( \frac{W}{L} \right) V_{eff} = \sqrt{2\mu_nC_{ox} \left( \frac{W}{L} \right)} I_D = \frac{2I_D}{V_{eff}} \]
\[ g_s = \frac{\partial I_D}{\partial VSB} = \frac{\gamma g_m}{2\sqrt{V_{SB} + 2\Phi}} \]
\[ r_{ds} = \frac{\partial V_{DS}}{\partial I_D} \cong \frac{1}{I_D} \]
\[ C_{gs} = \frac{3}{2}WLC_{ox} + WL_{ov}C_{ox} = \frac{3}{2}WLC_{ox} + WC_{gs-ov} \]
\[ C_{gd} = WL_{ov}C_{ox} = WC_{gd-ov} \]
\[ C_{sb} = (A_s + W/L)C_{js} + P_sC_{j-sw} \]
\[ C_{js} = \frac{C_{10}}{\sqrt{1 + \frac{2V_D}{\Phi_0}}} \]
\[ C_{db} = A_dC_{jd} + P_dC_{j-sw} \]
\[ C_{jd} = \frac{C_{10}}{\sqrt{1 + \frac{4V_D}{\Phi_0}}} \]

3.3 Default values for MOSFET (0.8 \mu m)

n-channel \quad T = 300K (Room temperature) \quad p-channel

\[ \mu_nC_{ox} = 92 \mu A/V^2 \quad \text{(30)} \]
\[ V_{thn} = 0.8V \quad \text{(V_{tp-0} = -0.9V)} \]
\[ \gamma = 0.5V^{1/2} \quad \text{(0.8)} \]
\[ r_{ds} (\Omega) = 8000L (\mu m) / I_D (m A) \text{ in active region} \quad \text{(12000)} \]
\[ C_{js} = C_{jd} (= C_{j}) = 2.4 \times 10^{-4} \text{pF/(}\mu\text{m})^2 \quad \text{(4.5 \times 10^{-4})} \]
\[ C_{j-sw} = 2.0 \times 10^{-4} \text{pF/}\mu\text{m} \quad \text{(2.5 \times 10^{-4})} \]
\[ C_{ox} = 1.9 \times 10^{-3} \text{pF/(}\mu\text{m})^2 \quad \text{(1.9 \times 10^{-3})} \]
\[ C_{gs-ov} = C_{gd-ov} = 2.0 \times 10^{-4} \text{pF/}\mu\text{m} \quad \text{(2.0 \times 10^{-4})} \]
4 Design rules

The design rules are expressed in terms of a quantity, $\lambda$, where $\lambda$ is $\frac{1}{2}$ the minimum permitted gate length ($L = 2\lambda$). The corresponding layout of the active, polysilicon, and contact masks of the smallest transistor that can be realized in a given process when a contact must be made to each junction is summarized hereafter.

The n well surrounds the p-channel MOST, by at least $3\lambda$. The minimum spacing between the n well and the junctions of n-channel MOST is $5\lambda$. Therefore, the closest an n-channel MOST can be placed to a p-channel MOST is $8\lambda$. The minimum widths of poly, metal 1, and metal 2 are $2\lambda$, $2\lambda$, and $\lambda\bar{3}$, respectively.

5 Filters

5.1 First order

General form $H(s) = \frac{k_1s + k_0}{s + \Omega_0}$

Low Pass $H(s) = \frac{\Omega_0}{s + \Omega_0}$

High Pass $H(s) = \frac{s}{s + \Omega_0}$

5.2 Second order (Biquad)

General form $H(s) = \frac{k_2s^2 + k_1s + k_0}{s^2 + (\Omega_0/Q)s + \Omega_0^2}$

Low Pass $H(s) = \frac{\Omega_0^2}{s^2 + (\Omega_0/Q)s + \Omega_0^2}$

Band Pass $H(s) = \frac{\Omega_0^2}{s^2 + (\Omega_0/Q)s + \Omega_0^2}$

Band Stop $H(s) = \frac{s^2 + \Omega_0^2}{s^2 + (\Omega_0/Q)s + \Omega_0^2}$

High Pass $H(s) = \frac{s^2}{s^2 + (\Omega_0/Q)s + \Omega_0^2}$

6 Z transform

<table>
<thead>
<tr>
<th>Exact transform</th>
<th>Bilinear transform</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z = e^{j\omega T}$</td>
<td>$s = \frac{z - 1}{z + 1}, z = \frac{1 + s}{1 - s}$</td>
</tr>
<tr>
<td>$z \simeq 1 + j\omega T$ if $\omega T \ll 1$</td>
<td>$\Omega_{s-domain} = \tan \left( \frac{\omega T}{2} \right)$</td>
</tr>
</tbody>
</table>

7 Switched-capacitor circuits

7.1 Signal-Flow-Graph Analysis

![Diagram](image)

8 Data converters

Number of bits: $N$, number of levels: $L = 2^N$, quantization: $\Delta = \frac{V_{ref}}{L}$, RMS error: $\epsilon_{rms} = \Delta/\sqrt{2}$, oversampling rate: $OSR = \frac{f_s}{2f_0}$.

<table>
<thead>
<tr>
<th>Converter type</th>
<th>Signal to noise ratio $SQNR_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nyquist rate ($OSR = 1$)</td>
<td>$6.02N + 1.76$</td>
</tr>
<tr>
<td>Oversamp., no noise shaping</td>
<td>$6.02N + 1.76 + 10\log OSR$</td>
</tr>
<tr>
<td>Oversamp., 1\textsuperscript{st}-order noise shaping</td>
<td>$6.02N + 1.76 - 5.17 + 30\log OSR$</td>
</tr>
<tr>
<td>Oversamp., 2\textsuperscript{nd}-order noise shaping</td>
<td>$6.02N + 1.76 - 12.9 + 50\log OSR$</td>
</tr>
</tbody>
</table>

These formulae are valid (1) for an input sine wave (otherwise remove the $+1.76$ term), and (2) when the input signal spans the full range of the converter.

8.1 first-order $\Sigma\Delta$ modulator

![Diagram](image)

The state equations of the first-order $\Sigma\Delta$ modulator are given by:

- $y(n) = Q(x(n))$
- $e(n) = y(n) - x(n)$
- $x(n + 1) = x(n) + u(n) - y(n)$

9 Noise Analysis and Modeling

Spectral Density: $V_n^2(f) = \frac{[V^2/Hz]}{[V^2/Hz]}$.

Root Spectral Density: $V_n(f) = \frac{[V/\sqrt{Hz}]}{[V/\sqrt{Hz}]}$.

Total noise power: $V_n^2 = \int_0^\infty V_n^2(f) df$.
Sum of two noise sources

\[ V_n^2 = V_{n1}^2 + V_{n2}^2 + 2CV_{n1}V_{n2}, \]

\[ P_n = P_{n1} + P_{n2} + 2C\sqrt{(P_{n1}P_{n2})}. \]

White noise: \( V_n^2 \) (f) = \( k_w^2 \)

Pink (Flicker or 1/f) noise: \( V_n^2 \) (f) = \( k_f^2 f \)

Filtered noise: \( V_{n1}^2 \) (f) = \( |A(f)|^2 V_{n1}^2 \) (f)

Voltage noise across a resistor: \( V_R^2 \) (f) = 4\( kT R \)

Accumulated Voltage noise across a capacitor: \( V_C^2 = \frac{kT}{C} \)

Accumulated Current noise across an inductor: \( I_L^2 = \frac{kT}{L} \)

10 Miscellaneous

Matching accuracy for capacitors

We desire to match \( C_1 \) and \( C_2 \), such that \( K = \frac{C_2}{C_1} \geq 1 \).

Analysis gives the condition \( \frac{P_{n1}}{A_1^2} = \frac{P_{n2}}{A_2^2} \).

Therefore \( K = \frac{C_2}{C_1} = \frac{A_2}{A_1} = \frac{P_{n2}}{P_{n1}} \).

If \( C_1 \) is a square of size \( x_1 \times x_1 \), and \( C_2 \) has size \( x_2 \times y_2 \),
we have:

\[ y_2 = x_1 \left( K \pm \sqrt{(K^2 - K)} \right) \]
\[ x_2 = K \frac{x_1^2}{y_2} \]

Square resistance

\[ R = \rho = \frac{1}{\eta q \mu N_{p} H} \]
\[ R = R_{\square} \frac{L}{W} \]

Signal to noise ratio (SNR), decibels

\[ SNR = 10 \log \left( \frac{P_{\text{signal}}}{P_{\text{noise}}} \right) \quad [dB] \]

Conversion from power to dB: 10 log (P)

Conversion from power to dBm (dB mW): 10 log \( \left( \frac{P}{1\text{mW}} \right) \)

Conversion from voltage to dB: 20 log (V)

Conversion from voltage to dBm (dB mV): 20 log \( \left( \frac{V}{1\text{mV}} \right) \)

Steady state percentage value of first order filter versus time constant \( \tau \)

<table>
<thead>
<tr>
<th>Time</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau )</td>
<td>63%</td>
</tr>
<tr>
<td>2( \tau )</td>
<td>86%</td>
</tr>
<tr>
<td>3( \tau )</td>
<td>95%</td>
</tr>
<tr>
<td>4( \tau )</td>
<td>98%</td>
</tr>
<tr>
<td>5( \tau )</td>
<td>99%</td>
</tr>
</tbody>
</table>